Physical causes and modeling challenges of anomalous diffusion of sediment tracers

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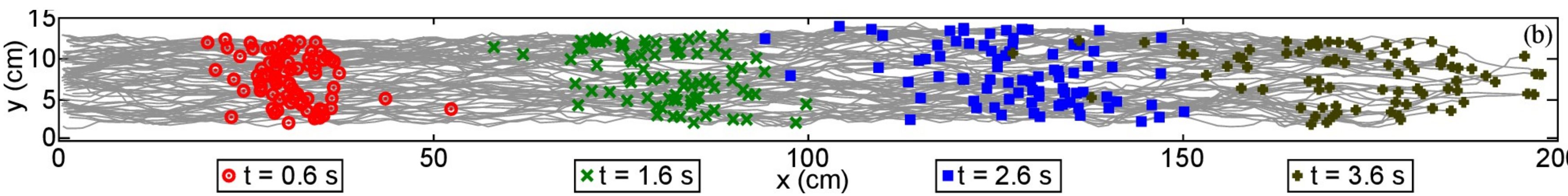
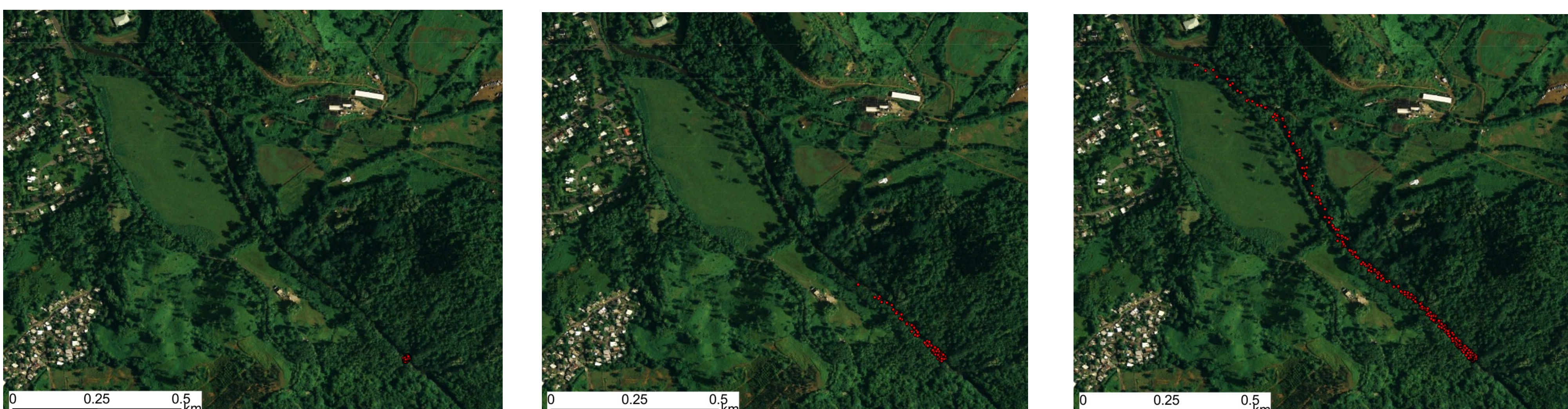
"Bridging the Gap", Princeton U., 2 Oct. 2012



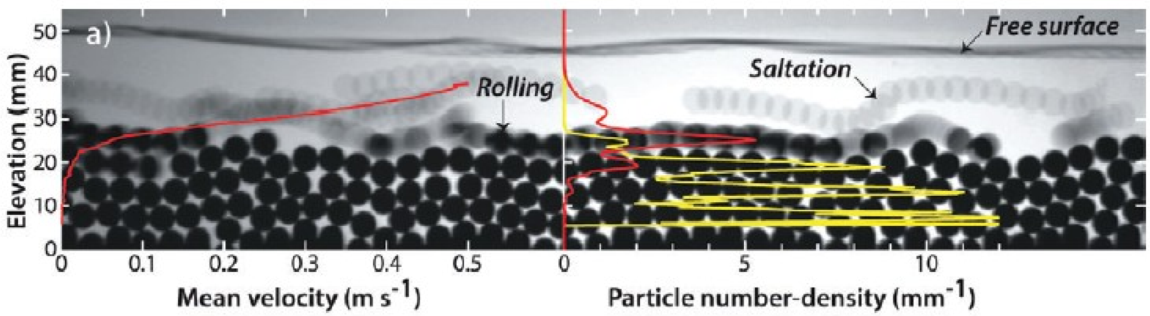


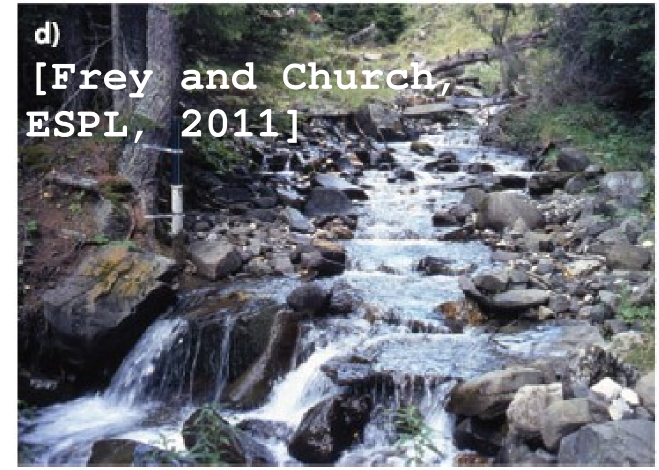
"Bed load" transport: multi-phase flow problem

Grains supported by and in frequent contact with the bed.
A well defined "layer" describable by:

1. Particle volume, δv [L^3].
2. Average velocity, u_s, of bed load sediment [L/T].
3. Surface density, n, of moving particles [#/L^2].

