

**Advances in Novel Numerical Methodologies for the Geosciences:  
An Avenue for Developing Collaboration Environments Towards  
Scientific Discovery**

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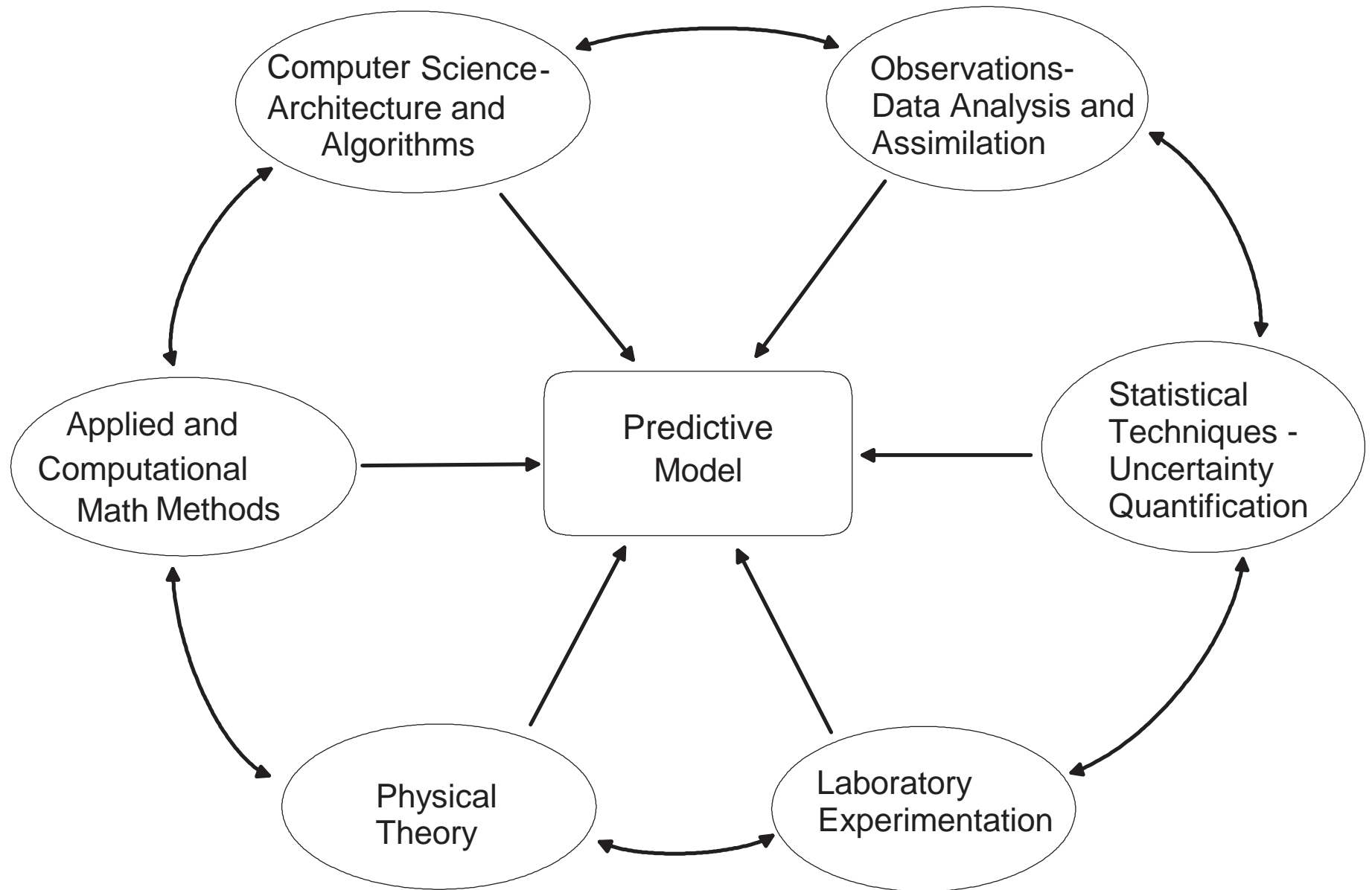
## Why We Are Here?

