0000 # SEMBLANCE-BASED ANISOTROPY-PARAMETER ESTIMATION IN LAYERED VTI MEDIA USING RATIONAL INTERPOLATION

H. DOUMA[1] and M. VAN DER BAAN[2]
[1] Center for Wave Phenomena and Department of Geophysics, Colorado School of Mines, Golden, USA.
[2] School of Earth and Environment, Earth Sciences, Univ. of Leeds, Leeds LS2 9JT, UK.

Summary

The τ-p domain is the natural domain for anisotropy-parameter estimation in horizontally layered media. However, the need to transform the data to the τ-p domain or to pick traveltimes in the t-x domain, is a practical disadvantage. To overcome this, we combine a τ-p domain inversion technique with rational interpolation of traveltimes in the t-x domain. This combination results in a highly accurate and efficient semblance-based method for anisotropy parameter-estimation from the moveout of P-waves in layered transversely isotropic (TI) media with a vertical symmetry axis (VTI).

Introduction

In the past two decades seismic processing has gradually developed to allow for estimation of anisotropy parameters from seismic data. In the presence of anisotropy, the moveout of P-waves in horizontally layered media as observed in common-midpoint (CMP) gathers, deviates from being hyperbolic. It is common practice to get (initial) estimates of the relevant anisotropic parameters by ascribing the nonhyperbolic character of the moveout to the presence of anisotropy. For P-waves in TI media these parameters are the anellipticity parameter η and the normal moveout velocity v_{nmo}. Early techniques based on Taylor-series approximations to the traveltimes include [HHM84] and [TsTh94]. However, these techniques lack accuracy at intermediate and long offsets.

The τ-p transform is the natural domain for anisotropy parameter estimation in layered media [Hake86, VaKe02], since the horizontal slowness is preserved upon propagation through such media. Since the τ-p transform is a plane-wave decomposition, the relevant velocity is the phase velocity instead of the group velocity. The latter velocity is mathematically more complex and often follows from approximations to already approximated phase velocities. Hence, anisotropy-parameter estimation in the τ-p domain renders the extra approximation for the group velocity obsolete, thus leading to more accurate estimates of the relevant parameters. The disadvantage of the τ-p domain inversion technique is that we either need to (1) transform the t-x gathers to the τ-p domain and pick the τ-p curves in this domain, or (2) pick the $t(x)$ moveout curves and transform these to the τ-p domain. Most interpreters will prefer the second option since they have more experience viewing data in the t-x domain. [VaKe02] thus recommended picking traveltimes in the t-x domain for practical purposes.

Semblance-based analyses are significantly faster and probably more robust than manual picking of traveltimes. This requires that traveltimes are computed at each recorded offset. In this paper we combine the τ-p domain inversion technique of [VaKe02] with the rational interpolation method of [DoCa06] to create a highly accurate semblance-based approach in the t-x domain for anisotropy-parameter estimation using P-waves in layered VTI media.

Nonhyperbolic moveout as a function of horizontal slowness

The slant stack relates traveltime t and offset x to intercept time τ and horizontal slowness p by

$$t = px + \tau. \qquad (1)$$

For P-wave reflections in horizontally layered VTI media the total intercept time τ is a linear combination of the interval zero-offset traveltimes $\Delta t_{0,i}$, the vertical P-wave velocity $v_{0,i}$, and the vertical slowness q_i in each layer i [Hake86, VaKe02]. That is,

$$\tau = \sum_i \Delta t_{0,i} v_{0,i} q_i. \tag{2}$$

[Alkh98] derived a reduced-parameter expression for the vertical P-wave slowness q_i expressed in terms of the interval NMO velocity $v_{nmo,i} = v_{0,i}[1+2\delta_i]^{1/2}$ and the interval anellipticity parameter η_i; see his equation A-10. [GrTs98] showed that the horizontal velocity $v_{hor,i} = v_{nmo,i}[1+2\eta_i]^{1/2}$ is better constrained than the anisotropy parameter η in a semblance analysis. Rewriting Alkhalifah's equation (A-10) in terms of $v_{nmo,i}$ and $v_{hor,i}$, and substituting the result in equation (2), gives

$$\tau = \sum_i \Delta t_{0,i} \left[\frac{1 - p^2 v_{hor,i}^2}{1 - p^2 (v_{hor,i}^2 - v_{nmo,i}^2)} \right]^{1/2}. \tag{3}$$

