Physical causes and modeling challenges of anomalous diffusion of sediment tracers

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Grains supported by and in frequent contact with the bed. A well defined “layer” describable by:
1. Particle volume, $\delta v \ [L^3]$. 
2. Average velocity, $u_s$, of bed load sediment $[L/T]$. 

$$q_s = \delta v \ n \ u_s$$

Grains clump together – bed forms
→ “Force chains” form and break
→ Slow, stochastic shearing of granular bed
→ Spatial sorting and clusters
Near threshold transport: “avalanching” and spatially heterogeneous dynamics?

Turbulence, granular collisions, grain size dispersion.

Intermittent transport under steady flow.

Laminar flow, uniform beads.

Intermittent transport → collective grain motion.
One solution: CFD coupled to DEM

Direct numerical simulation of bedload transport using a local, dynamic boundary condition

MARK W. SCHMEECKLE* and JONATHAN M. NELSON†

Numerical simulation of turbulent sediment transport, from bed load to saltation.

Orencio Durán, Bruno Andreotti, and Philippe Claudin

Direct observation of the full transition from ballistic to diffusive Brownian motion in a liquid

Rongxin Huang, Isaac Chavez, Katja M. Taute, Branimir Lukić, Sylvia Jeney, Mark G. Raizen and Ernst-Ludwig Florin

Inertial timescale

\[ T_p = \frac{m}{b} \sim 10^{-6} \text{s} \]

Ballistic transport at short time

\[ \gamma = \text{slope/2} = 1 \]

Experimental VACF and theoretical description. The VACF
Bed load: Brownian motion with drift?

Momentum balance for particle in a turbulent shear flow:

\[ T_p = \frac{4\rho_s D}{3\rho C_d} \left( \frac{1}{u_f - v_x} \right) \sim 10^{-1} \text{s} \]

Velocity autocorrelation:
Controlled by inertia

Dispersion: inertial at short time

Short timescales: yes, diffusive particle transport:

Statistical mechanics may be used to derive macroscopic transport laws from stochastic particle motions.
But long-time dynamics governed by power-law waits

Particle transport: Exponential flights, power-law waits

Waiting times >> fluid timescales:
  → Driven by burial and excavation of bed

Rice pile [Christensen et al., PRL, 1996] shows power-law residence times due to burial and excavation
Particle transport in real rivers: Radio Frequency Identifier (RFID) Tags

- Intermittent floods drive particle motion.
- Measure position of “radio rocks” after each flood.
Mean square displacement → Superdiffusion

- Flights are thin tailed ($\mu = \infty$).

- Transport is strongly asymmetric.

::: Power-law wait times.

Mobile particles → Momentum conservation.

Particles get stuck in bed.

Mobile ↔ immobile transitions w/ power-law waiting.
Tracer particles spend much more time at rest than in motion.

Stochastic modeling approach:

Direct solution of fADE, if known, to determine dispersion.

Lagrangian particle tracking to determine dispersion from collection of particle motions.

But how to assess, a priori, what particle waiting times and hop lengths are?

→ Need better understanding of physics

Fractional advection-dispersion equations for modeling transport at the Earth surface

Rina Schumer,1 Mark M. Meerschaert,2 and Boris Baeumer3

\[
\frac{\partial^\gamma C(x,t)}{\partial t^\gamma} = -v \frac{\partial C(x,t)}{\partial x} + D \frac{\partial^2 C(x,t)}{\partial x^2}.
\]

Figure 9. When governed by a fractional-in-time ADE, particles have memory of the time that they arrive at a given point. Their probability of release decays as a power law from arrival time.

Anomalous Dispersion

[Benson et al., 2001]
Summary and directions

**Thresholds of motion:** stick-slip dynamics, stochastic transport

**Direct simulation:** possible path forward, difficult for natural systems

**Statistical mechanics:** useful framework for deriving transport equations, but mobile/immobile partitioning complicates application

**Fractional ADEs and Random Walk models:** flexible for modeling anomalous diffusion, but must be informed by physics