To explore venues on developing and funding a national framework for collaboration in mathematics and geosciences research

General questions to be addressed:

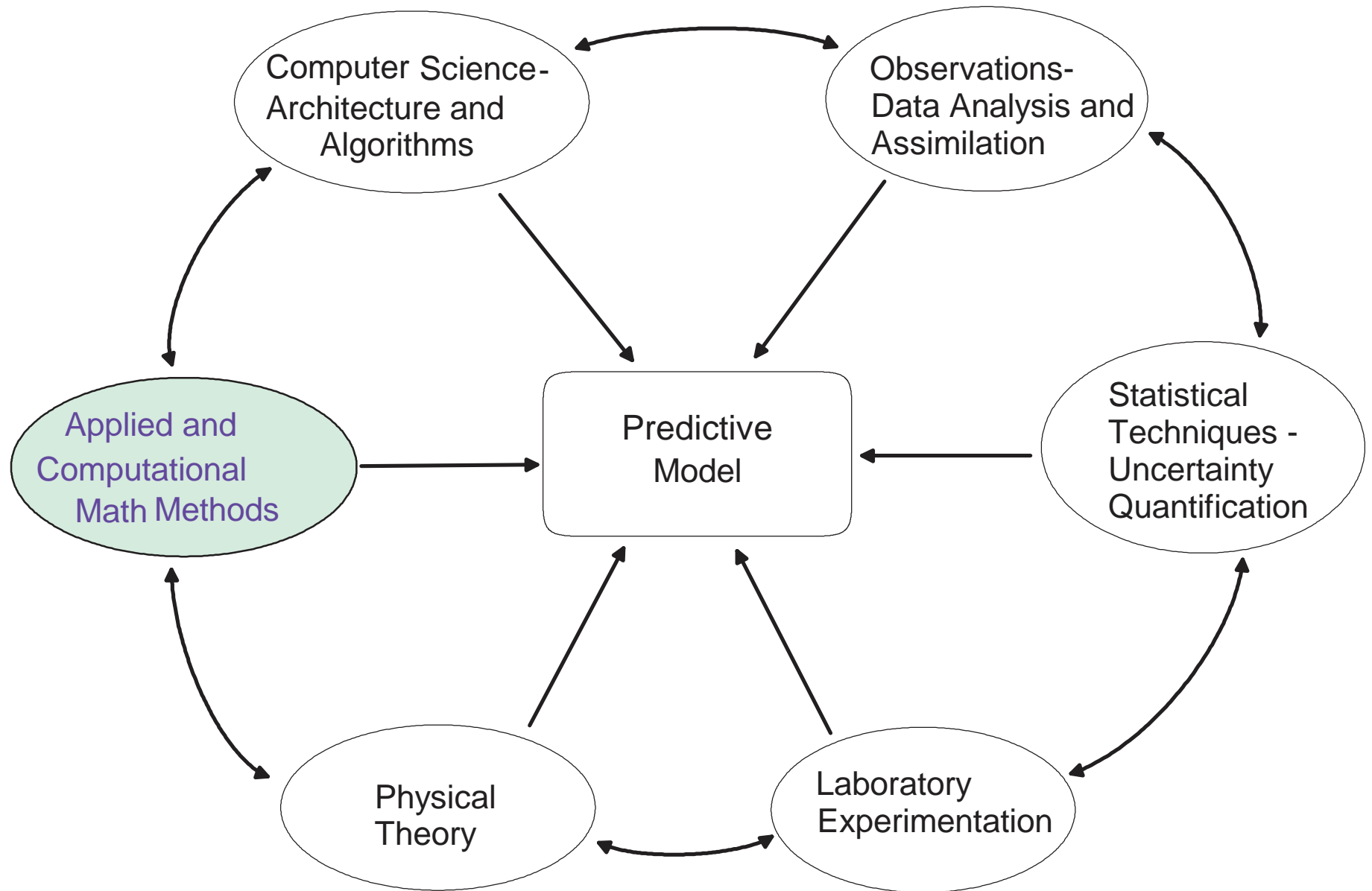
- What form would this national framework take?
- How would it transform the frontier of math-geoscience collaboration?
- How does our individual research interests fit into this national framework?
- Given existing collaborative solicitations, what avenues are we to pursue?
- As individuals, how much effort and time are we willing to contribute to making this initiative happen?