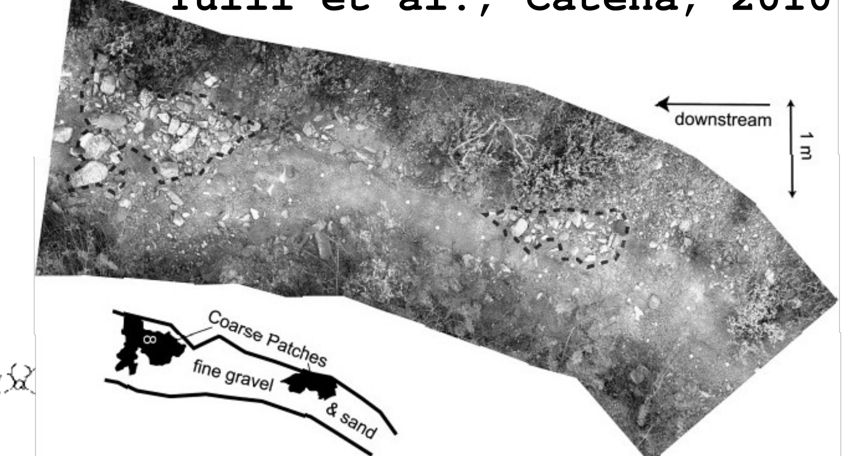
$$q_s = \delta v \;\; n \;\; u_s$$

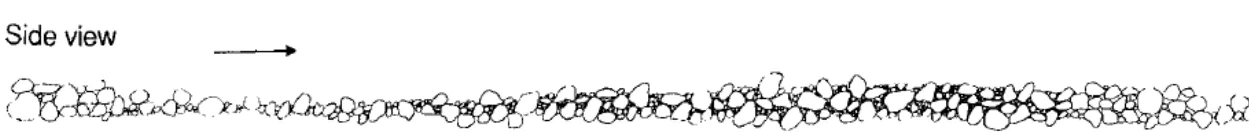




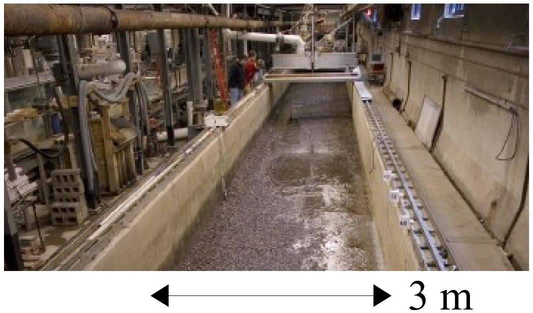
Grains clump together – bed forms
→ "Force chains" form and break
→ Slow, stochastic shearing of granular bed
→ Spatial sorting and clusters

Yuill et al., Catena, 2010

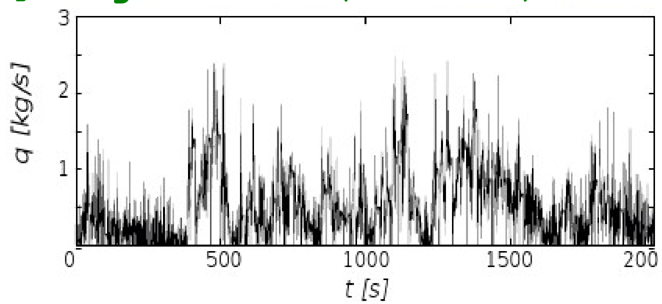




Near threshold transport: "avalanching" and spatially heterogeneous dynamics(?)

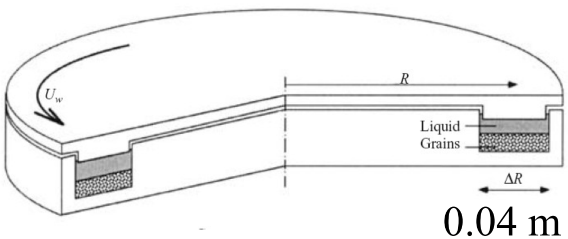


[Singh et al.,JGR-ES, 2009]

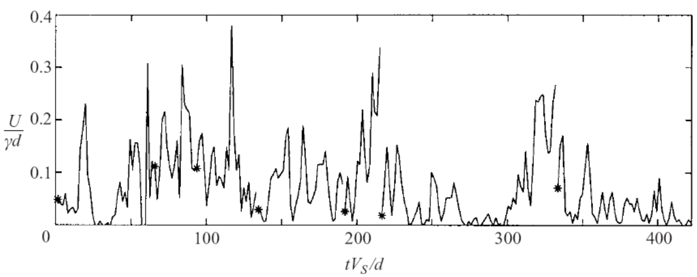


Turbulence, granular collisions, grain size dispersion.

Intermittent transport under steady flow.



[Charru et al., JFM, 2004]



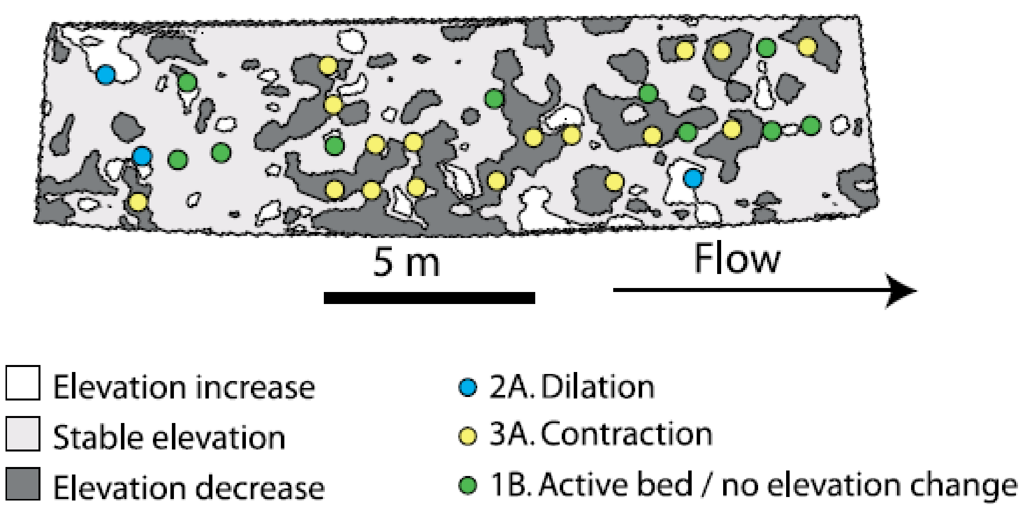
Laminar flow, uniform beads.

Intermittent transport → collective grain motion.

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 117, F01014, doi:10.1029/2011JF002120, 2012

Using multiple bed load measurements: Toward the identification of bed dilation and contraction in gravel-bed rivers

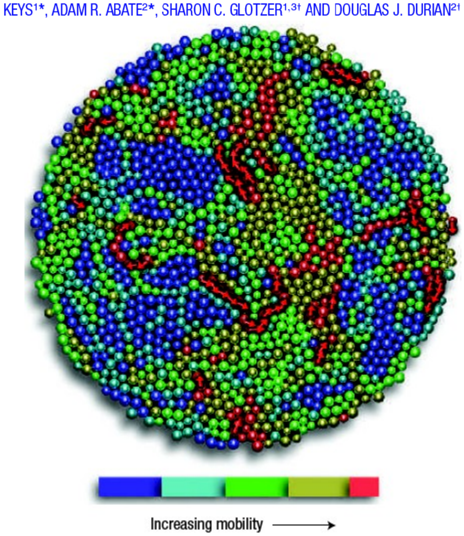
G. A. Marquis[1] and A. G. Roy[1]



