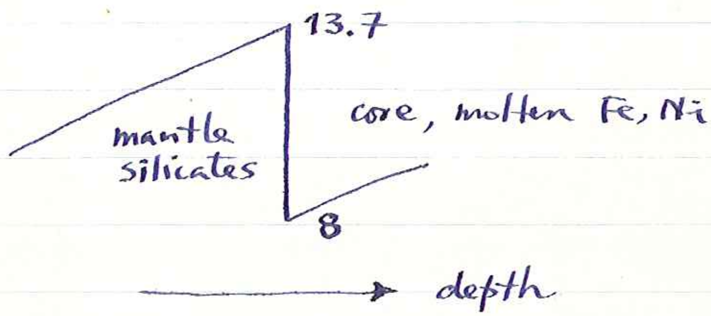
Before considering the inverse problem we must pause to consider the complications which may arise in $T(\Delta)$ and $p(\Delta)$ curves.

First, the most important features caused by the fluid core.

The P velocity decreases from ~ 13.7 km/s to ~ 8 km/s at core-mantle bdry



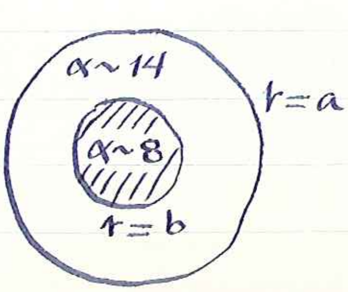
Main reason: fluid $\mu \to 0$, $\alpha = \left(\frac{K + \frac{4}{3}\mu}{\rho}\right)^{1/2}$

↑ not only reason, also major compositional difference.

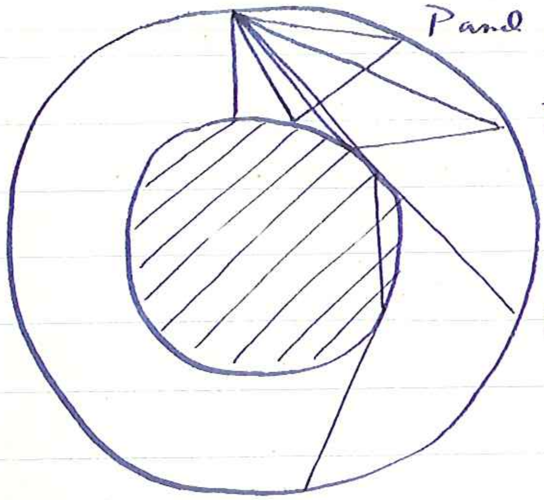
$\alpha_{core} < \alpha_{mantle}$

$\alpha_{core} > \beta_{mantle}$, but $\approx \beta_{mantle} = 7.2$ km/s

Consider simplest example: homogeneous core and mantle, e.g.

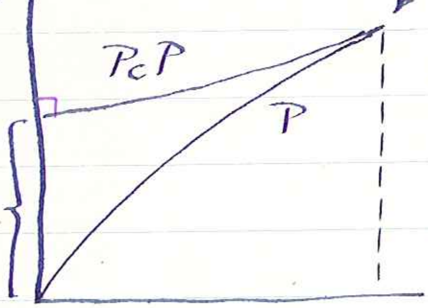


Surface focus quake: consider first P and PcP.



P and PcP merge into one grazing ray, edge of core shadow at $\Delta \sim 105^\circ$ for ⊕'s core.

refracted ray at grazing incidence, emerges at $\Delta \lesssim 180^\circ$ for ⊕



$T \sim 14$ min for ⊕

core shadow $\Delta \sim 105^\circ$.

two times mantle transit time (total ~ 8.5 m for ⊕)

$\frac{a}{\alpha}$ (since $i = \frac{\pi}{2}$ at surface)

sharp point at junction

$\cancel{\frac{b}{\alpha}}$ $\frac{R_c}{\alpha}$ (since $i = \pi/2$ at core)

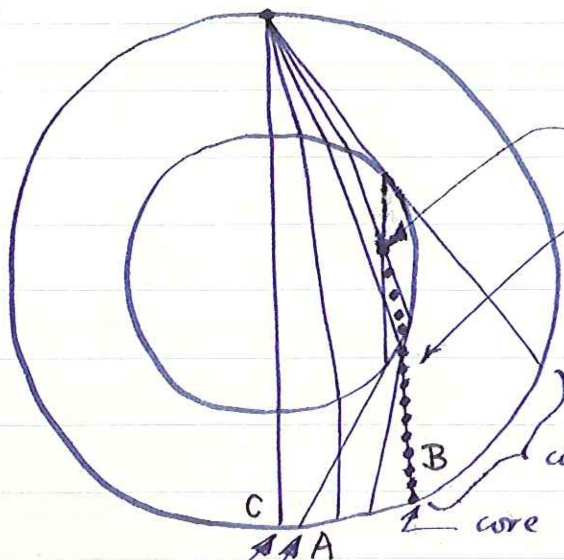
p of grazing ray for ⊕ is 4.45 sec/degree

$dT/d\Delta$ decreases for P, increases from 0 for PcP

$p = \frac{dT}{d\Delta}$

$\frac{dT}{d\Delta} = 0$, waves are coming straight up

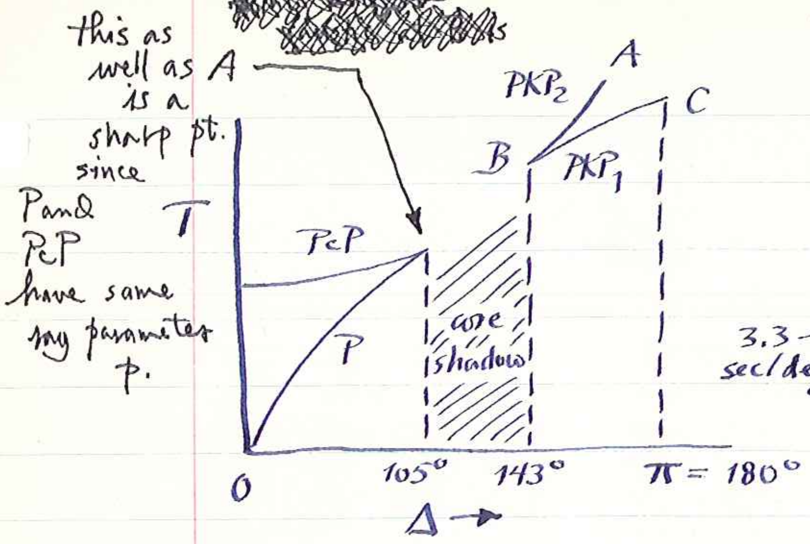
For Δ beyond the core shadow: PKP waves refracted at core bdry.

