Self-similar Network Approach to Modeling Complex Geophysical Systems

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Brief summary

- We propose a sustained research effort with the focus on Tokunaga self-similar structure of the observed geophysical networks and predictive understanding of dynamical processes on networks
- The research will have direct impact on
  - Hydrology: environmental transport on river and hillslope networks, scaling of river discharge peaks, variability of discharge for basins of a given size, connection between geographic/climatic characteristics of a basin and its geometric and transport properties
  - Seismology: objective detection and analysis of seismic clustering, its relations to the physical characteristics of the lithosphere, and ultimately improving seismic hazard assessment and forecast techniques
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- Feasibility of the project is supported by the ongoing collaboration of the PIs, with concrete results in each of the above areas
- Intellectual merit is in novel theoretical framework quite distinct from that addresses by the bulk of the mathematical theory to date, and in the respective applied methodology
- Broader impact is in immediate applicability of the results to a wide variety of processes described by tree graphs and dynamical processes on networks: biology, aerosols, chemical reactions, transport, social interactions, etc.

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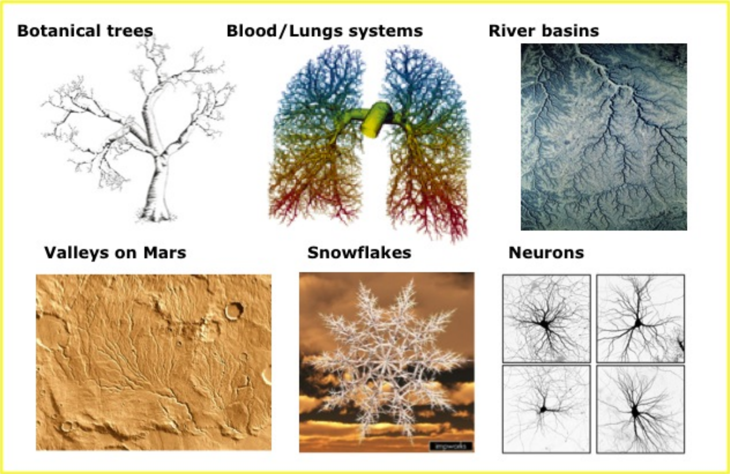
Trees are ubiquitous in Nature



Dynamic process as a tree

If one focuses on cluster formation, a dynamic processes on networks (graphs) can be represented as a tree

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Self-similar trees

- A surprising variety of trees in observed and modeled systems can be closely approximated by a two-parametric class of *Tokunaga self-similar trees* [Tokunaga, 1978; Peckham, 1995; Newman et al., 1997]
- Tokunaga self-similarity ensures existence of the *Horton laws*, heavily used in hydrology since the 60-s
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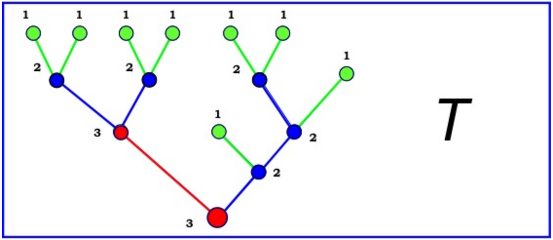
- Formally,
  - *Pruning* $\mathcal{R}(T)$ of a finite tree T cuts leaf-branches.



Definition (Burd et al., *Bernoulli*, 2000)

A random tree T with distribution P on a space of finite trees is self-similar if P is invariant with respect to the pruning operation $\mathcal{R}$:

$$\mathsf{P}(\cdot | T \neq \emptyset) \circ \mathcal{R}^{-1} = \mathsf{P}(\cdot)$$

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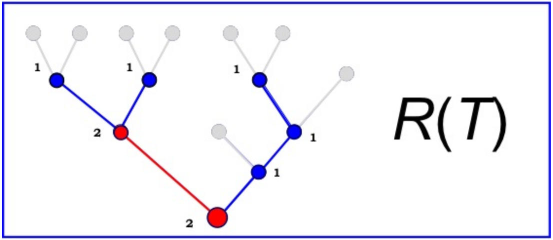

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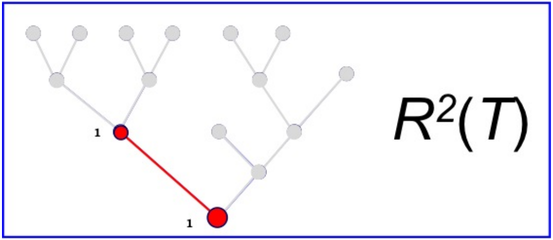

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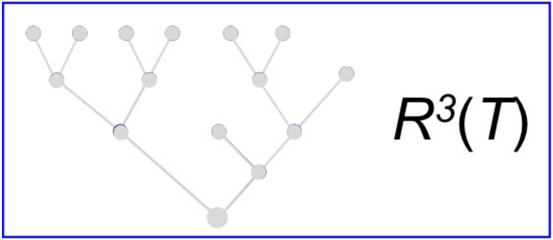

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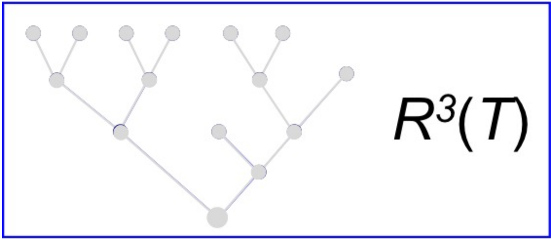

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Tokunaga trees everywhere

Horton and Tokunaga self-similarity, with a broad range of respective parameters, have been empirically or rigorously established in numerous observed and modeled systems:

- River networks: static and dynamic
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- Hillslope drainage networks: static and dynamic
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Models of Tokunaga trees

The PI's research of the last 3 years has significantly expanded the range of models that produce self-similar trees, with a range of parameters:

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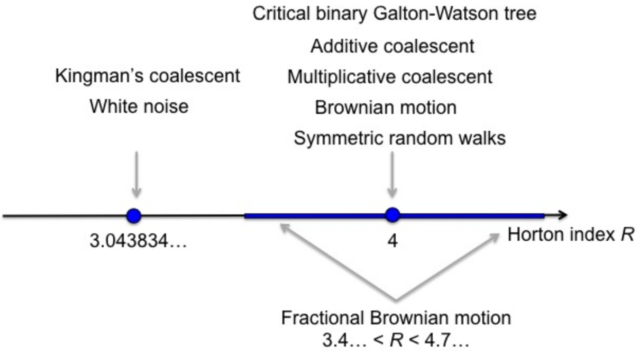
Summary of SS results



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- Large scale validation of Tokunaga self-similarity in hydrology
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Existing results: The point of departure

- Tokunaga and Horton self-similarity in essential probability models: branching, coalescence, time series
  [Zaliapin and Kovchegov 2012, Kovchegov and Zaliapin, 2012, Tejedor and Zaliapin, 2012]
- Statistical inference: self-similar tests, parameter estimation
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  - Reconstruction and inversion (estimation)
- ODEs, PDEs: Horton-Smoluchowski equations
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  - Using Tokunaga universality classes for cross-application of various techniques. For instance, using the existence of gelation phase (phase transition) in multiplicative cascade to establish similar phenomena in other Tokunaga trees from the same class.
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