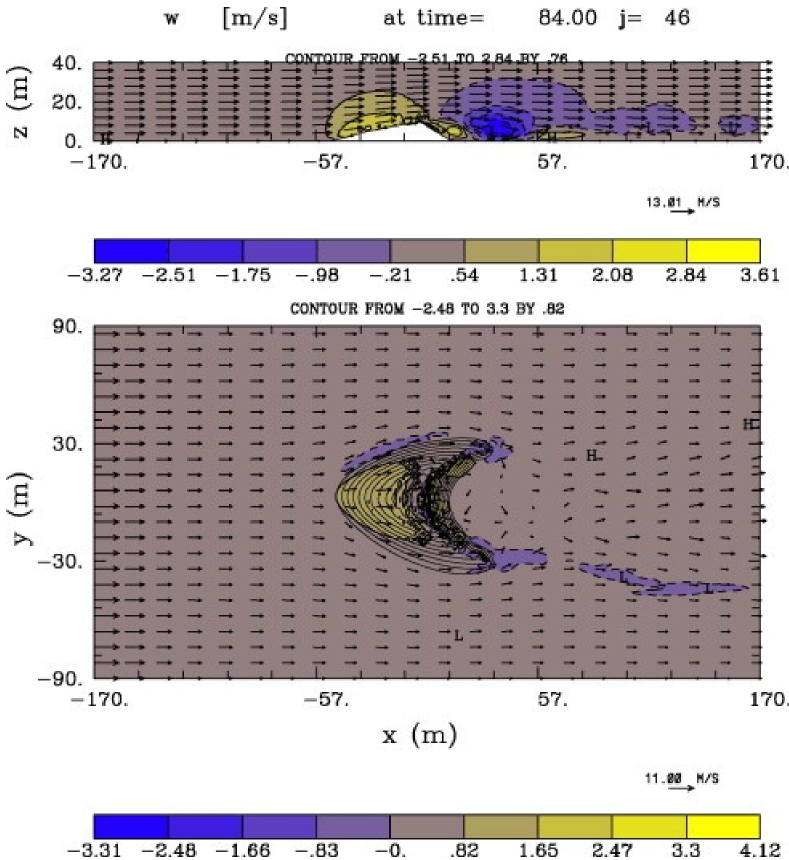
nature physics | VOL 3 | APRIL 2007 | www.nature.com/naturephysics

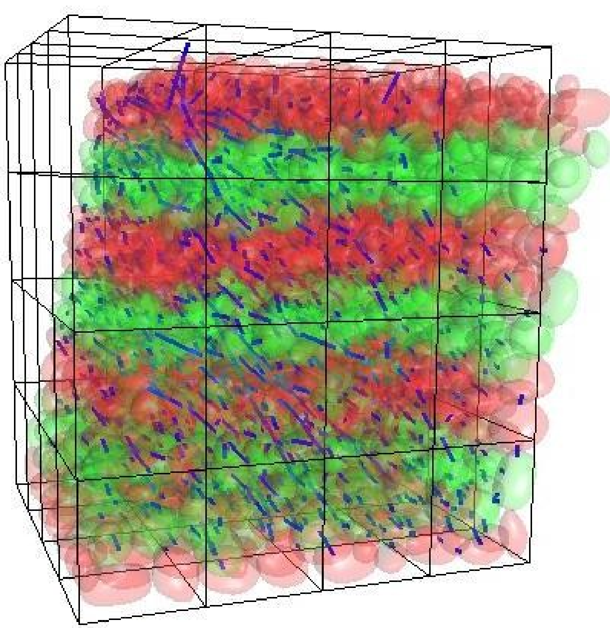
Measurement of growing dynamical length scales and prediction of the jamming transition in a granular material

AARON S. KEYS[1*], ADAM R. ABATE[2*], SHARON C. GLOTZER[1,3†] AND DOUGLAS J. DURIAN[2†]



One solution: CFD coupled to DEM





Sedimentology (2003) **50**, 279–301

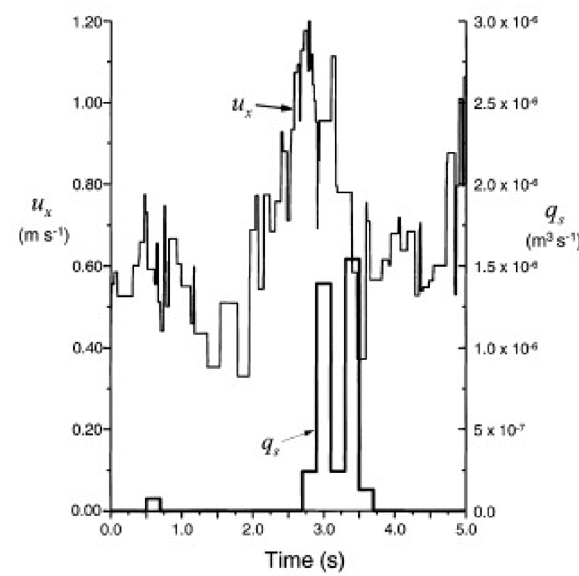
Direct numerical simulation of bedload transport using a local, dynamic boundary condition

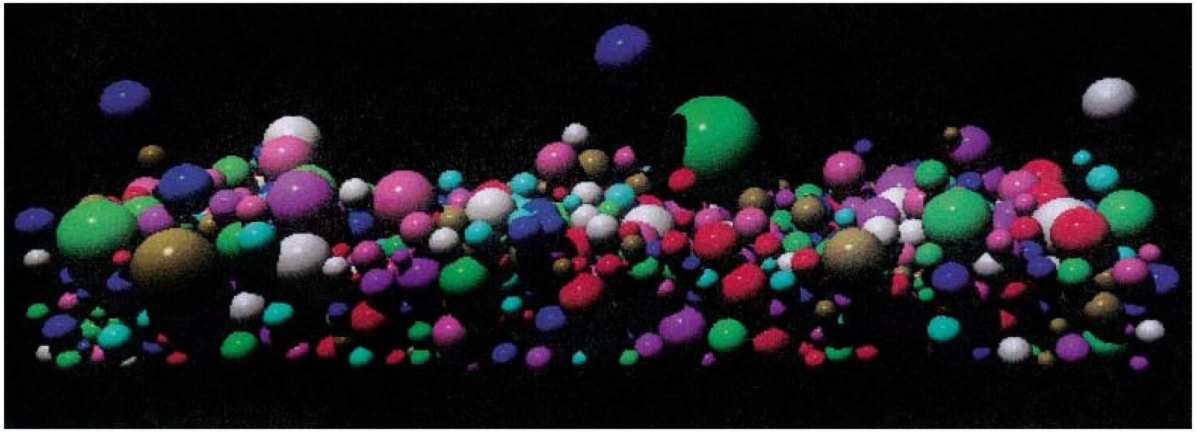
MARK W. SCHMEECKLE* and JONATHAN M. NELSON†

Numerical simulation of turbulent sediment transport, from bed load to saltation.

Orencio Durán,[1,a)] Bruno Andreotti,[1] and Philippe Claudin[1]

arXiv:1111.6898v2 [cond-mat.soft] 29 May 2012





Stochastic particle transport: diffusion?

ARTICLES
PUBLISHED ONLINE: 27 MARCH 2011 | DOI:10.1038/NPHYS1953
nature physics

Direct observation of the full transition from ballistic to diffusive Brownian motion in a liquid

Rongxin Huang[1], Isaac Chavez[1], Katja M. Taute[1], Branimir Lukić[2], Sylvia Jeney[2], Mark G. Raizen[1] and Ernst-Ludwig Florin[1]*

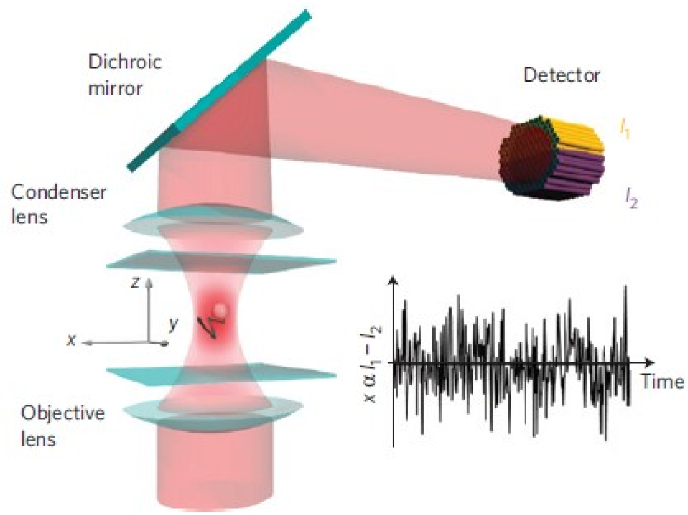


Figure 1 | Schematic diagram of the experiment. A single micrometre-size particle in water is undergoing Brownian motion in the observation volume given by an optical trap.