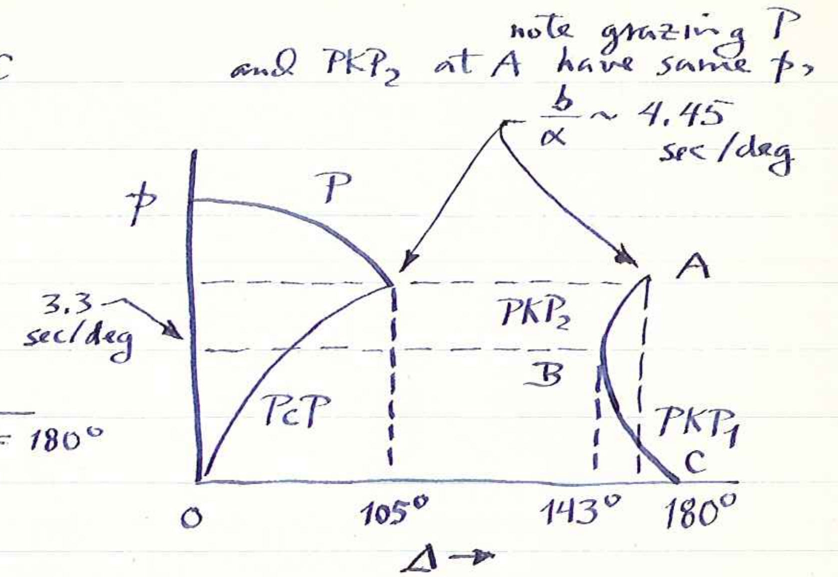


The locus of crossing points of adjacent rays is called the core caustic surface. It can be visualized better in Fig. 9 from Julian + Anderson, shows P, PKP ray paths for a realistic ⊕ model. I've dotted the crossing pts. of adjacent rays.

The $T(\Delta)$ and $p(\Delta)$ curves for the PKP waves have two branches in this simple model PKP_1 and PKP_2.

[redacted]





Note: for PKP$_2$ both dT and $d\Delta$ are negative, so $p = dT/d\Delta$ is still positive.

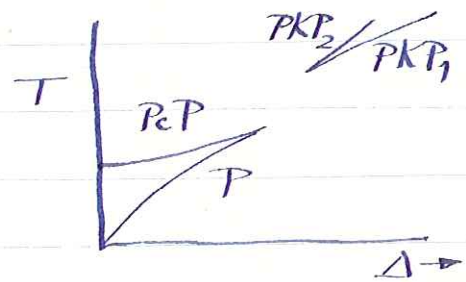
<u>Caustics</u> are recognizable by two features:

1. neighboring rays cross (actually become tangent, the caustic surface is the <u>envelope</u> of all possible rays)
2. $p(\Delta)$ turns <u>vertical</u>, not sharp point as at junction of P and PcP.

Geometrical ray theory, in its simplest form as described here, fails to adequately describe signal at shadow bdry, the caustic point B and points A and C (the antipode). <u>Diffraction</u> effects become important in vicinity of all these pts, e.g. the shadow bdry is <u>not sharp</u>.

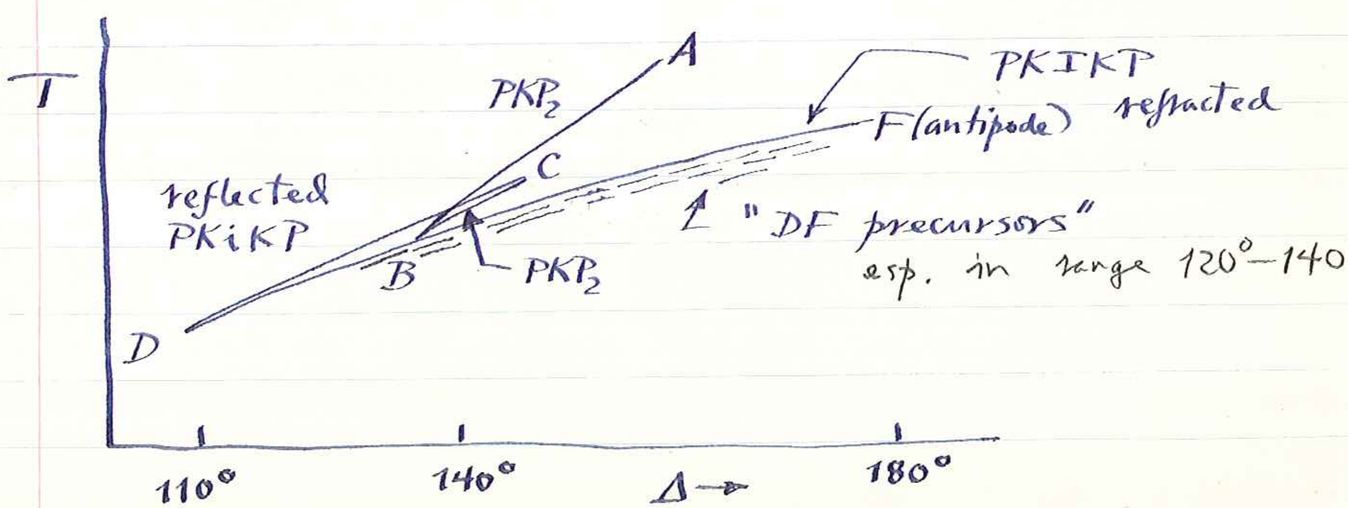
PKP in actual Earth:

Oldham (1906): first evidence for low velocity core, observations of P, PcP, P_{diff} near the shadow. $T(\Delta)$ curve thus thought for many years to look like

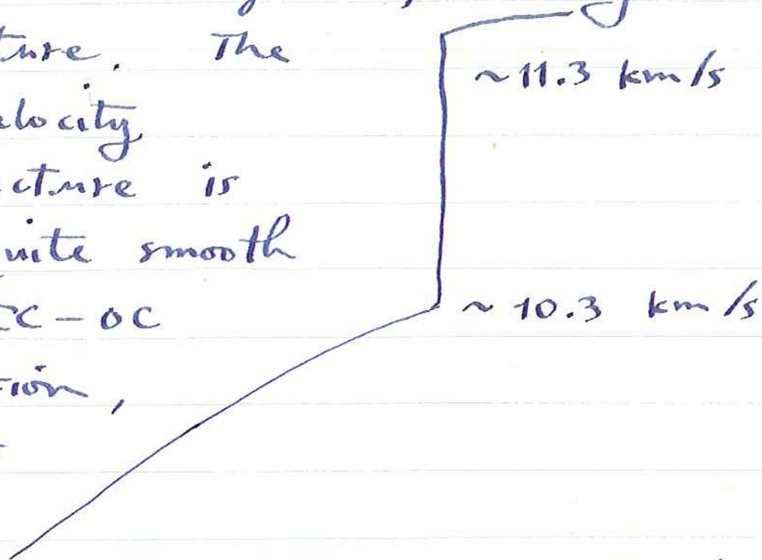


1936 Inge Lehman observed arrivals between 110° and 140° which could not be explained on this basis (they would be in the core shadow). Interpreted as refractions from a high velocity inner core, also reflections from this bdry, Lehman the discoverer of the inner core.

