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Comment on 'On sensitivity kernels for 'wave-equation' transmission tomography' by de Hoop and van der Hilst

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SUMMARY

The finite-frequency sensitivity kernels developed by Dahlen *et al.* are an appropriate tool for inverting traveltimes of non-triplicated *P*, *S*, *PP* and *SS* waves, measured by cross-correlation with a synthetic pulse. The critique of this theoretical methodology in a recent paper by de Hoop & van der Hilst is based upon the incorrect notion that one can account for errors in the synthetic pulse and/or origin time of an earthquake by a modification of the Fréchet kernels expressing the first-order dependence upon the perturbation in the wavespeed.

Key words: body waves, Fréchet derivatives, global seismology, source time functions, tomography, traveltime.

GJI Seismology

1 INTRODUCTION

The recent paper on finite-frequency traveltime sensitivity kernels by de Hoop & van der Hilst (2005)—hereafter referred to as H&H—contains several statements which we consider to be erroneous or misleading, which may give the impression that the Fréchet kernels developed by Dahlen *et al.* (2000)—hereafter referred to as DHN—are inadequate or incorrect. The purpose of this comment is to correct these misstatements, and to reiterate that the DHN kernels properly account for diffractive wavefront healing effects that are ignored in linearized ray theory, and enable improved imaging of adequately sampled, small-scale, mantle wavespeed anomalies, as intended.

2 GENERAL COMMENTS

Many of the doubts that H&H seek to sow are concerned with the method of measuring finite-frequency traveltimes by finding the maximum of the cross-correlation with an unperturbed synthetic pulse. They contend that 'whereas the cross-correlation criterion may work very well for propagating delta pulses . . . it can break down when more general wave fields are considered.' In fact, cross-correlation is an optimal matched filter for any synthetic pulse in the presence of noise (e.g. Robinson & Treitel 1980, chapter 14), and, since the advent of digital seismology, many global tomographers have turned to cross-correlation traveltime measurements for this and a variety of other practical reasons (e.g. Su & Dziewonski 1992; Grand 1994; Masters *et al.* 1996; Ritsema & van Heijst 2002). Hung *et al.* (2000, 2001) and Baig *et al.* (2003) have conducted extensive numerical validations of the DHN kernels, by comparison with synthetic 'ground-truth' cross-correlation traveltime measurements, for both isolated spherical wavespeed anomalies and random 3-D heterogeneous media.

H&H also express concern that 'the maximum of the cross-correlation will not match the wavefront arrival.' Obviously, finite-frequency, cross-correlation and infinite-frequency, ray-theoretical traveltimes will differ; indeed, this is what originally motivated development of the DHN sensitivity kernels, which are expressly tailored to measurements made using cross-correlation of broadband or bandpass-filtered pulses. H&H evidently feel that the term 'traveltime' should be used exclusively to refer to the arrival time of a singular wavefront; DHN use the term more loosely, in accordance with common seismological parlance.

In their section 4, after recasting the main result of DHN in the mathematical language of distribution theory, H&H make the remarkable claim that 'the 'wave-equation' tomography and traveltime tomography share the same sensitivity kernel.' In fact, an infinite-frequency, ray-theoretical kernel is a delta function confined to the geometrical ray, whereas a finite-frequency traveltime exhibits off-ray sensitivity, with zero sensitivity along a non-triplicated, turning *P* or *S* ray. To call these two kernels the 'same' because one can be regarded as a 'mollified' version of the other is, in our opinion, misleading if not plainly wrong. Real seismic waveforms are fundamentally different from 'propagating delta pulses' and have a bandlimited frequency content imposed by the finite spatial dimensions and duration of the source, attenuation, and limitations in instrumental sensitivity. One of the few statements of H&H with which we wholeheartedly agree is that finite-frequency sensitivity kernels 'allow internally consistent 'data fusion', that is, the integration and joint interpretation of data measured at different frequencies.'

Much of the discussion in H&H seems intended to cause an uncritical reader to question the existence of a ‘doughnut hole’ in traveltime sensitivity along a turning *P* or *S* ray. In their introduction, they assert: ‘We will show, however, that the kernel itself does not have a zero on the unperturbed ray.’ In their concluding discussion, they attempt to clarify this with the statement that ‘. . . a banana-doughnut structure appears in the regularization of a sensitivity kernel as the result of a *pointwise* evaluation of its values. However, an evaluation of this kernel in the sense of generalized functions leads to a structure without the ‘hole’.’ It is true that a DHN finite-frequency kernel can be regarded as a distribution, or continuous linear functional acting upon a space of test functions; however, in that context, it is a regular (non-singular) distribution, i.e. simply a smooth ordinary function, and pointwise evaluation of such a regular convolution kernel is both appropriate and commonplace. The characterization of a finite-frequency kernel as nothing more than a ‘regularization’ of an underlying, ray-theoretical kernel that is singular on the unperturbed ray seems to us to be a perverse point of view.

