Nonlinear Methods for Geophysical Imaging

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Bridging the Gap Workshop

October 2, 2012

Biological Applications

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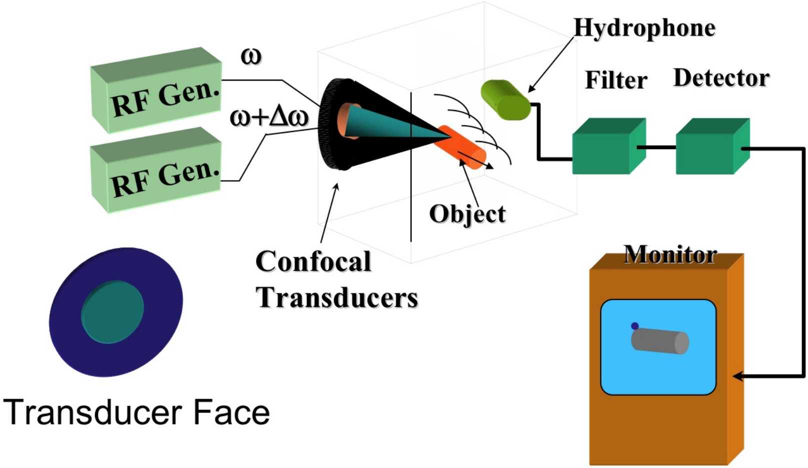
Two Examples

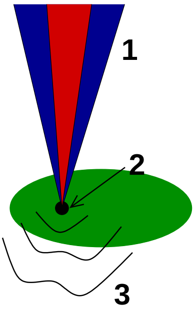
1. **Nonlinear elasticity imaging**
   - **Sensing one wave with another**
2. **FWI starting models**

The Experiment

Fatemi & Greenleaf (1998)

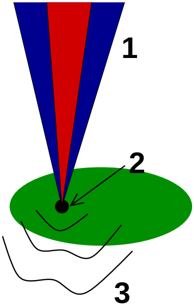


A (very) simple model



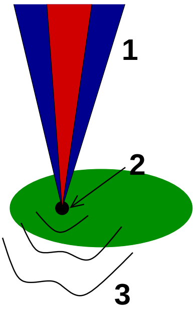
1. linear wave propagation
2. nonlinear interaction

A (very) simple model



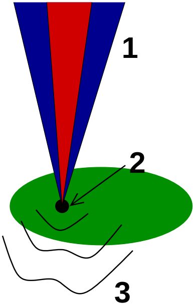
1. linear wave propagation
2. nonlinear interaction
   - $p(\rho) = p_0 \left(\rho/\rho_0\right)^{\gamma}$

A (very) simple model



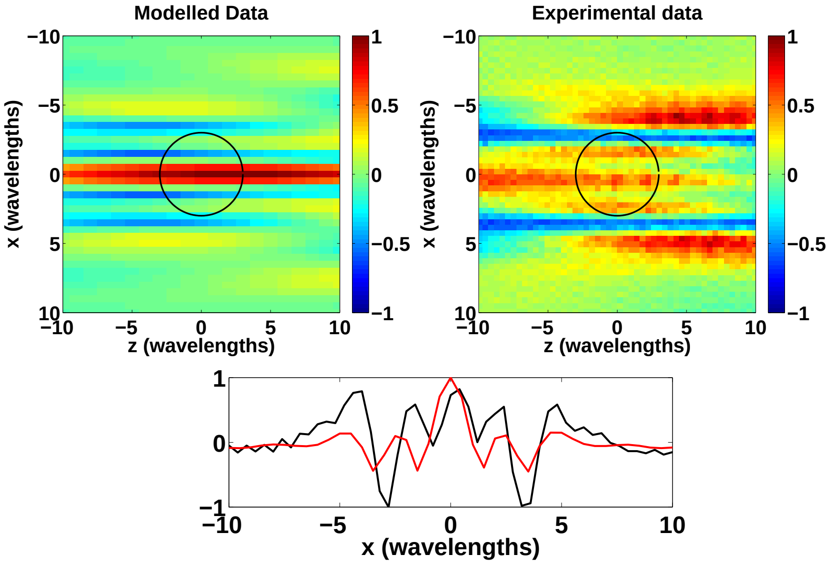
1. **linear wave propagation**
2. **nonlinear interaction**
   - $p(\rho) = p_0 \left(\rho/\rho_0\right)^\gamma$
   - $\partial_t^2 \psi - c(x)^2 \triangle \psi =$
     $$\partial_t \left[|\nabla \phi|^2 + \frac{\gamma - 1}{2c(x)^2} |\partial_t \phi|^2 \right]$$