Travel time curve now known to look like:

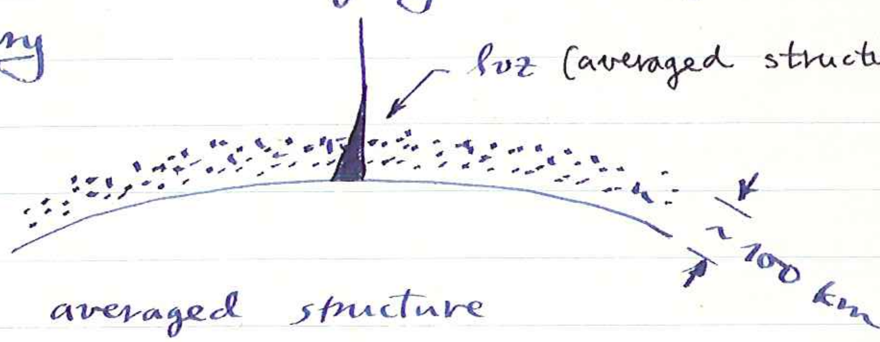


core-mantle bdry. Above $T(\Delta)$ curve now thought to be quite reasonable for spherically averaged structure. The velocity structure is quite smooth at IC-OC transition, jumps from

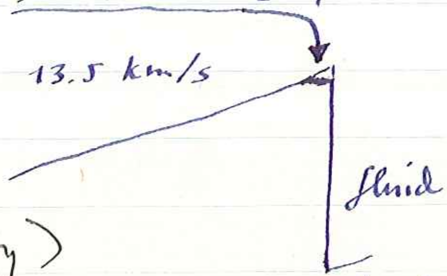


about 10 to about 11 km/s, much as envisaged by Ms. Lehman.

In contrast, core-mantle bdry, current picture: highly inhomogeneous above bdry

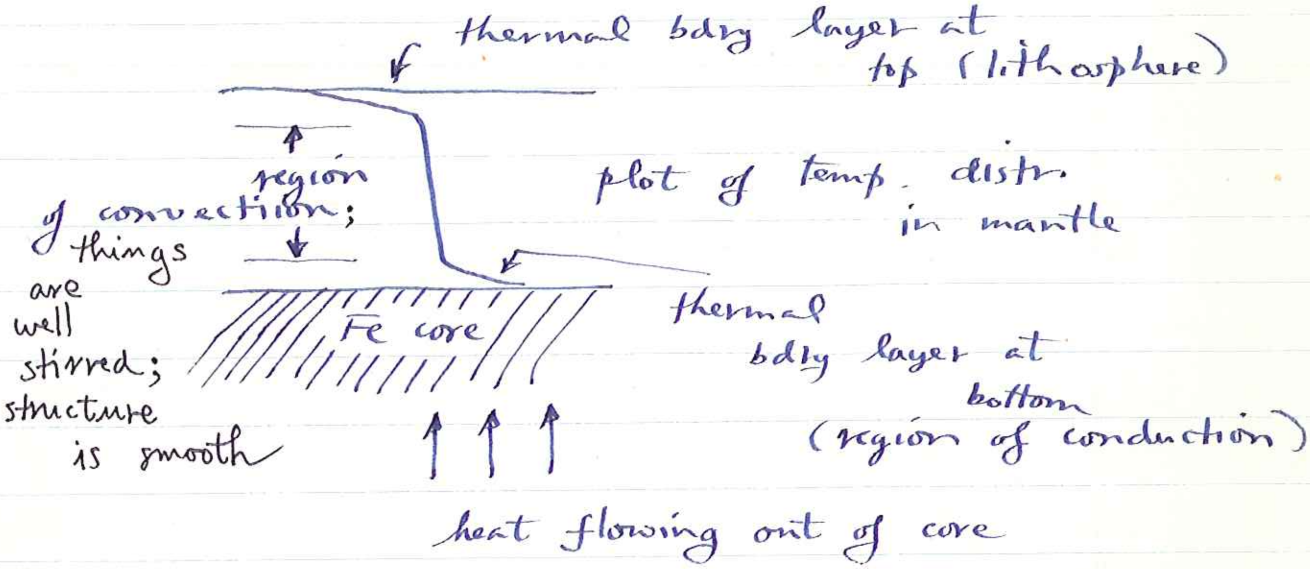


averaged structure seems to have a weak lvz, see e.g. model 1066A



Also suggestion ∃ a Thorne layer (discontinuity)

Current interpretation, <u>thermal bdry layer</u> above core mantle bdry.



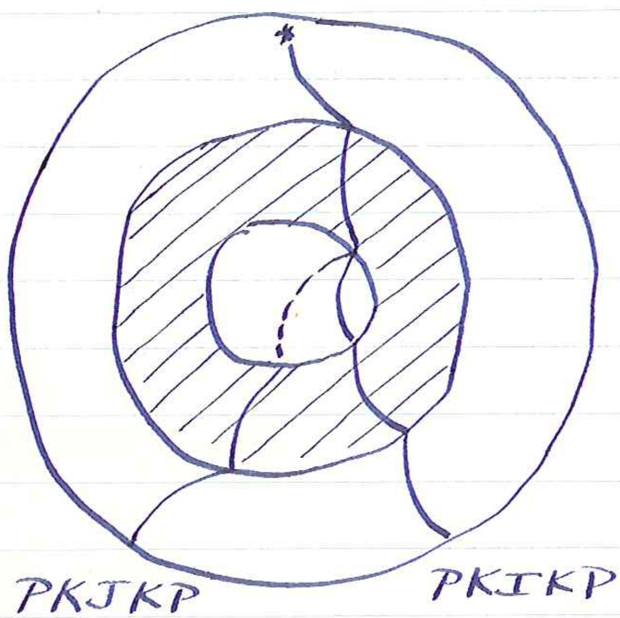
<u>Underside</u> of core-mantle bdry appears on other hand to be <u>very</u> smooth because travel times of P4KP, P7KP, P9KP, P11KP (!) etc. show no increasing scatter, arrivals are sharp and impulsive, see example of P4KP cover of EOS. Any <u>bumps</u> on inside of core must be small ≤ 1 km, whether this large enough to perturb dynamo is debated.

no min. marks ⇒ whole → length shown is less than a minute; this is a short-period record unlike those in our lab.

PnKP rays really get thrown around, see e.g. P7KKP from Chapman, narrow solid angle at source gets sprayed out over 3/4 ⊕ surface ⇒ amplitude small.

Inner core long thought to be solid since this an easy way to ↗ $\alpha(r)$.

Not until 1972 that a claim was made for observation of PKJKP (Julian, Sheppard, Davies) which would be direct proof of solidity.



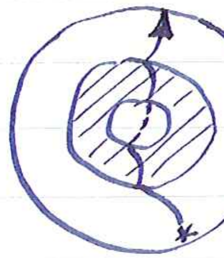
Look for in range $\Delta \sim 230° - 290°$ $T \sim 1/2$ hr. They found a phase in ~ right place.

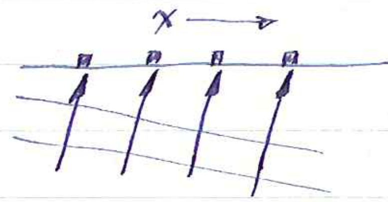
∃ a major unresolved problem with interpretation. They find $\beta \sim 3$ km/s, cannot be reconciled with $\beta \sim 3.5$ km/s from inversion of free oscillations. Has been suggested they may have seen SKJKP, but they checked dT/dh using quakes of different depths. Its a puzzle.

How are weak body phases looked for in general? Two tricks.