A few sentences later in their discussion, H&H assert that ‘there are obstructions to the formation of the ‘hole’ in the kernel’ because ‘caustics form generically in a heterogeneous medium.’ In fact, the proliferation of caustics in a strongly heterogeneous medium is irrelevant in any linearized global tomographic study that seeks to determine 3-D wavespeed perturbations with respect to an unperturbed, spherically symmetric earth model. The well-known *PP* and *SS* caustics that arise in such a spherical model were fully accounted for by DHN; the associated, extensive, saddle-shaped sensitivity can be exploited to improve upper-mantle coverage in traveltime inversions. H&H go on to say that ‘one must be careful with the use of approximate finite-frequency kernels to linearize tomographic inversions, because the kernels calculated in the starting model (usually simple, with a ‘banana-doughnut’ feature) may well differ significantly from the sensitivity kernels implied by the heterogeneous model produced by the inversion.’ That is of course true—although, given the current generation of mantle models, we would not expect the updated kernels to differ ‘significantly’—but it is a well-known feature of any non-linear, iterative inversion. Proper treatment of non-linearities and full utilization of all the information contained in recorded waveforms are the long-term research goals of all seismic tomographers, and the multi-resolution analysis of finite-frequency sensitivity kernels in section 5 of H&H may be a step in this direction. DHN had a more modest goal: to improve the theoretical basis of present-day global inversions that are linearized with respect to a spherically symmetric starting model, by accounting for finite-frequency diffraction effects upon measured, long-period, cross-correlation traveltimes.

3 SOURCE TIME ERRORS

In their section 4, H&H present an analysis of the effect of uncertainty in what they call the ‘source signature’ (which we take to mean the shape of the unperturbed far-field pulse). We conclude this comment with our own analysis of this uncertainty in the source, using a simple perturbation-theoretic approach, which we believe illuminates the salient issues. Let $u(t)$ be the true pulse shape in the unperturbed earth model, with wavespeed c, and let $\delta u_1(t) + \delta u_2(t) + \cdots$ be the perturbation in this pulse due to a perturbation in the model, $c \to c + \delta c$. The subscripts 1, 2, . . . denote the order of the perturbation, so that $\delta u_1(t)$ depends linearly upon the wavespeed perturbation δc, whereas $\delta u_2(t)$ depends quadratically upon δc, etc. The observed pulse will be denoted by $o(t) = u(t) + \delta u_1(t) + \delta u_2(t) + \cdots$. Suppose that one seeks to measure a finite-frequency traveltime residual δT by cross-correlation of $o(t)$ with a synthetic pulse $s(t) = u(t) + \varepsilon(t)$, where $\varepsilon(t)$ is an error in the unperturbed pulse $u(t)$ due to imprecise knowledge of the source. The criterion defining δT in this scenario is

$$\int s(t - \delta T)\, o(t)\, dt = \text{maximum}, \tag{1}$$

where the integration is carried out over the time interval in which $o(t)$ is non-zero. Assuming that the waveform perturbation $\delta u_1(t)$ and the error $\varepsilon(t)$ are of the same order, and using eq. (1) to expand δT in a regular perturbation series, we find

$$\begin{aligned}\delta T = {} & \overbrace{\frac{\int \dot u\,\delta u_1\,dt}{\int \ddot u u\,dt}}^{\text{DHN term}} + \overbrace{\frac{\int \dot\varepsilon u\,dt}{\int \ddot u u\,dt}}^{\text{origin-time error}} + \overbrace{\frac{\int \dot u\,\delta u_2\,dt}{\int \ddot u u\,dt} - \frac{\left(\int \dot u\,\delta u_1\,dt\right)\left(\int \ddot u\,\delta u_1\,dt\right)}{\left(\int \ddot u u\,dt\right)^2}}^{\text{terms of order } \delta c^2} + \overbrace{\frac{\int \dot\varepsilon\,\delta u_1\,dt}{\int \ddot u u\,dt}}^{\text{H H term, order } \varepsilon\,\delta c} \\ & \underbrace{- \frac{\left(\int \dot\varepsilon u\,dt\right)\left(\int \ddot u\,\delta u_1\,dt\right)}{\left(\int \ddot u u\,dt\right)^2} - \frac{\left(\int \ddot\varepsilon u\,dt\right)\left(\int \dot u\,\delta u_1\,dt\right)}{\left(\int \ddot u u\,dt\right)^2}}_{\text{additional terms of order } \varepsilon\,\delta c} - \underbrace{\frac{\left(\int \dot\varepsilon u\,dt\right)\left(\int \ddot\varepsilon u\,dt\right)}{\left(\int \ddot u u\,dt\right)^2}}_{\text{order } \varepsilon^2} + \cdots,\end{aligned} \tag{2}$$