# The Predictive Model: Fostering Synergistic Activity between Disciplines



Funding allowed scientists to cross disciplinary barriers, stimulating novel collaboration modes

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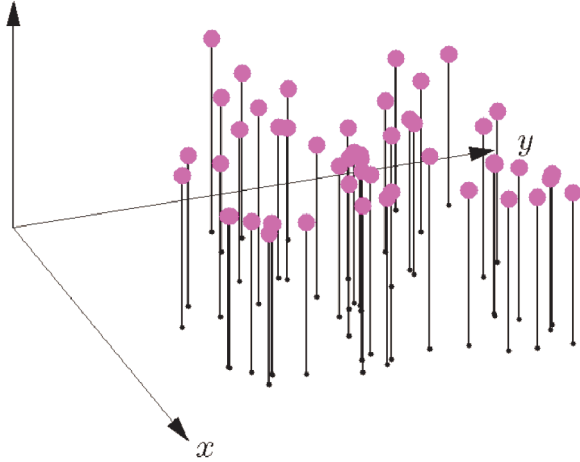


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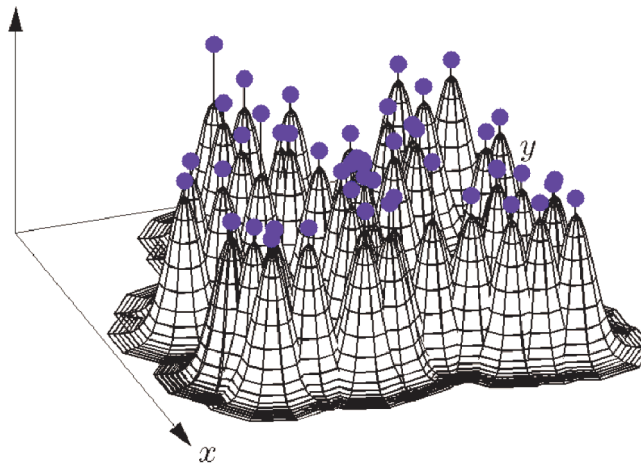
Focus of following discussion:

Describe a novel numerical method and how national funding for that has lead to math-geo collaborations that otherwise might not have taken place

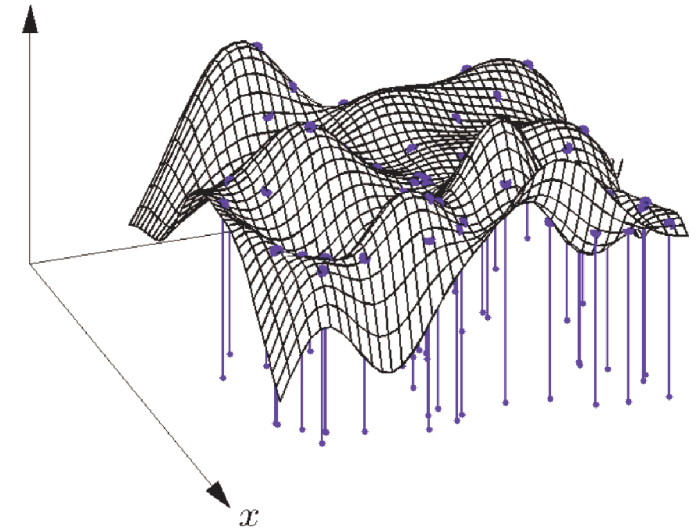
# RBF: An example of novel numerics stimulating new paths of collaboration



Scattered data within a 2-D region



Radial basis functions (RBF) here, 'rotated Gaussians'



Linear combination of the basis functions that fits all the data

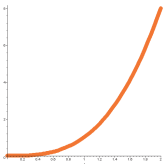
## Many types of RBFs are available

Piecewise smooth (Algebraic accuracy)

Infinitely smooth (Spectral accuracy)