1. use deep focus quakes, not obscured by larger surface waves, we do this in our lab.
2. use beam steering of arrays

say want to look for a phase with a certain ray parameter p



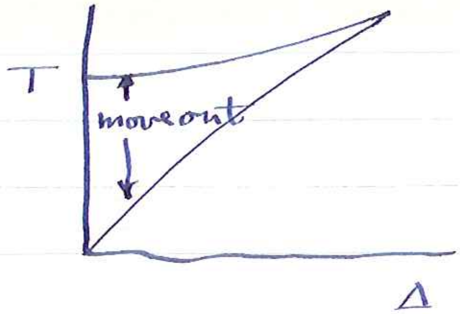


$$p = a\frac{dT}{dx}$$

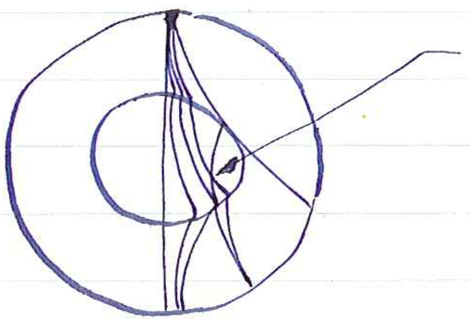
beam steering: simple delay and sum of signals at various seismometers.

<u>Two additional comments:</u>

1. PcP is generally quite a weak signal since <u>reflection coefficient</u> at CM bdry is small (~ 0.1) other 90% is transmitted to become PKP. See Fig. 12 from Buchbinder, disregard data, theoretical value from known contrast in properties, at near vertical incidence almost zero, we look for PcP in lab, can recognize by <u>moveout</u> at several stations, different Δ.

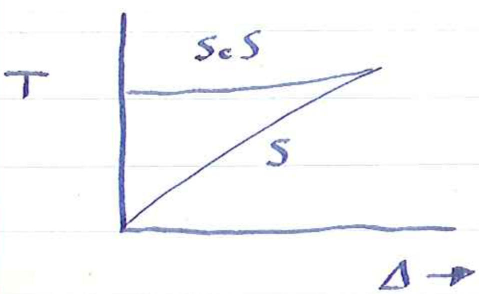


2. Note PKP does not bottom in upper part of core



A ray is most sensitive to velocity near bottoming point. This makes it very difficult to use PKP times to find $\alpha(r)$ in outer half of core. We shall see that SKS does not have this problem, and can be (and is) so used. Instead it has a different problem: SKKKS, etc. all come in at ~ same time.

The conclusions regarding S rays <u>in the mantle</u> are similar, get S and ScS merging at core shadow, but $(ScS)_{SH}$ can be a strong arrival since its reflection coefficient is necessarily 1 ($\exists$ no transmitted SH)

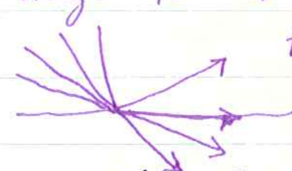


The <u>refracted</u> core waves have <u>quite</u> a different behavior since

$$\beta_{mantle} \sim 7.2 \text{ km/s}$$
$$\alpha_{core} \sim 8.0 \text{ km/s}$$

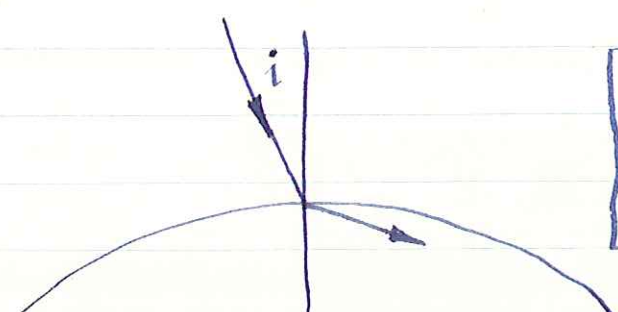
$$\boxed{\beta_{mantle} < \alpha_{core}}$$

discuss critical angle in more detail



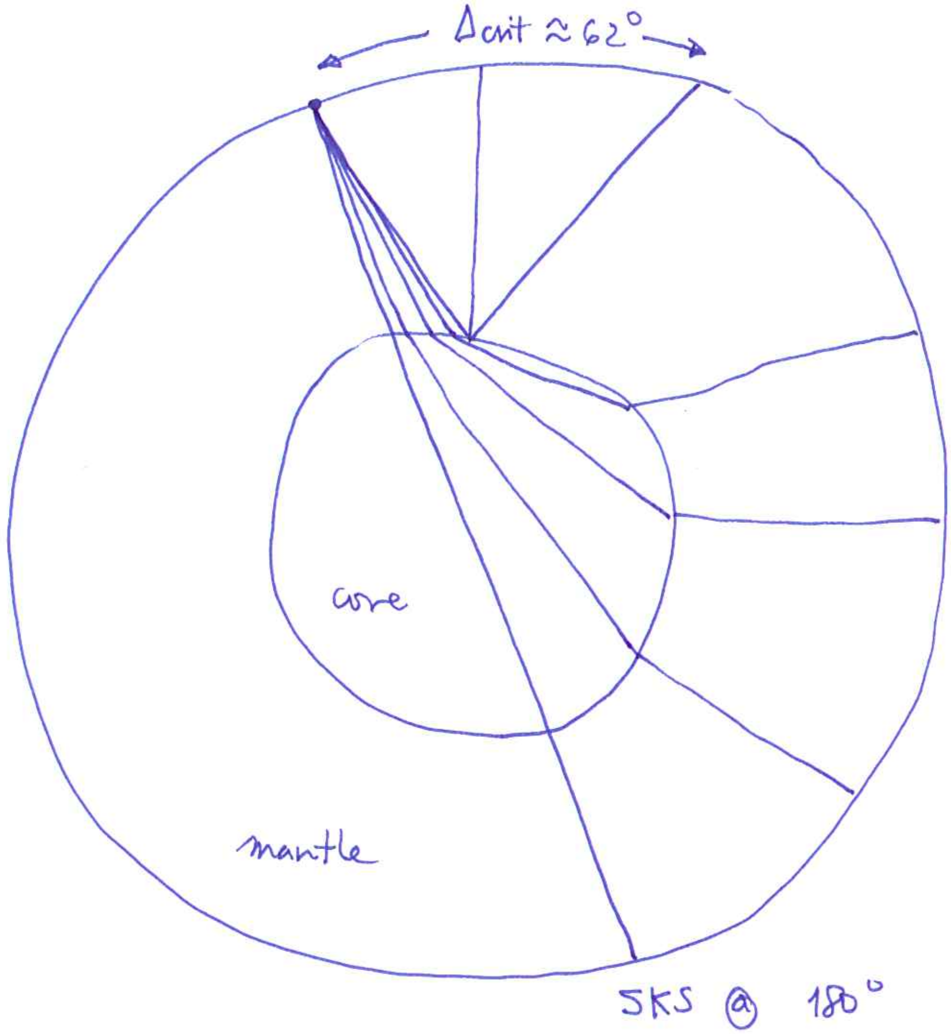
total internal reflection

PKP core rays existed for all incident angles, but now there is a <u>critical angle</u> i_c beyond which no rays are refracted into the core.