To allow semblance analysis to be done in the t-x domain, we need to know the traveltime for each recorded offset x. It follows from the definition of the τ-p transform (1) that $x = -\partial\tau/\partial p$. Therefore,

$$x = \sum_i \Delta t_{0,i} \left[\frac{p^2 v_{nmo,i}^2 / \{1 - p^2 v_{hor,i}^2\}^{1/2}}{\{1 - p^2 (v_{hor.i}^2 - v_{nmo,i}^2)\}^{3/2}} \right]. \tag{4}$$

Combining expressions (1), (3) and (4) results in the expression for the traveltime as a function of the horizontal slowness p, i.e.,

$$t = \sum_i \Delta t_{0,i} \left[\frac{p^2 v_{nmo,i}^2 / [1 - p^2 (v_{hor,i}^2 - v_{nmo,i}^2)] + 1 - p^2 v_{hor,i}^2}{\{(1 - p^2 v_{hor,i}^2)[1 - p^2 (v_{hor,i}^2 - v_{nmo,i}^2)]\}^{1/2}} \right]. \tag{5}$$

Equations (4) and (5) are explicit in p, whereas for a t-x based semblance analysis we require traveltimes for each offset acquired in the field. However, finding the p value that corresponds to a certain offset x in equation (4), corresponds to a simplified 2-point raytracing problem which can be solved using a straightforward bisection approach, since expressions (4) and (5) are single valued and monotonically increasing with increasing p-values. However, a more elegant and computationally less demanding procedure is available on by combining the τ-p inversion technique with the rational interpolation approach of [DoCa06].

Rational interpolation

A rational approximation to a function $t(x)$ is generally written as (see, e.g., [StBu93] p.58-63)

$$t(x) \approx \frac{N_L(x)}{D_M(x)}, \tag{6}$$

with N_L and D_M polynomials of degree L and M respectively. We denote such an approximation as [L/M]. The [L/M] rational interpolation is fully determined by $L+M+1$ support points $(t_j,\ x_j)$ satisfying $t_j = t(x_j) = N_L(x_j)/D_M(x_j)$. For a single horizontal VTI layer, [DoCa06] showed that a [2/2] rational interpolation yields highly accurate traveltime predictions for all offsets between the first and last support offsets. Here we extend their approach to horizontally layered VTI media.
Expressed as a Thiele continued fraction, the [2/2] rational interpolation for P-wave moveout becomes

$$\begin{aligned} t(x) \approx \frac{N_2(x)}{D_2(x)} = \quad & t_0 + \frac{x - x_0}{\rho(x_0,x_1) +} \frac{x - x_1}{\rho(x_0,x_1,x_2) - \rho(x_0) +} \frac{x - x_2}{\rho(x_0,\dots,x_3) - \rho(x_0,x_1) +} \\ & \frac{x - x_3}{\rho(x_0,\dots,x_4) - \rho(x_0,x_1,x_2)}, \end{aligned} \tag{7}$$

where we use the short-hand notation $\frac{a}{b+}\frac{c}{d} = \frac{a}{b+c/d}$. The reciprocal differences ρ are defined by

$$\rho(x_j) = t_j, \qquad \rho(x_j, x_k) = \frac{x_j - x_k}{t_j - t_k},$$
$$\rho(x_j, x_{i+1}, \ldots, x_{i+k}) = \frac{x_j - x_{i+k}}{\rho(x_j, \ldots, x_{i+k-1}) - \rho(x_{i+1}, \ldots, x_{i+k})} + \rho(x_{i+1}, \ldots, x_{i+k-1}). \tag{8}$$