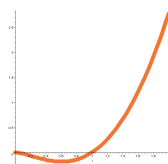
**Cubics**

$$r^3$$



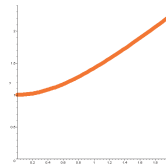
**TP splines**

$$r^2 \log r$$



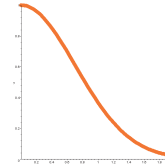
**Multiquadric**

$$\sqrt{1 + (\epsilon r)^2}$$



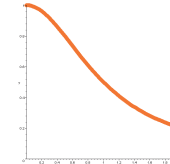
**Gaussian**

$$e^{-(\epsilon r)^2}$$



**Inverse quadratic**

$$\frac{1}{1 + (\epsilon r)^2}$$

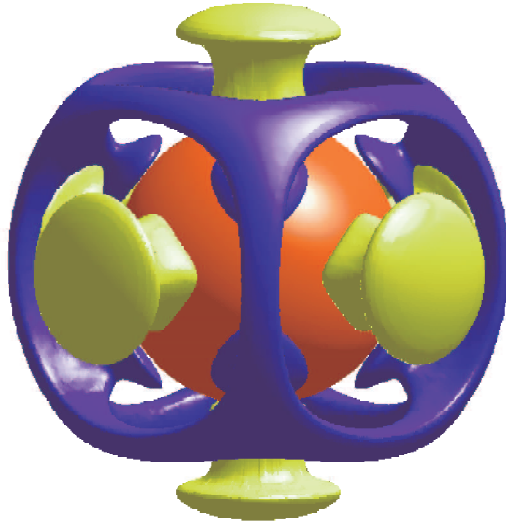


$$r = \sqrt{(x - x_k)^2 + (y - y_k)^2}, \quad (x_k, y_k) \text{ is where the RBF is centered}$$

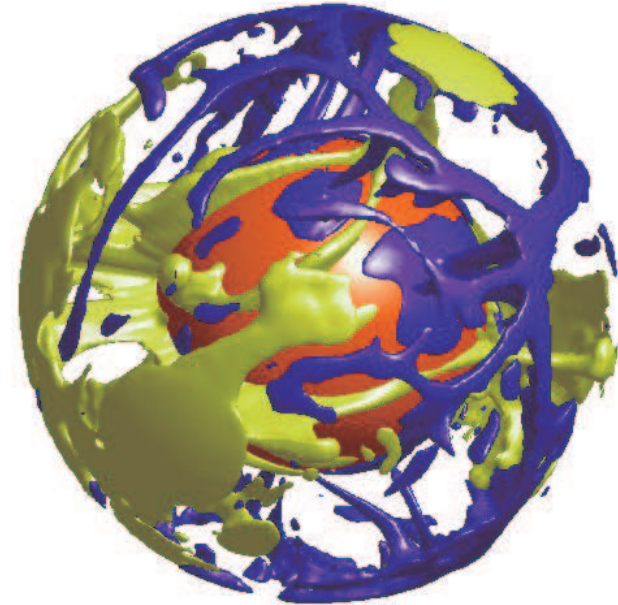
# First round of CMG funding got us to a stage of RBF development to interact with the Geosciences