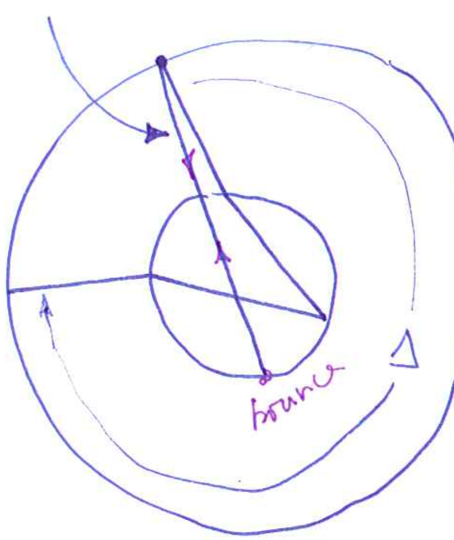


$$\boxed{\begin{array}{l} i_{refr} = 90^\circ \\ \sin i_c = \beta_{mantle} / \alpha_{core} \end{array}}$$

Consider possible SKS rays, starting from 180°:

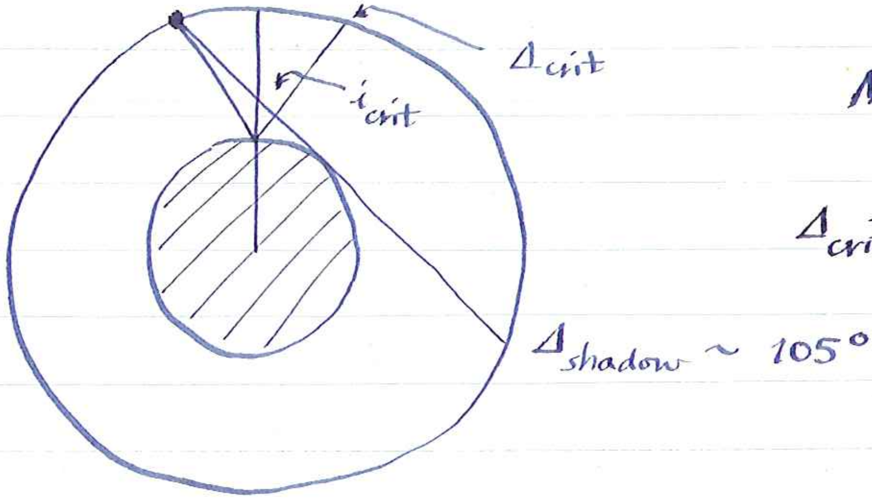


SKKS @ Δ = 360°



SKKS at angles > 180° look like this

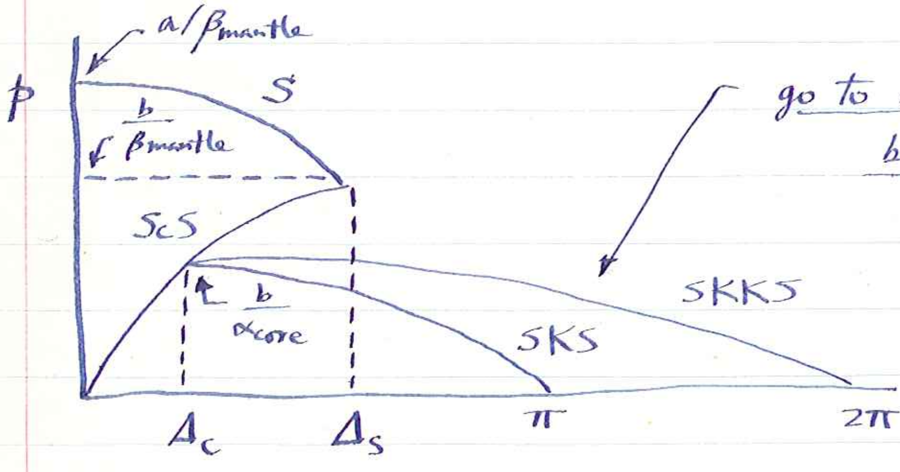
Thus we have, for the <u>homogeneous</u> core and mantle case considered before,



No SKS for $\Delta < \Delta_{crit}$

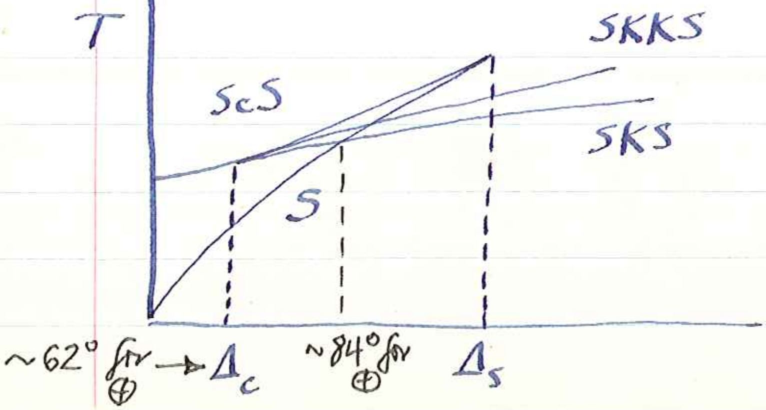
$\Delta_{crit} \sim$ 62° for $\oplus$

For $\Delta = \Delta_{crit}$, SKS and ScS coincide. Plot of p vs. Δ thus looks like:



go to next page before drawing SKKS, etc. in this figure and that below

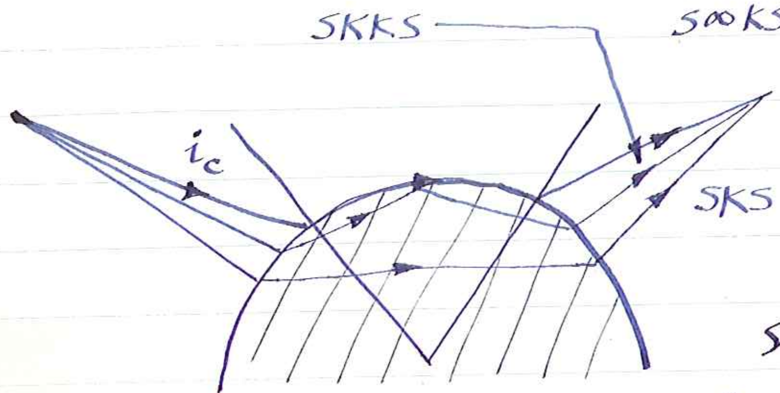
The travel time curve looks like:



$\Delta \sim 84°$ where S and SKS cross.

What about SKKS, SKKKS, etc?

At any $\Delta > \Delta_{crit}$ $\exists$ an ∞ family of rays SKS, SKKS, etc.



$S\infty KS$ suffers an ∞ no. of internal bounces, travels along core-mantle interface

Similar to the whispering gallery phenomenon in St. Paul's Cathedral, London.

The times of the rays SKKS, SKKKS, etc. approach that of $S\infty KS$ which appears to travel along the core-mantle interface

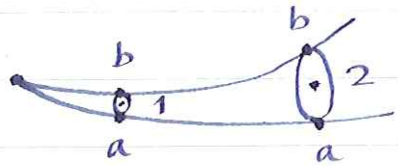
Geometrical ray theory clearly fails adequately to describe this coalescence of rays.

Ray tracing plot for SKKKS: note small azimuth range leaving source gets spread out considerably, amplitudes of SKKKS etc. thus weak.