Equation (7) with the reciprocal differences given in (8) can be used to find the traveltimes t at offsets x different from the support offsets x_j. The interpolation is highly efficient since only four support points need be calculated (t_0 is treated as a parameter with $x_0 = 0$). It is known that rational interpolation becomes degenerate in the case of common roots in the polynomials N_L and D_M (see [StBu93], p.61-62). Such degeneracy can be overcome by noise injection, i.e., small perturbation of the support traveltimes ($O[10^{-1}]$ ms or smaller) and offsets ($O[10^{-1}]$ m or smaller).

Parameter estimation by *t*-*x* semblance analysis

In order to perform semblance analysis in the *t*-*x* domain, we need to compute the traveltime for each offset acquired in the field for a specific combination of zero-offset time t_0, NMO velocity v_{nmo} and horizontal velocity v_{hor}. For a single horizontal VTI layer, [DoCa06] used 5 regularlyspaced offsets to act as the support points for the [2/2] rational interpolation. For the layered medium case, the traveltimes for such offsets can be found using a bisection approach applied to equation (4) to find the appropriate horizontal slownesses p_j, and subsequent use of the found p_j in equation (5) to calculatethe associated traveltime. Alternatively, a simple 'ray-shooting' method can be used to compute offsets for a range of p-values and extract the offsets (and the associated traveltimes) closest to the desired ones, to act as the support points. The thus obtained values of x_j and t_j ($j = 0, 1, \ldots, 4$) constitute the support points for [2/2] rational interpolation [equation (7)], which allows the calculation of the traveltime as a function of offset. The resulting moveout curve can be used for semblance analysis or to flatten a CMP gather.
Since the support points are expressed in terms of the interval velocities $v_{hor,i}$ and $v_{nmo,i}$, in principle a semblance based global inversion can be set up to determine the values of $v_{hor,i}$ and $v_{nmo,i}$ for all layers at the same time, or by applying the inversion in a layer-stripping fashion. However, in practice such approachesare not generally desirable, due to the trade-off between $v_{hor,i}$ and $v_{nmo,i}$ for limited offset acquisition geometries. Alternatively, once the layer of interest is determined, the overburden can be modeled as a single layer with certain effective values of v_{hor} and v_{nmo}. Once these values are established, the values of $v_{hor,i}$ and $v_{nmo,i}$ can be determined by finding the maximum semblance value over a range of values of $v_{hor,i}$ and $v_{nmo,i}$, while keeping the effective values of v_{hor} and v_{nmo} of the overburden fixed.

Numerical example

The same 4-layer model as in [DoCa06] is used to demonstrate the accuracy of the inversion approach to estimate interval parameters. The anellipticity increases with depth from none to extreme (see annotated values of η in Fig.1), and the maximum offset-to-depth ratio is 4 for all events. Exact traveltimes are computed by means of raytracing. Fig 1a displays the computed CMP gather and shows the true values of η, v_{hor} and v_{nmo}.
Fig 1c-f show the semblance scans for the first through the fourth event as a function of the interval values $v_{hor,i}$ and $v_{nmo,i}$. For the purpose of this numerical example, the semblance scans are obtained with a layer-stripping approach since the trade-off relations between $v_{hor,i}$ and $v_{nmo,i}$ were only small due to the relatively large employed offset-to-depth ratio of 4. The white triangle indicates the location of maximum semblance and the grey circle shows the values of the true velocities. Note the excellent agreement between true and retrieved values. The accuracy of the inversion technique, and thus the obtained moveout curves, is also confirmed by the high semblance maxima s_{max} (i.e., close to one). Fig 1b shows the traveltime curves predicted by the proposed approach from the recovered interval horizontal and NMO velocities superposed on the true moveout curves. Discrepancies are negligible demonstrating again the high accuracy of the proposed method. The black dots indicate the support points used in the rational interpolation.