## Ex. Solid Earth

Ra = 7000



Ra = 1,000,000



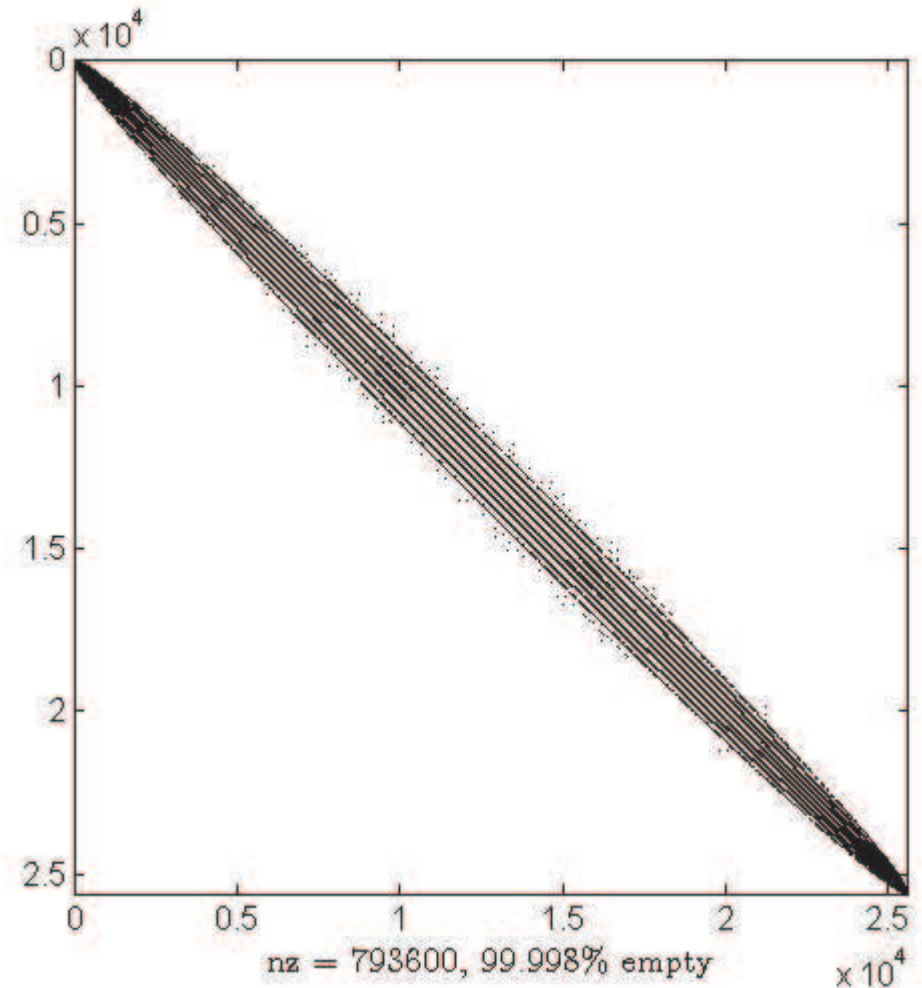
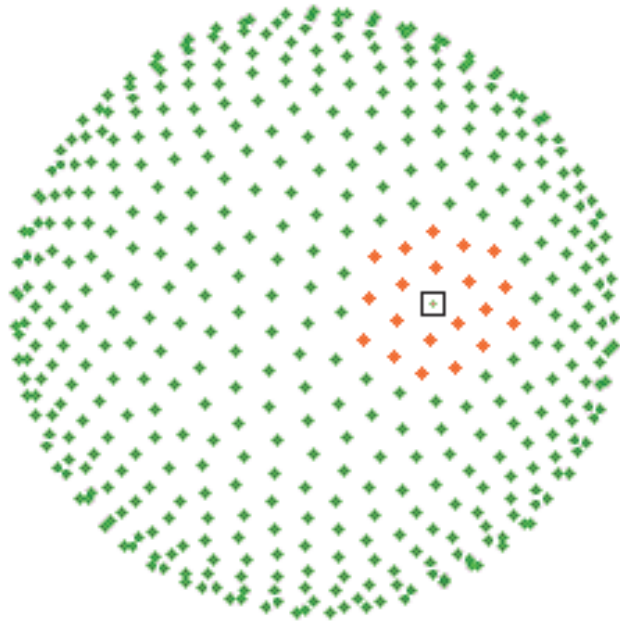
### Benchmark at Ra = 7000

Method	No of nodes	$Nu_{outer}$	$Nu_{inner}$	$\langle V_{RMS} \rangle$	$\langle T \rangle$
Finite volume	663,552	3.5983	3.5984	31.0226	0.21594
Finite elements (CitcomS)	393,216	3.6254	3.6016	31.09	0.2176
Finite differences (Japan Earth Simulator)	12,582,912	3.6083		31.0741	0.21639
Spherical harmonics -FD	552,960	3.6086		31.0765	0.21582
Spherical harmonics -FD	Extrapolated	3.6096		31.0821	0.21577
RBF-Chebyshev	36,800	3.6096	3.6096	31.0820	0.21578

Second round of CMG funding addressed scalability expanding collaborations to government agencies, industry, and academia