SnKS has $n-1$ caustics

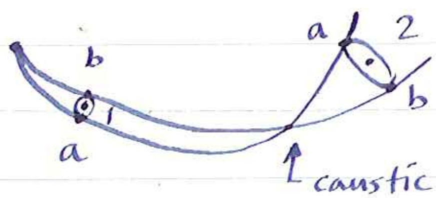
Note caustic surface inside core: locus of crossing points of nearby rays, this surface does not intersect surface of $\oplus$.

Passage of a ray through a caustic gives rise to a <u>$\pi/2$ change in phase</u>.
Reason: consider ray bundle



Amplitude law:
$A_2/A_1 \sim (\text{area }1/\text{area }2)^{1/2}$

Caustic surface is where nearby rays <u>cross</u>



The ray bundle gets turned inside out. The area at 2 is <u>negative</u>.

Thus (this actually just a mnemonic device, not a rigorous derivation):

$$A_2/A_1 = \left(-\left|\frac{\text{area }1}{\text{area }2}\right|\right)^{1/2}$$

$$= i\left|\frac{\text{area }1}{\text{area }2}\right|^{1/2}, \quad i = e^{i\pi/2}$$

a $\pi/2$ <u>phase advance upon passage thru caustic</u>

This not easy to observe with SKKS, SKKKS, etc. since $\exists$ phase shifts at CM bdry as well which depend upon angle i.

But another such situation of an <u>internal caustic</u> is PP, SS etc.

See ray tracing plot for PP; SS would look similar. If attention restricted to SH phase shift upon surface reflection is nil and SS should be phase advanced relative to S by exactly $\pi/2$.

Given a <u>pulse</u> $s(t)$ if every constituent Fourier component is phase shifted uniformly by $\pi/2$ the result is said to be the <u>Hilbert transform</u> of $s(t)$. We may think of passage thru a caustic as a black box which converts $s(t)$ into $H[s(t)]$, its <u>Hilbert transform</u>.

$$\sin\omega t \rightarrow \boxed{\text{Hilbert transform}} \rightarrow \cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

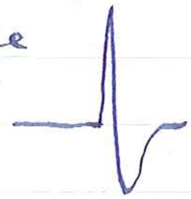
Passage thru 2 such boxes advances phase by π, equivalent to multiplication by -1

$$s(t) \rightarrow \boxed{\text{Hilbert transform}} - \boxed{\text{Hilbert transform}} \rightarrow -s(t) = \sin(\omega t + 180^\circ)$$

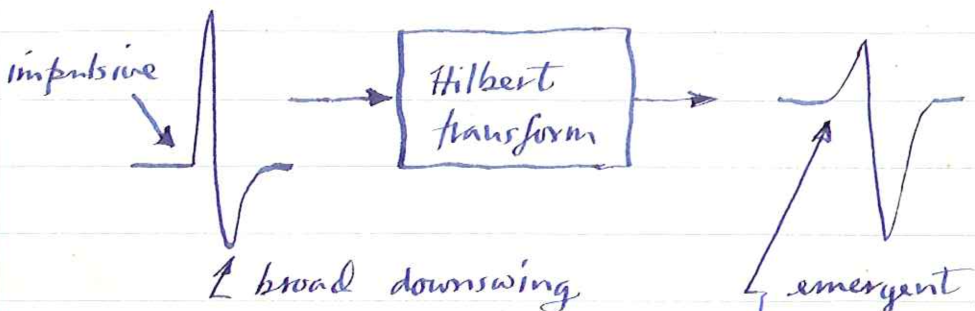
in the case above

Figs. 9 and 10 from Choy + Richards show two transverse S + SS seismograms.

The Hilbert transform of a seismograph-filtered impulse



looks like the signal shown, viz.



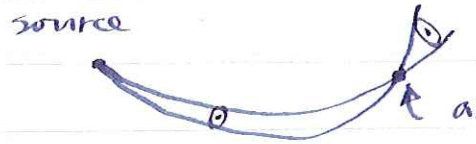
emergent, hard to "pick" first arrival — this associated with fact that SS not a least time path

But Hilbert transformation of the seismogram, accomplished easily in computer, has precisely the expected effect:

1. H[S] takes on the emergent character of SS
2. H[SS] looks like −S, but weaker due to shear attenuation, finite Q_β.

This a particularly clear example of caustic phase shift.

Note that at the caustic itself ray theory predicts an ∞ amplitude



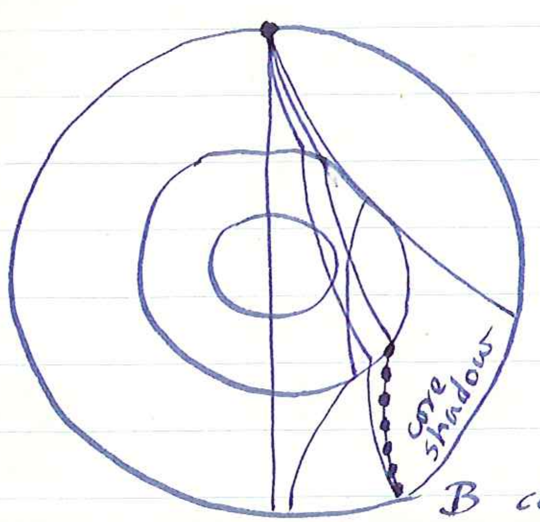
area of ray bundle → zero at caustic

amplitude $\sim 1/\sqrt{\text{area}} \to \infty$

This an example of failure of ray theory, fails near caustics, focal points and shadow boundaries.

At all such locations the finite frequency of the waves must be taken into account.

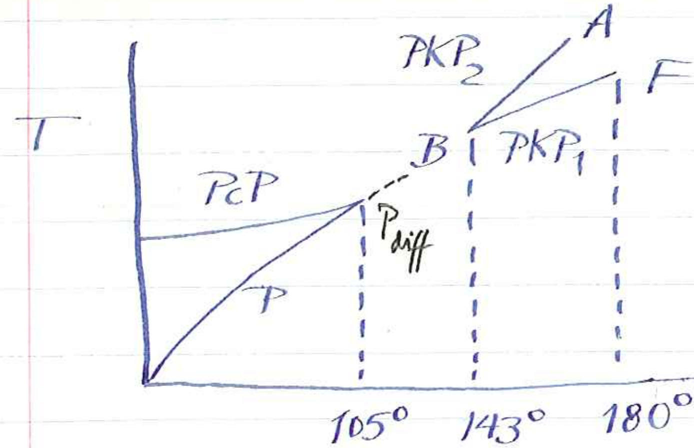
Above problem does not arise for SS etc. since caustic does not intersect ⊕ surface, but PKP caustic does, at $\Delta \sim 143°$.