$$T_p = m/b \sim 10^{-6}\,\mathrm{s}$$

Inertial timescale — Particle mass — Stokes drag

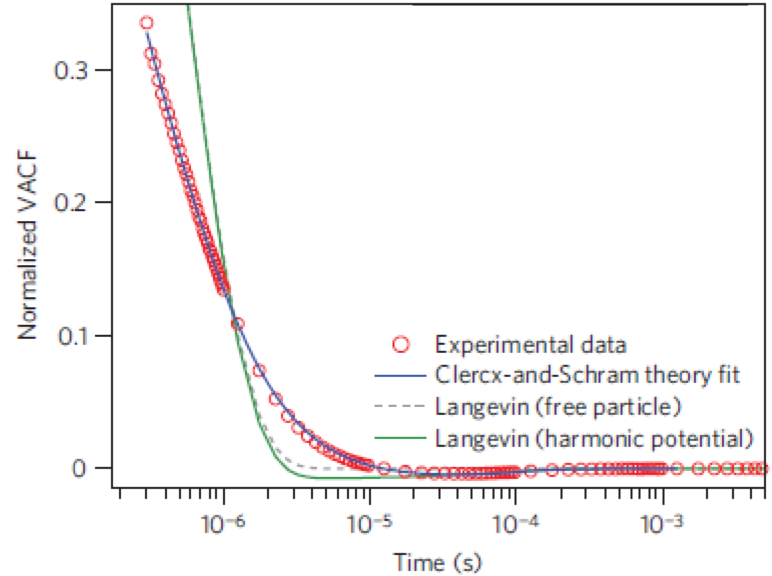


Figure 4 | Experimental VACF and theoretical description. The VACF

Ballistic transport at short time

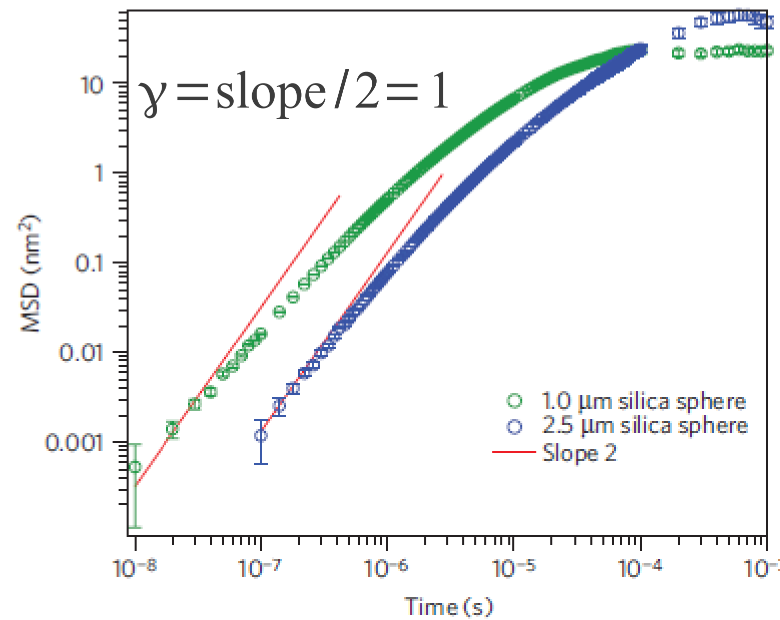
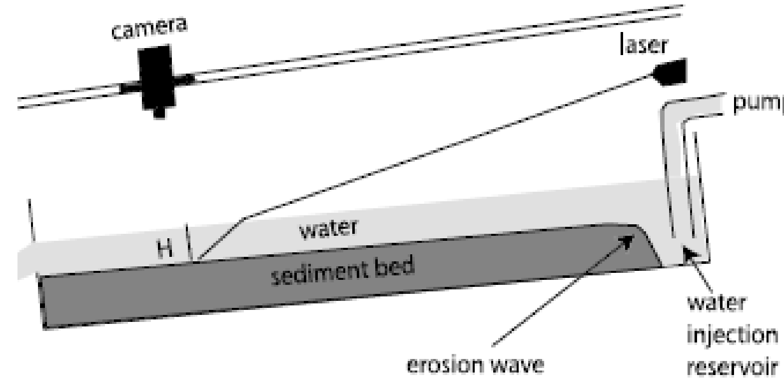


Figure 2 | Example MSD for silica particles 1 µm and 2.5 µm in diameter.

Bed load: Brownian motion with drift?

PennSeD
UNIVERSITY of PENNSYLVANIA SEDIMENT DYNAMICS LAB



(a)

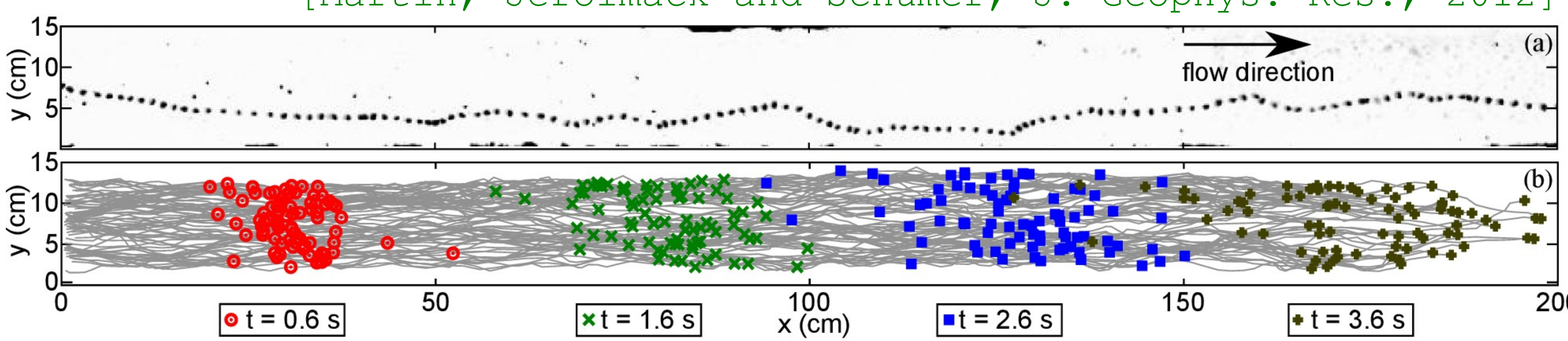
Momentum balance for particle in a turbulent shear flow:

$$T_p = \frac{4\rho_s D}{3\rho C_d}\left(\frac{1}{u_f - v_x}\right) \sim 10^{-1}\,\mathrm{s}$$