## RBF-generated Finite Differences Stencils (RBF-FD)

$N = 25K$ ,  $n = 31$ , 99.998% Empty, 5th-6th Order



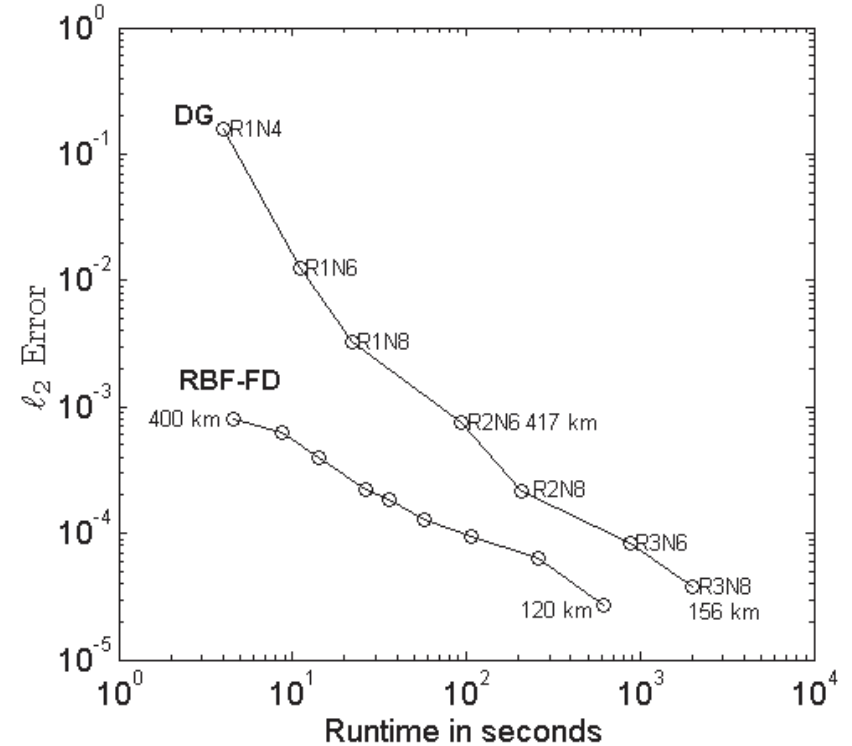
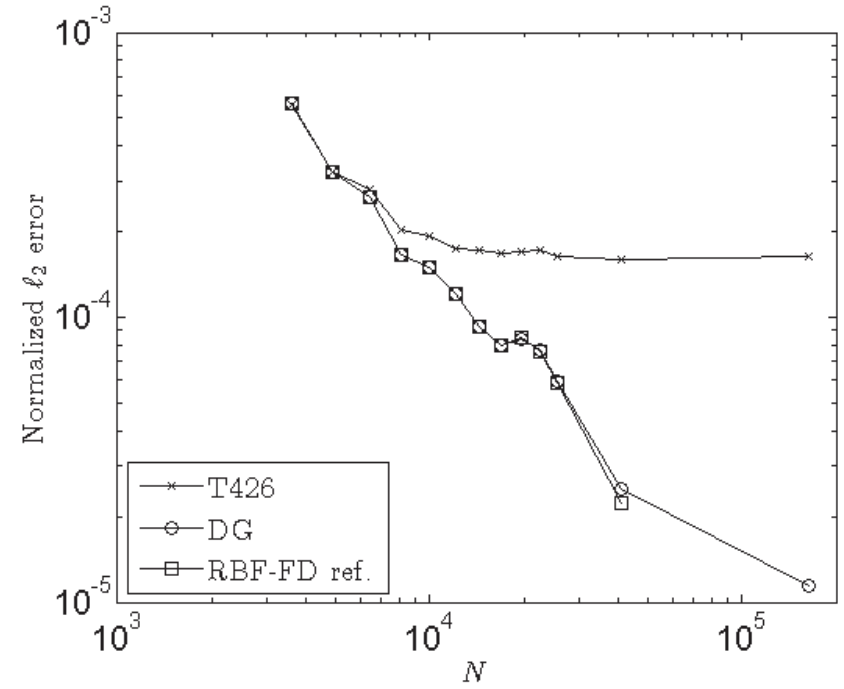