A (very) simple model



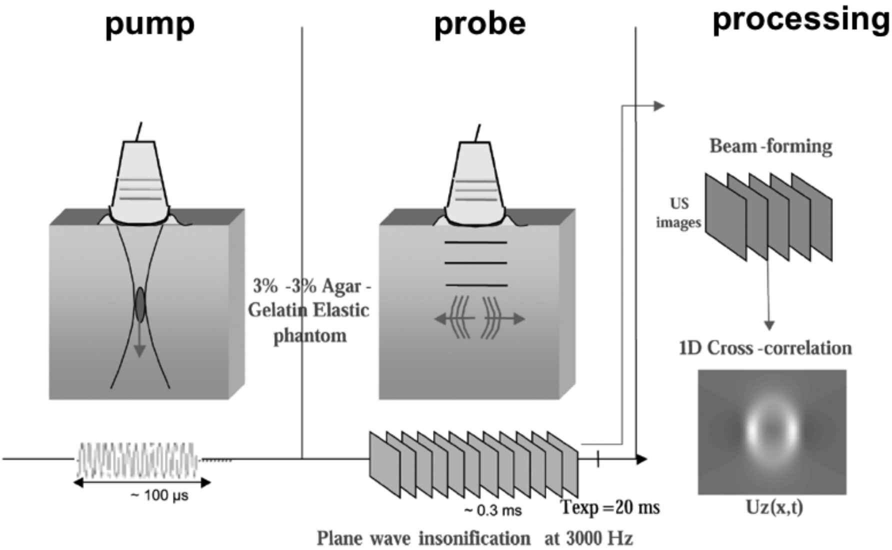
1. linear wave propagation
2. nonlinear interaction
   - $p(\rho) = p_0 \left(\rho/\rho_0\right)^\gamma$
   - $\partial_t^2 \psi - c(x)^2 \triangle \psi = \partial_t \left[|\nabla \phi|^2 + \frac{\gamma - 1}{2c(x)^2} |\partial_t \phi|^2 \right]$
3. image is $|\psi|$(focus point)

Experimental comparison



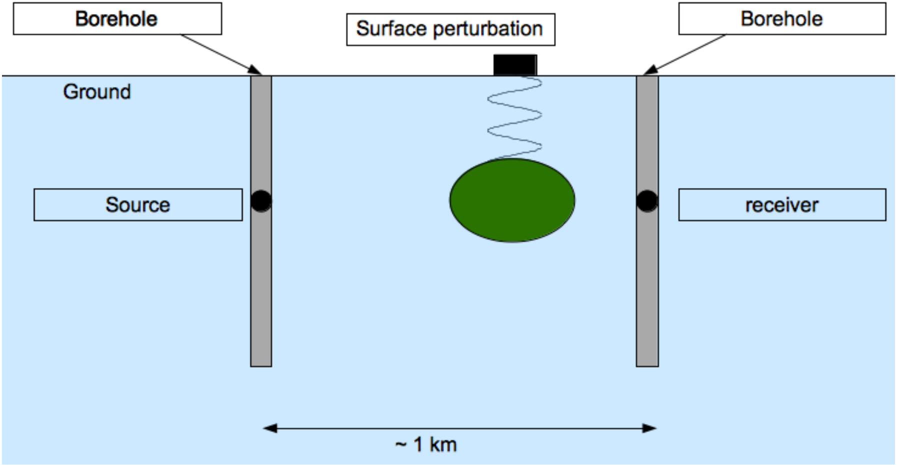
AM et. al. (08)

Elastography



Bercoff et al 2004

Field Scale Rough Idea

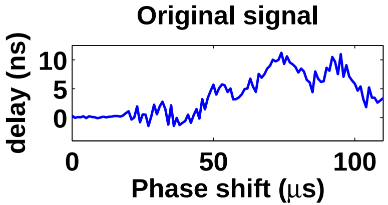


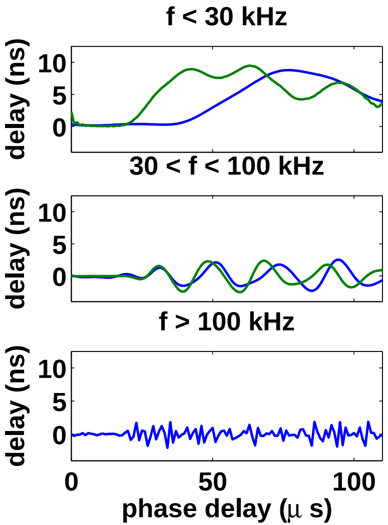
Experimental Setup

Gallot, AM et al (in prep)