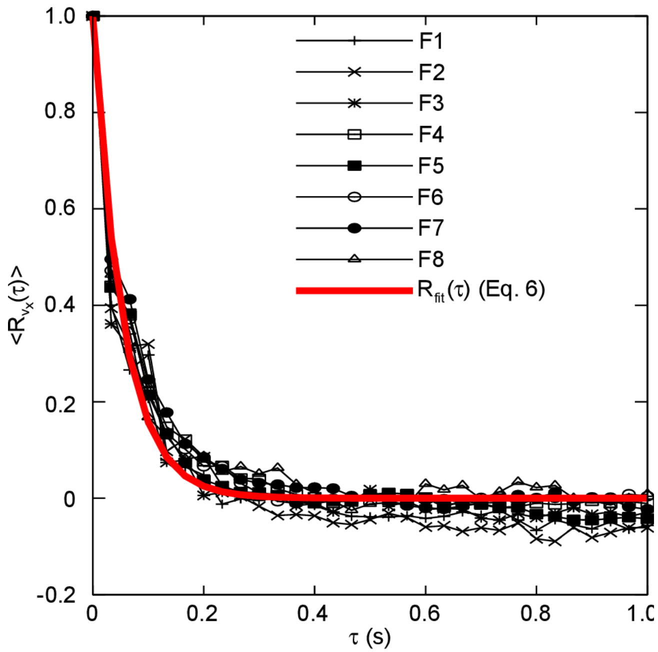
Drag, inviscid limit

Slip velocity

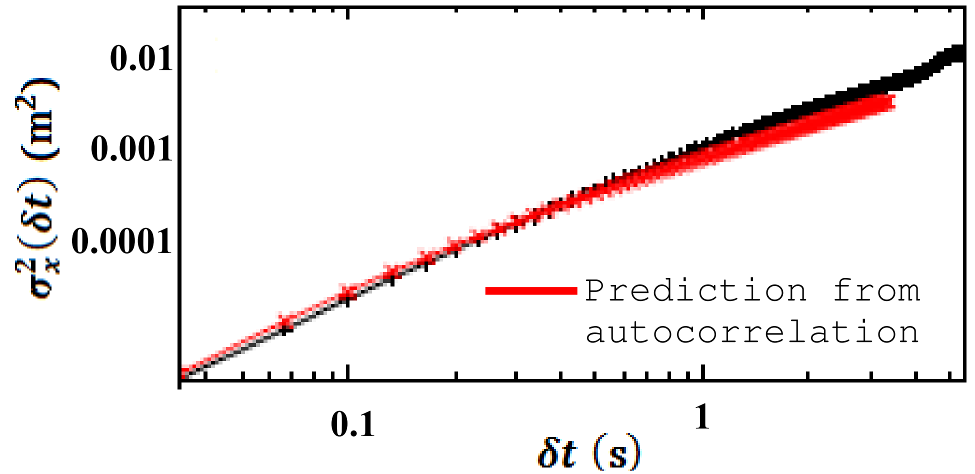
[Martin, Jerolmack and Schumer, J. Geophys. Res., 2012]



Velocity autocorrelation:
Controlled by inertia



Dispersion: inertial at short time



Short timescales: yes, diffusive particle transport:

Statistical mechanics may be used to derive macroscopic transport laws from stochastic particle motions.

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 117, F03031, doi:10.1029/2012JF002352, 2012

A probabilistic description of the bed load sediment flux: 1. Theory

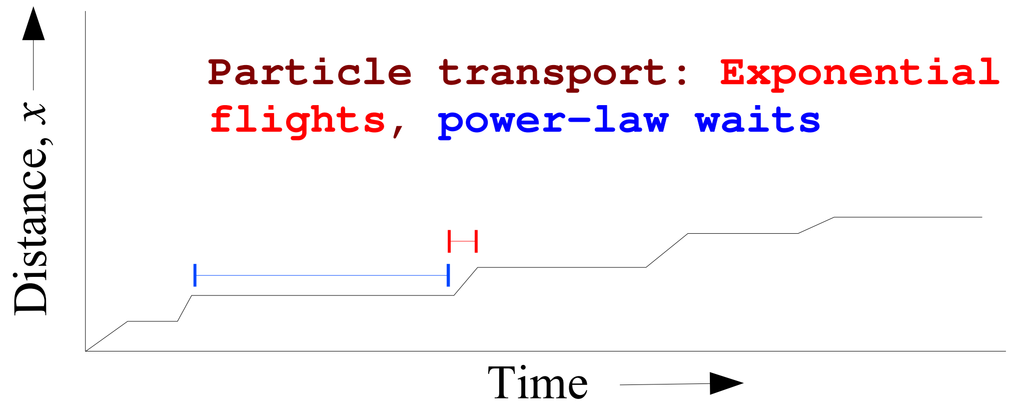
David Jon Furbish,[1] Peter K. Haff,[2] John C. Roseberry,[1] and Mark W. Schmeeckle[3]

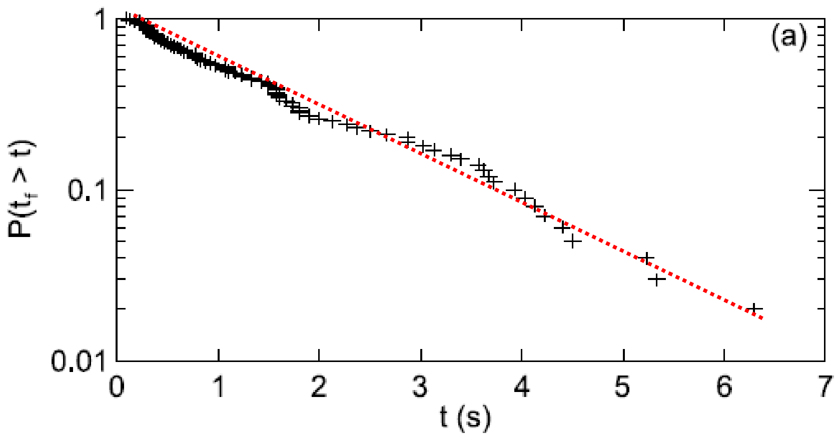
JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 117, F03034, doi:10.1029/2012JF002356, 2012

A probabilistic description of the bed load sediment flux: 4. Fickian diffusion at low transport rates

David Jon Furbish,[1] Ashley E. Ball,[2] and Mark W. Schmeeckle[3]

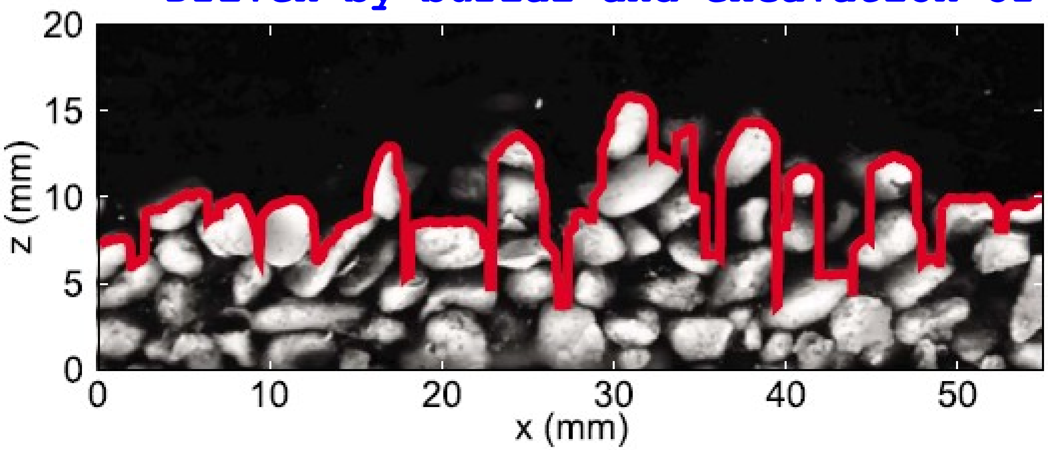
But long-time dynamics governed by power-law waits