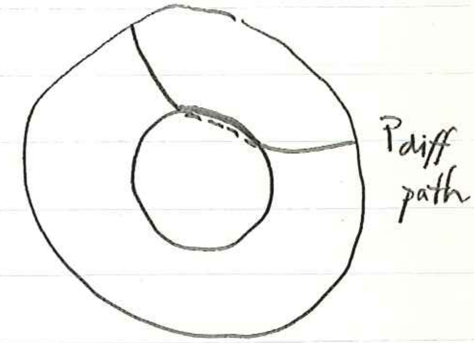


A core shadow $\Delta \sim 105°$

B caustic $\Delta \sim 143°$

F antipode $\Delta = 180°$



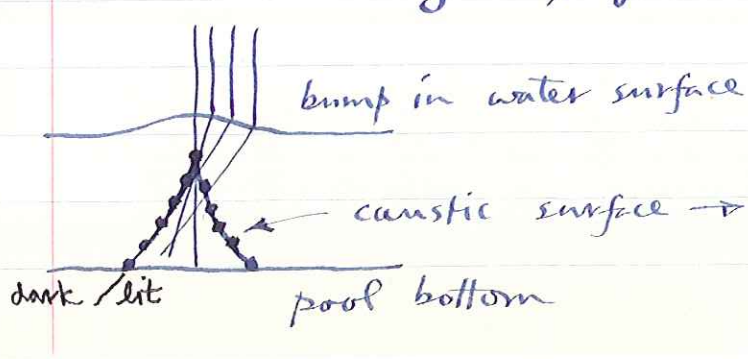


Ray theory predicts ∞ amplitude arrivals at point B, PKP caustic, also predicts sharp shadow bdry at $\Delta \sim 105°$.
Actually diffracted P_{diff} is seen far into the shadow, particularly at long periods.

Amplitude at B is large but not ∞.
Theory for behavior of waves in vicinity of caustic due to Airy, whom we have met in connection with isostasy.

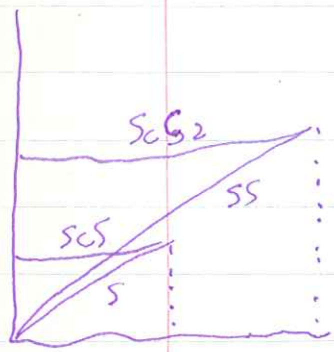
An everyday example of caustics : patterns of dark and light on bottom of a swimming pool on sunny day, due to refraction at air-water interface, focusing and defocusing

term "caustic" $\Rightarrow$ burning, intense



Travel time plot for <u>surface</u> focus shows the various phenomena associated with the <u>core</u> we have been discussing :

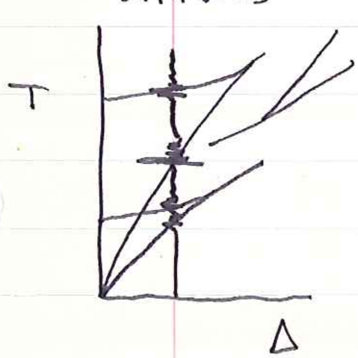
1. P and PcP joining at core shadow , P_{diff} extending way into core shadow , makes it difficult to determine location of shadow boundary



2. PKP_1 and PKP_2 with CD branch (PKiKP) going into core shadow
3. notice also PP , PPP , etc .
4. S and ScS , SKS , SKKS , etc .

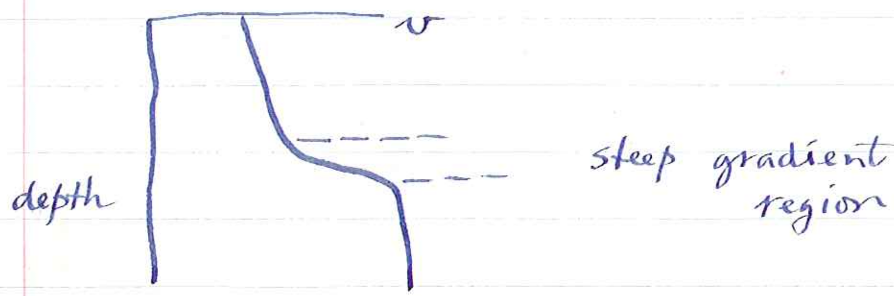
All the above are empirical . Many of the phases shown not necessarily observed . Easy to determine expected times of compound phases e.g. PKPPKP by adding relevant segments together

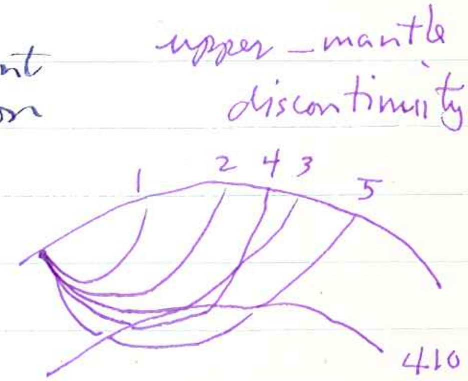
5. LQ and LR , straight lines , constant velocity are approximate arrival times of 20 s period Love and Rayleigh surface waves , dominant signal for oceanic paths , shallow focus events .

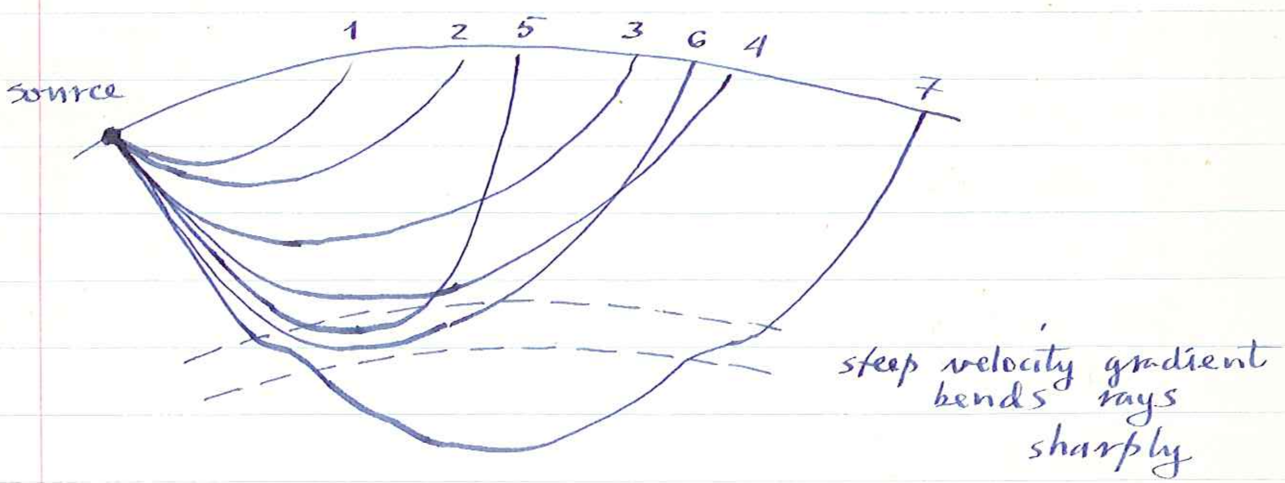


Consider now effect of continuous but rapid changes in velocity.
Also important in ⊕, two cases of ~~[illegible]~~ interest, both occur in upper mantle.