Conclusions

The τ-p domain is the natural domain for anisotropy-parameter estimation in horizontally layered media. However, quality control of processing results and interpretation of seismic data is preferably done in the t-x domain. We combine the τ-p domain inversion technique of [VaKe02] with the rational interpolation method of [DoCa06] to allow a highly accurate semblance-based interval parameter inversion for layered VTI media to be done in the t-x domain. This method overcomes the need to pick traveltimes in the t-x domain or processing of the data in the τ-p domain. The efficiency of the method stems from the fact that only few support points need be calculated to achieve highly accurate moveout curves with rational interpolation.

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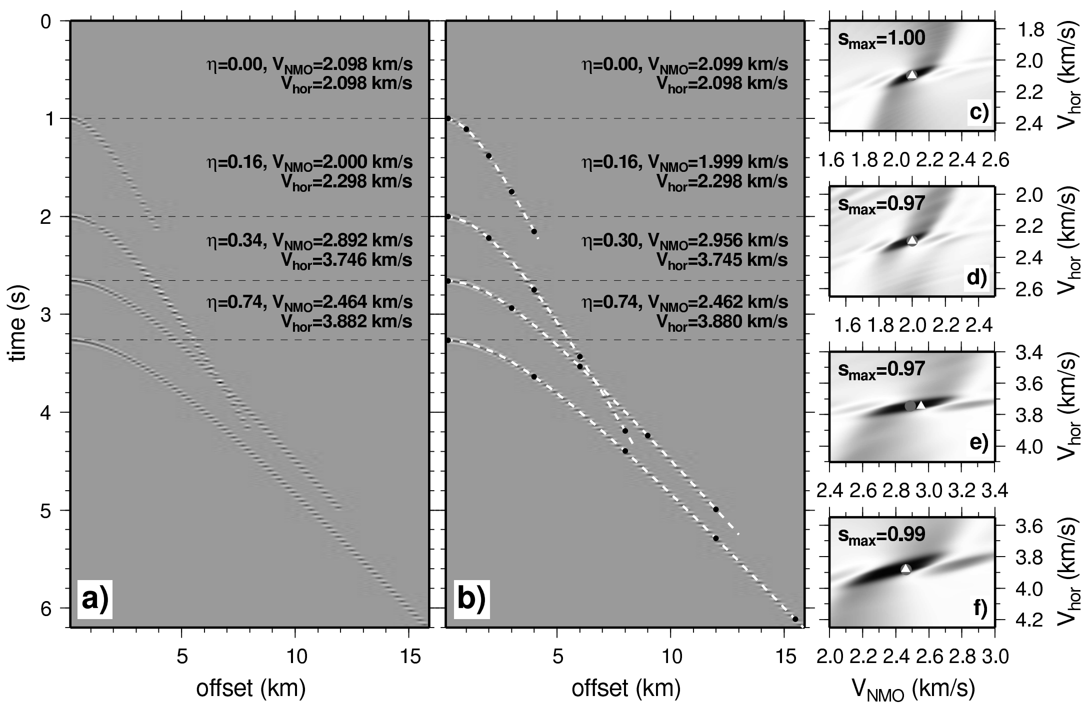

Fig. 1. Interval semblance analysis for VTI anisotropy estimation. (a) CMP gather in a strongly anisotropic 4-layer medium. (b) Traveltimes calculated from the estimated interval velocities are overlain on the true moveout curves. Black dots indicate the support points used for the rational interpolation. (c)-(f) Semblance scans for the *interval* horizontal and NMO velocities for layers 1 to 4, respectively. Grey circle: true values; white triangle: position of maximum semblance. Both the high semblance values s_{max} and the closely overlapping true and obtained interval parameters demonstrate the accuracy of the presented approach.