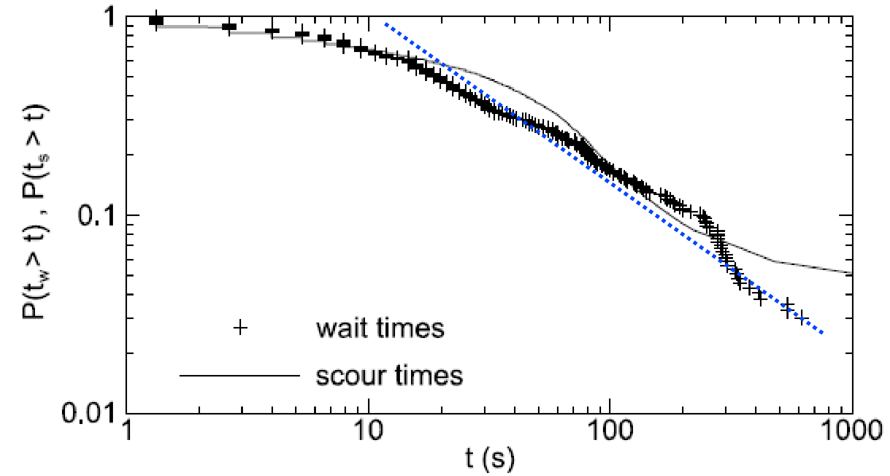




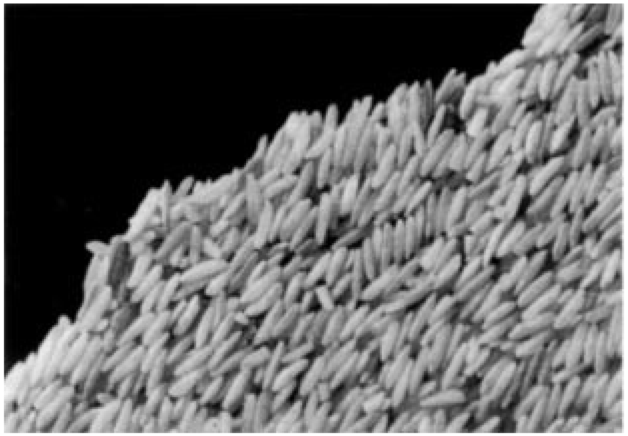
Waiting times >> fluid timescales:

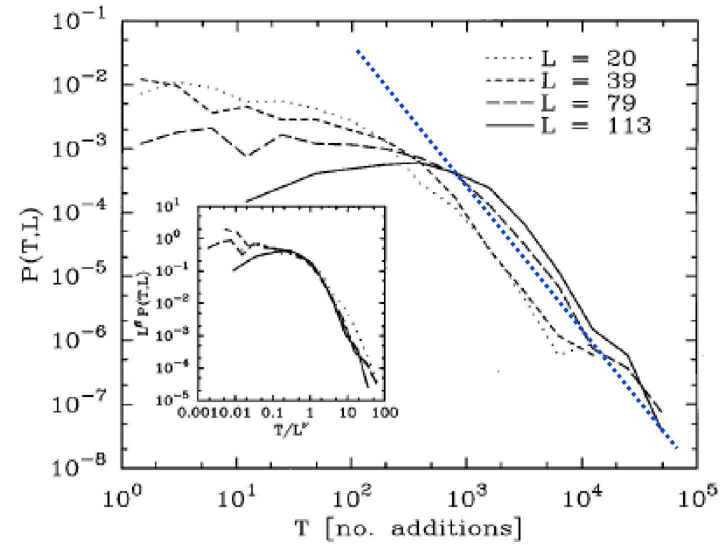
→ Driven by burial and excavation of bed





Rice pile [Christensen et al., PRL, 1996] shows power-law residence times due to burial and excavation

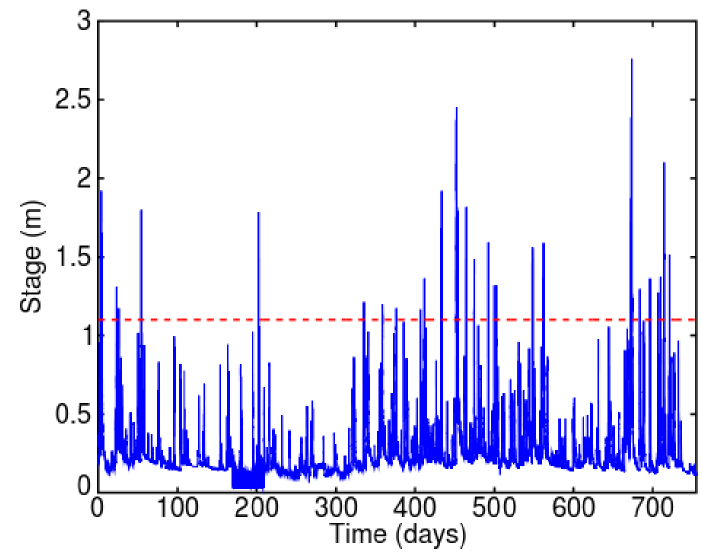


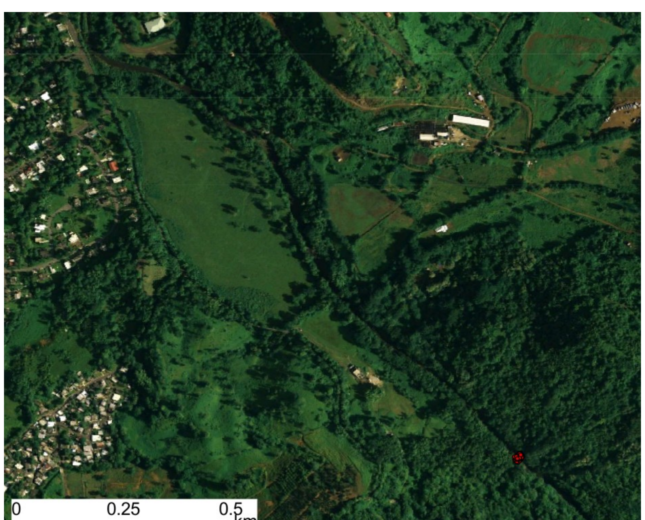


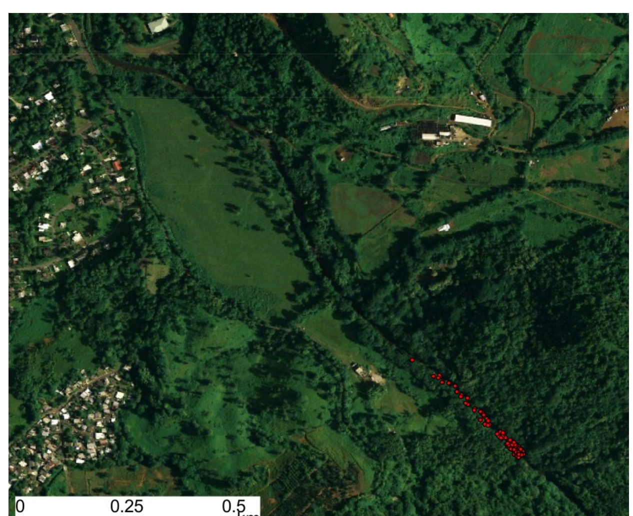
Particle transport in real rivers: Radio Frequency Identifier (RFID) Tags