Results

Berea

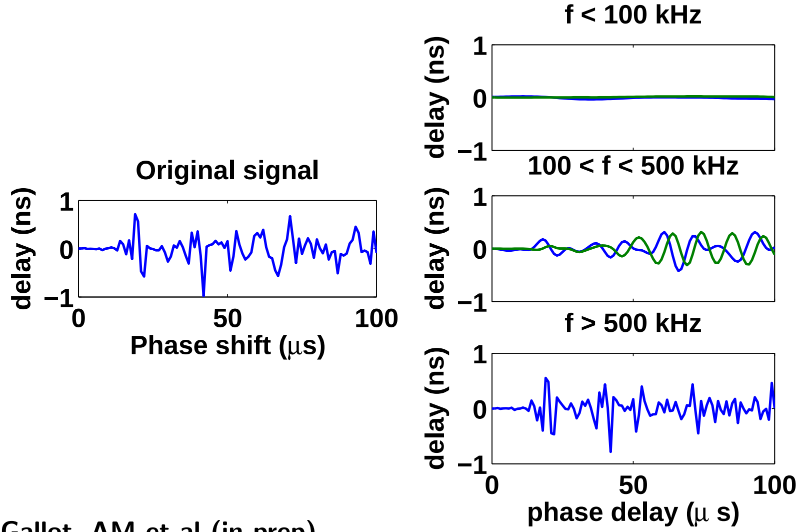




Gallot, AM et al (in prep)

Results

Aluminum



Gallot, AM et al (in prep)

Questions

- **How can we focus in an unknown, variable soundspeed material?**
- **Can we recover the nonlinear elasticity parameters?**
- **Can we efficiently model the nonlinear interactions?**

Two Examples

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Full Waveform Inversion

$$\text{argmin}\,\mathcal{J}(\mathbf{m}) = \|\mathbf{F}\mathbf{m} - \mathbf{d}\|_2 + \text{regularization}$$

- $\mathcal{J}$ – objective function
- **F** – wave operator
- **m** – soundspeed model
- **d** – recorded data

What $\mathbf{m}_0$ should we use?

Topological Derivative Methods

What m_0 should we use?

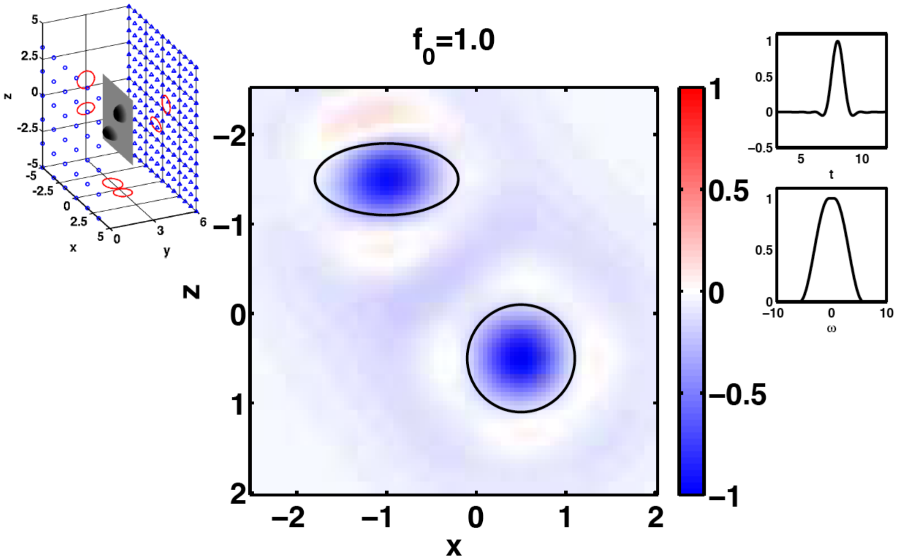
Find points x where

$$\frac{\partial \, \text{argmin} \, \mathcal{J}(m(x))}{\partial s} < 0$$

s – -ϵ size spherical inclusion

e.g. Guzina & Bonnet (04,06)

Example



AM & Guzina (08)

Questions

- Can we do something similar with layers?
- Will this improve our final models?
- What about interpretation?
  - Statistical change detection?
  - Automate?

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