where the ellipsis denotes terms of third order, and a dot denotes differentiation with respect to time t. Endpoint terms that arise in the integrations by parts needed to derive this result vanish by virtue of the assumption that the observed pulse $o(t)$ is completely contained within the cross-correlation interval. The first term in eq. (2) is identical to eq. (66) of DHN and eq. (12) of H&H. The second term, which we shall call $-\delta t_1$, is minus the first-order error in the origin time of the earthquake, due to the error $\varepsilon(t)$ in the assumed source pulse $s(t)$. This first-order contribution to the traveltime residual δT is independent of the wavespeed perturbation, and such origin-time errors are routinely taken into account in both ray-theoretical and finite-frequency traveltime tomography, by simultaneous inversion for shifts in the hypocentral parameters in addition to the wavespeed perturbations δc. The remaining six terms are all of second order, depending upon either δc^2, $\varepsilon\delta c$ or ε^2. The third and fourth terms, which are independent of ε, have been analyzed in a simplified random-medium context by Baig & Dahlen (2004), and used to place a weak constraint upon the small-scale granularity of the earth's mantle.

In their eqs (14)–(16), H&H ignore the first-order origin-time error $-\delta t_1$ and all of the second-order terms except the fifth, labelled H&H, which is of order $\varepsilon\delta c$. Rather than basing a body-wave traveltime inversion upon the systematically linearized relation

$$\delta T = \frac{\int \dot{u}\,\delta u_1\,dt}{\int \ddot{u}u\,dt} + \frac{\int \dot{\varepsilon}u\,dt}{\int \ddot{u}u\,dt} = \int_{\oplus} K\,\delta c\,dV - \delta t_1, \tag{3}$$

they contend that origin-time and other source-related errors should be accommodated in inversion using the (DHN first-order plus H&H second-order) 'asymmetric mollifier' relation

$$\delta T = \frac{\int \dot{u}\,\delta u_1\,dt}{\int \ddot{u}u\,dt} + \frac{\int \dot{\varepsilon}\,\delta u_1\,dt}{\int \ddot{u}u\,dt} = \frac{\int \dot{s}\,\delta u_1\,dt}{\int \ddot{u}u\,dt} = \int_{\oplus} K_\varepsilon\,\delta c\,dV. \tag{4}$$

In eqs (3) and (4), spatial integration is carried out over the 3-D earth $\oplus$, and K is the sensitivity kernel of DHN. H&H mysteriously associate their 'data' and 'modelled' source signatures W_d and W_m with our $s(t)$ and $\delta u_1(t)$ rather than vice-versa, as we do, but that distinction does not alter the equivalence of eq. (4) and their eqs (14)–(16). It is true, as they note, that the error-dependent ($W_d \neq W_m$) kernel K_ε will differ from K, and that in general the modified sensitivity will not vanish along a turning P or S ray; however, there are a number of practical reasons for strongly preferring eq. (3) to eq. (4). First, as we have shown above, eq. (4) is inconsistent, inasmuch as it ignores the dominant, order ε, origin-time error term, $-\delta t_1$, and retains only one of several terms of order $\varepsilon\,\delta c$ or ε^2. Secondly, simultaneous inversion for $-\delta t_1$ as well as for δc will eliminate large time-shift errors between $s(t)$ and $u(t)$, such as the one illustrated in the curiously upside-down fig. 4(b) of H&H. Thirdly, since there is no reason to expect any bias in the error $\varepsilon(t)$, the expected value of the H&H term in eq. (4) will be zero, so that $\langle K_\varepsilon\rangle = K$, correct to first order in ε. Fourthly, and most importantly, computation of K_ε depends upon knowledge of $\varepsilon(t)$, and if one actually possesses such knowledge, then it is obviously far simpler to eliminate the pulse-shape error at the outset, by altering the unperturbed pulse shape, $u(t) \to u(t) + \varepsilon(t)$.

In practice, errors in cross-correlation traveltime measurements are small provided that adequate experimental precautions are taken. In the measurement methodology of Masters *et al.* (1996) the same bandpass filter is applied to both the observed and synthetic seismograms prior to cross-correlation, and filtered waveforms $o(t)$ that do not closely resemble $s(t - \delta T)$ are stringently rejected from the data set. Differential *PP-P* or *SS-S* traveltimes are commonly measured by cross-correlation of two observed pulses $o(t)$ in the same seismogram (e.g. Kuo *et al.* 1987; Woodward & Masters 1991), so that errors in a synthetic $s(t)$ are not an issue. Cross-correlations done over local arrays (e.g. VanDecar & Crosson 1990) generally deal with waveforms that are very similar, and clustering techniques (e.g. Rowe *et al.* 2002) can be used to ensure that phase mismatches are small even in global, multi-station correlation analyses. In summary, we categorically reject the assertion of H&H that the sensitivity kernels of DHN are incorrect or inadequate because they do not properly account for uncertainties in the unperturbed source pulse.

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