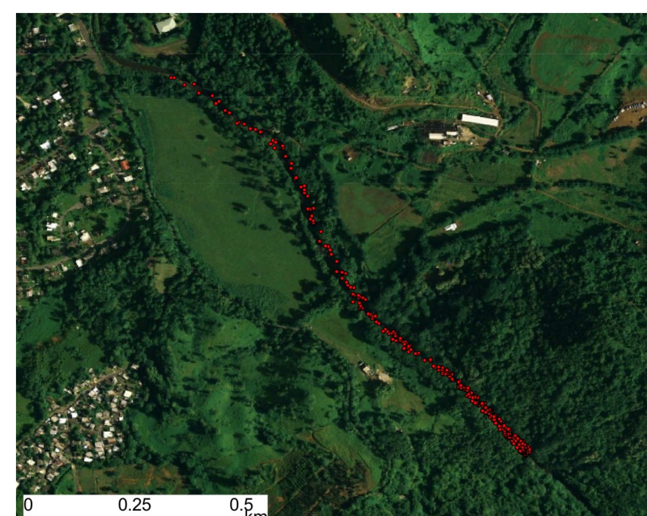
- Intermittent floods drive particle motion.

- Measure position of "radio rocks" after each flood.



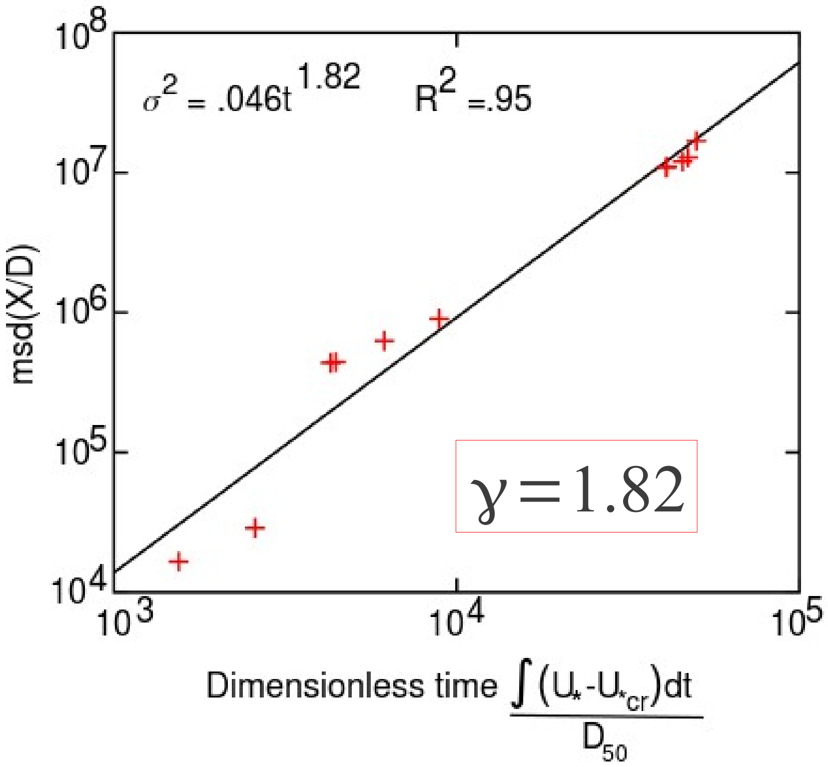


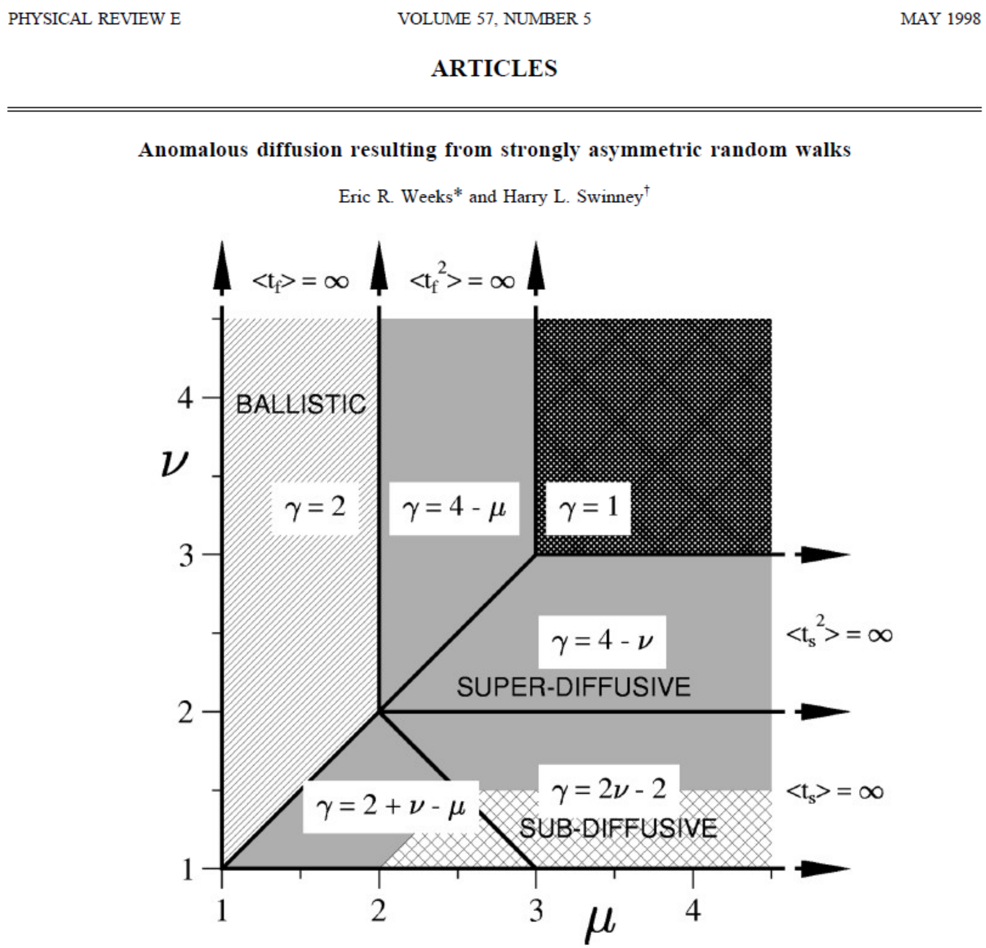




Dispersion: Superdiffusion due to power-law waits + drift

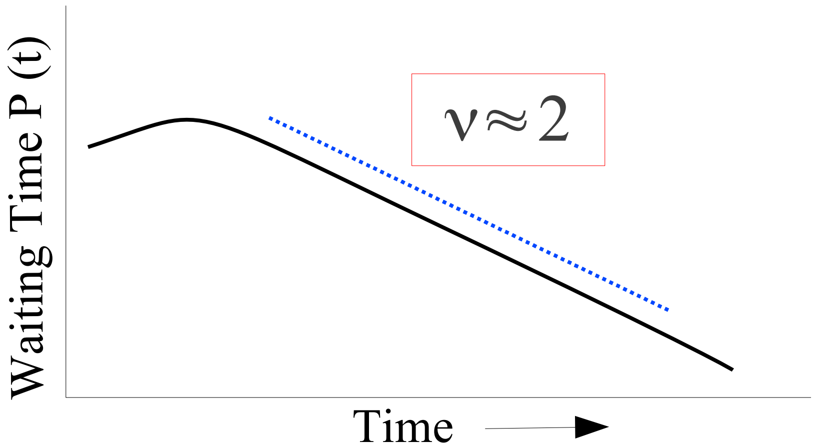
Mean square displacement → Superdiffusion





- **Flights are thin tailed** ($\mu = \infty$).
- **Transport is strongly asymmetric**

∴ **Power-law wait times.**



Mobile particles → Momentum conservation.

Particles get stuck in bed.

Mobile ↔ immobile transitions w/ power-law waiting.

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 114, F00A07, doi:10.1029/2008JF001246, 2009

Fractional advection-dispersion equations for modeling transport at the Earth surface

Rina Schumer,[1] Mark M. Meerschaert,[2] and Boris Baeumer[3]

$$\frac{\partial^\gamma C(x,t)}{\partial t^\gamma} = -v\frac{\partial C(x,t)}{\partial x} + \mathcal{D}\frac{\partial^2 C(x,t)}{\partial x^2}.$$

Tracer particles spend much more time at rest than in motion.