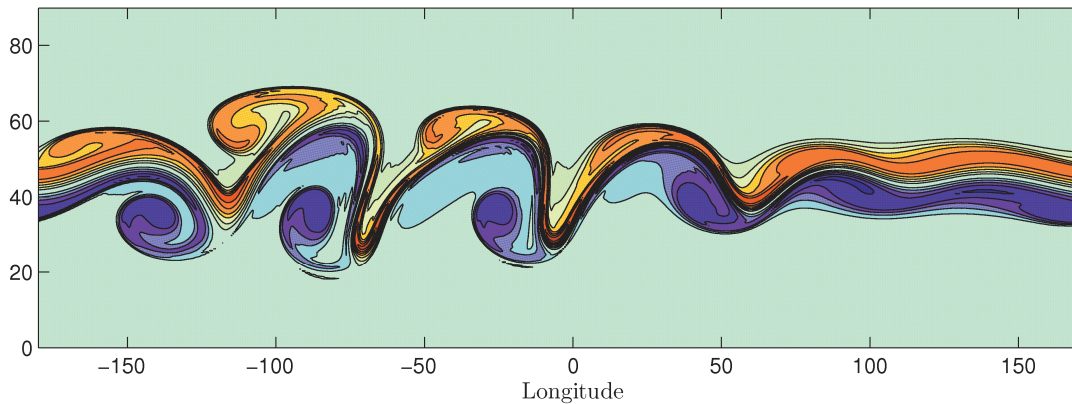
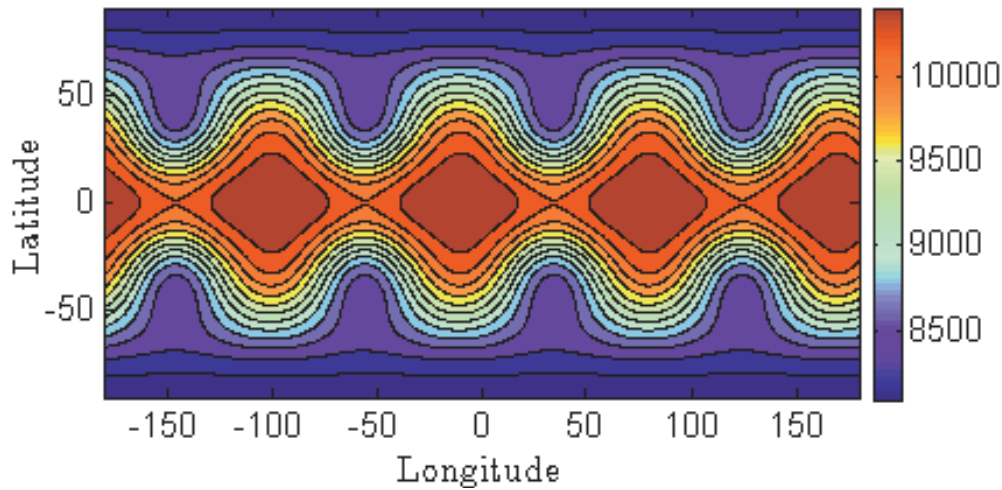
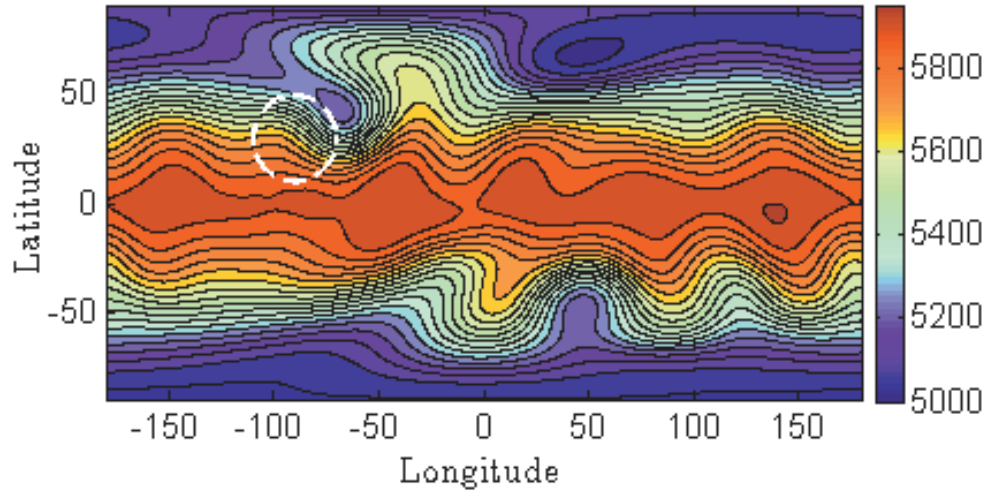
Graphical display of RBF-FD stencil