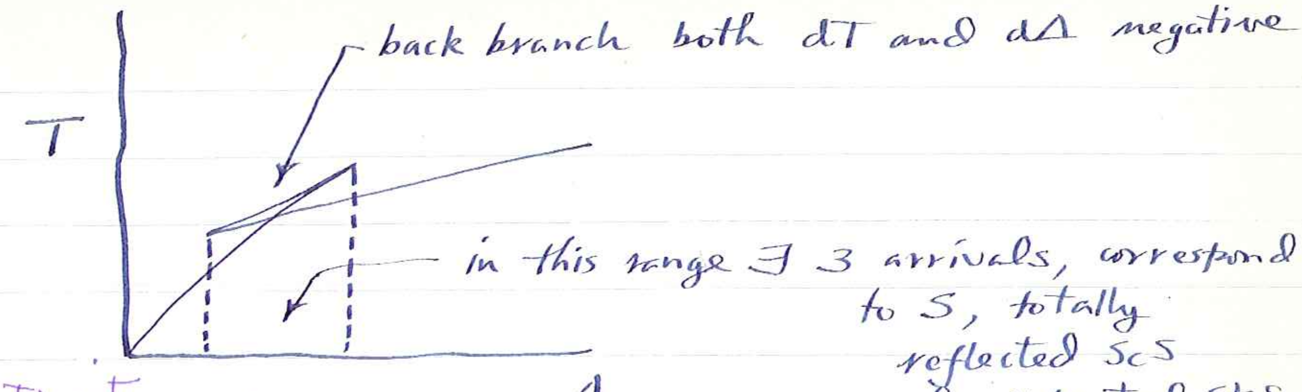
High velocity gradient: similar to S, SKS case. Suppose $v(r)$ looks like

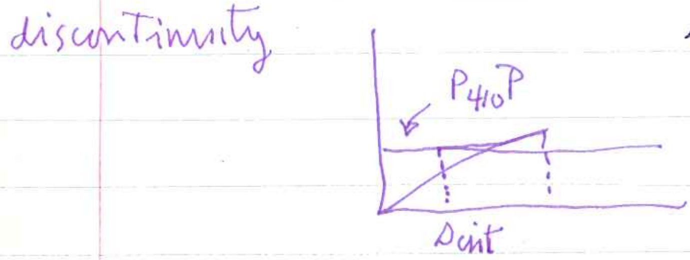




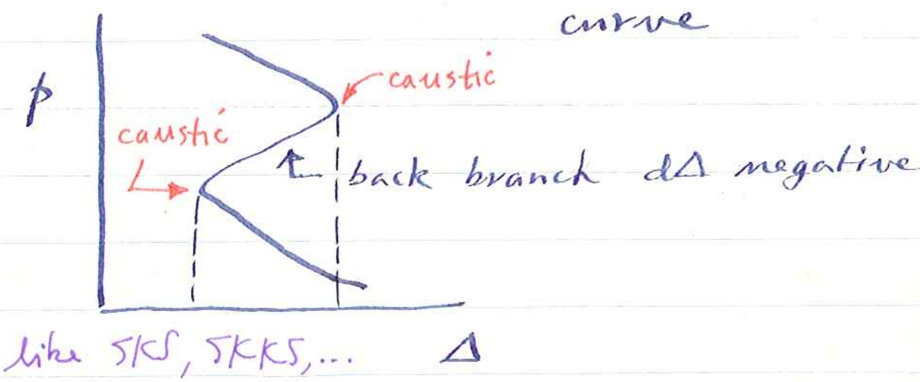


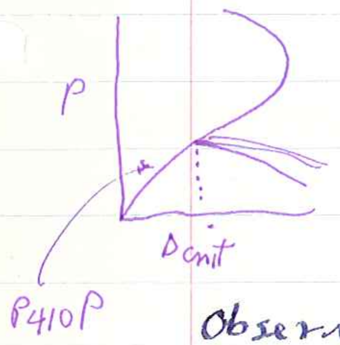
Travel time plot thus looks like:
called a triplication.





$p(\Delta)$ thus looks like, smooth continuous curve





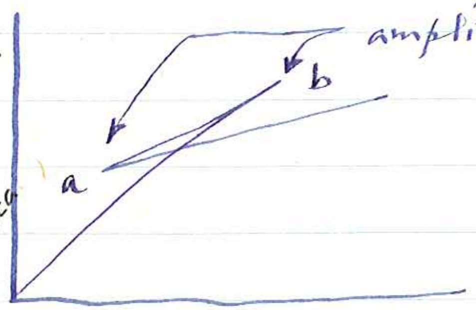
Observed triplications suggest ∃ two major high velocity gradient zones in upper mantle at depths of ~ 410 km and ~ 650 km. Easiest to observe using arrays. Recall Fig. 4 from TFSO study by Lane Johnson, location of epicenters used shown in Fig. 5

One triplication in range $15^\circ \lesssim \Delta \lesssim 20^\circ$ and another in range $20^\circ \lesssim \Delta \lesssim 25^\circ$.

His interpretation CIT204 shown in Fig. 24, two steep gradient zones at ~400 km and ~650 km depth.

To actually see all three arrivals is not easy. Look at ray tracing diagram for Δ out to 40° for model CIT204 from Julian and Anderson Fig. 25.

Spacing of rays at surface an indication of expected amplitude (if shot out from source in equal solid angles). In general for a triplication it is found that



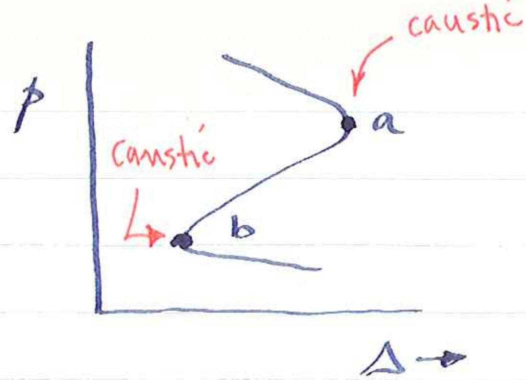
amplitudes largest at point a (corresponds essentially to total internal reflection) next largest at b, weaker between a and b

Look also at Walck & Clayton record section — Scarlet Array — Middle America quakes

Both a and b are caustics (for a smooth $v(r)$); $p(\Delta)$ is smoothly

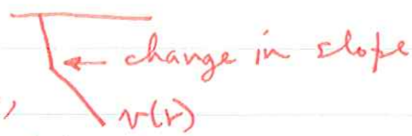
In fact this another example of failure of vertical ray theory, which predicts that

$$\text{amplitude} \sim (d^2T/d\Delta^2)^{1/2}.$$



Ray theory predicts ∞ amplitudes at points a and b since $d^2T/d\Delta^2 = dp/d\Delta \to \infty$ (p vs. Δ turns vertical)

For a model



get, instead,

<u>Modern</u> method of studying upper mantle structure is <u>using</u> <u>synthetic seismograms</u>, try to match <u>amplitudes</u> of later arrivals and other details of waveform.

Example of this method : work of Helmberger and Wiggins LRSM stations in western U.S. , nuclear explosions at NTS , Fig. 2 shows two record sections <u>reduced</u> to velocity of 10.8 degrees / second.
Large amplitude second arrival in range $\Delta \sim 15^\circ - 20^\circ$ very clear , see e.g. station WNSD from Aardvaark , due to "discontinuity" at 400 km depth.

Fig. 16 shows their suggested refinement of CIT204 (__) , p vs. Δ curve with more structure