Stochastic modeling approach:

Direct solution of fADE, if known, to determine dispersion.

Lagrangian particle tracking to determine dispersion from collection of particle motions.

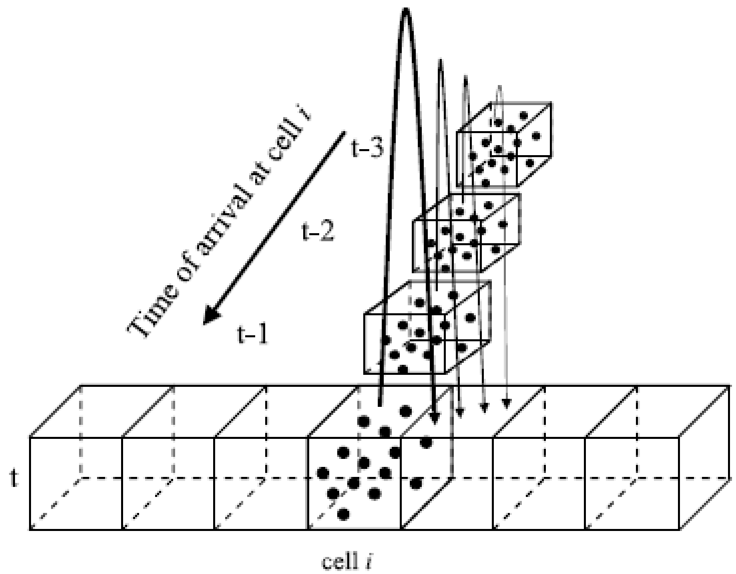
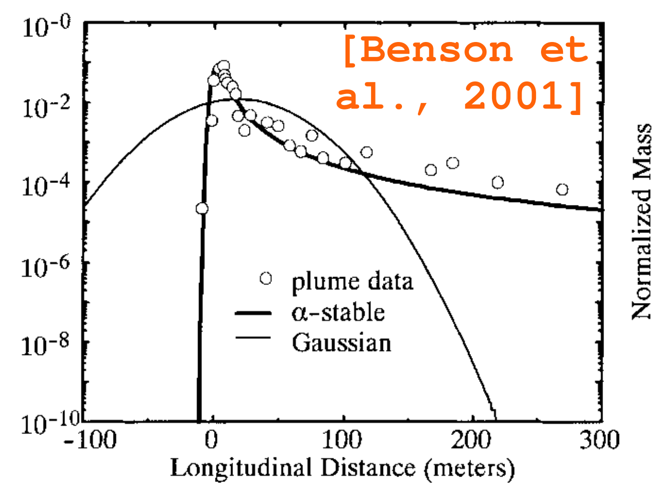


Figure 9. When governed by a fractional-in-time ADE, particles have memory of the time that they arrive at a given point. Their probability of release decays as a power law from arrival time.

Anomalous Dispersion



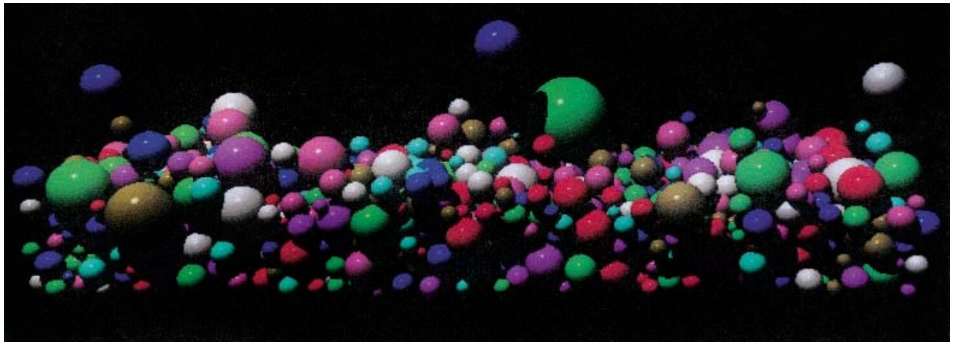
But how to assess, *a priori*, what particle waiting times and hop lengths are?

→ Need better understanding of physics

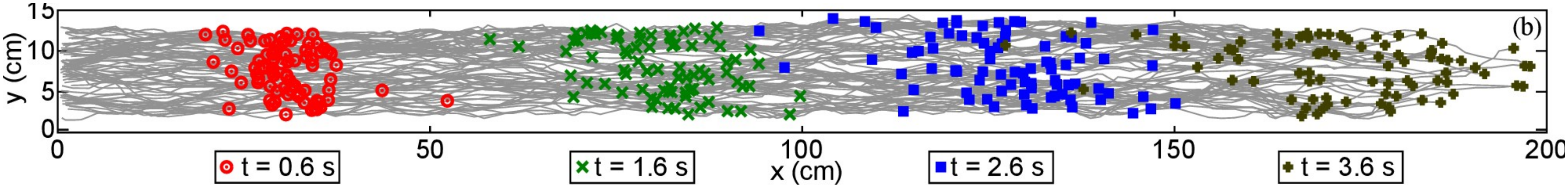
Summary and directions

Thresholds of motion: stick-slip dynamics, stochastic transport

Direct simulation: possible path forward, difficult for natural systems



Statistical mechanics: useful framework for deriving transport equations but mobile/immobile partitioning complicates application



Fractional ADEs and Random Walk models: flexible for modeling anomalous diffusion, but must be informed by physics