$\frac{\partial}{\partial \theta}$  is approximated at square

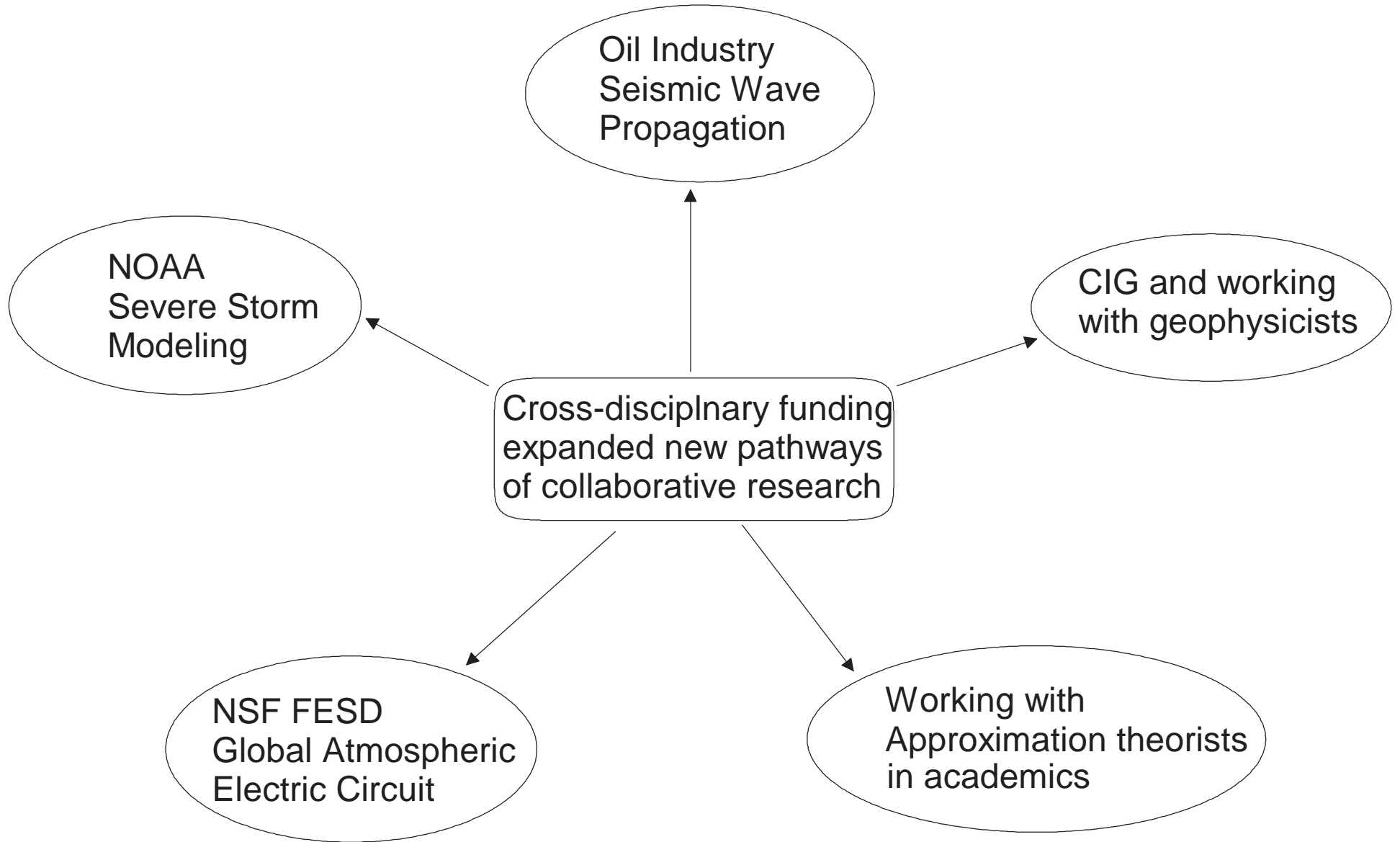
Each row is a stencil to approx.  $\frac{\partial}{\partial \theta}$  at a node



# RBF-FD for the Atmospheric Sciences: Important Benchmarks



# RBF-FD: Extensions into new collaborative environments



Question?

How and by what means are we to build new and sustain such collaborative and interdisciplinary connections ?