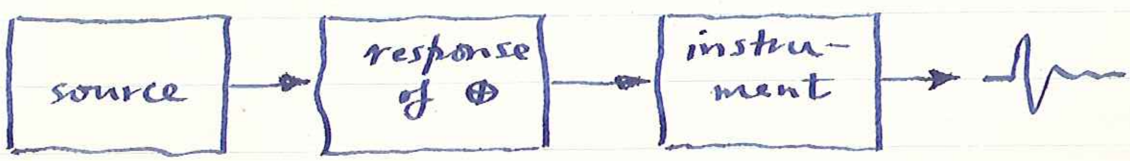
Seismology : introduction

By far our most important tool for probing the $\oplus$'s interior. Many different "types" of seismology, all share same basic picture, namely



The source may be natural (earthquakes) or artificial (explosions, nuclear, quarry blasts, or specially for seismic exploration, Vibroseis®)

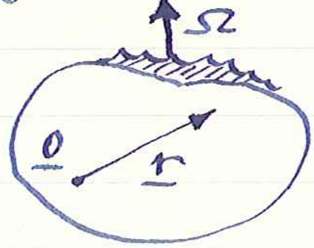
It generates waves which propagate through the $\oplus$ or from an alternative point of view it excites free oscillations of Earth. Response of $\oplus$ to source can be considered linear since seismic displacements are quite small.

The instrument has its own response, hopefully if well designed, linear also. Also its effect is presumably known (instrument response calibrated).

General program : given 1 or N wiggly lines infer elastic and anelastic properties of ⊕ and infer nature of seismic source if it is a quake.

Linear elasticity : a continuum approach

For seismological purposes perfectly elastic and isotropic materials may be characterized by three parameters which may be functions of position within the ⊕



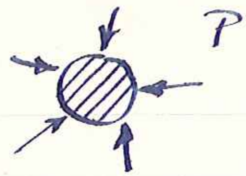
1. mass density $\rho(\underline{r})$
[gm / cm^3]

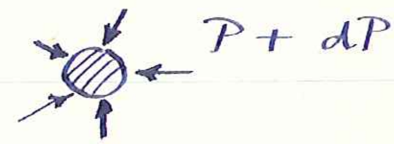
typical values:

granite : $\rho = 2.7$
basalt : $\rho = 2.9$
peridotite ~~dunite~~ : $\rho = 3.3$ (lots of Mg)
mean ⊕ : $\bar{\rho} = 5.517$
Fe : $\rho = 7.9$

There are also two elastic parameters which we have already introduced

2. bulk modulus or incompressibility $\kappa(\underline{r})$: a measure of ability to withstand compression





sample under <u>initial</u> pressure P, volume V (need not be spherical for defn * but if so it is easy to visualize deformation (purely radial).

augment pressure by dP, volume <u>change</u> dV (negative) i.e. new volume is $V + dV$

Then:

$$\boxed{{}^{*}\ \kappa = \left[- \frac{1}{V} \left(\frac{dV}{dP} \right) \right]^{-1}}$$

this makes $\kappa > 0$

this form

or rewritten

$$\boxed{dP = -\kappa \, (dV / V)}$$

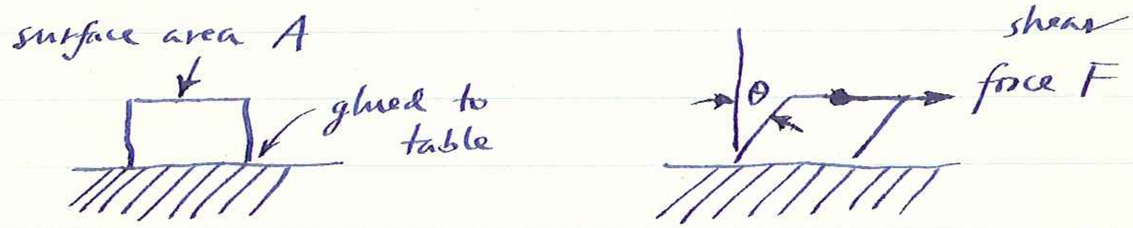
incremental change in pressure

fractional change in volume

	GPa	κ (dyne / cm^2)
granite	55	$5.5 \cdot 10^{11}$
basalt	75	$7.5 \cdot 10^{11}$
peridotite ~~dunite~~	100	$10. \cdot 10^{11}$
steel	170	$17 \cdot 10^{11}$
H_2O	2	$0.2 \cdot 10^{11}$

in general the denser a material the more incompressible

3. <u>rigidity</u> or <u>shear modulus</u>
$\mu(\underline{r})$ also dyne / cm^2 :
a measure of ability to
withstand <u>shear</u> stress, a
thought experiment



The block is sheared, angle θ
is a measure of shear

$$F/A = 2\mu\theta$$

conventional to put 2 here

shear force per unit area

shear of block

$$\boxed{\mu = \frac{F/A}{2\theta}}$$

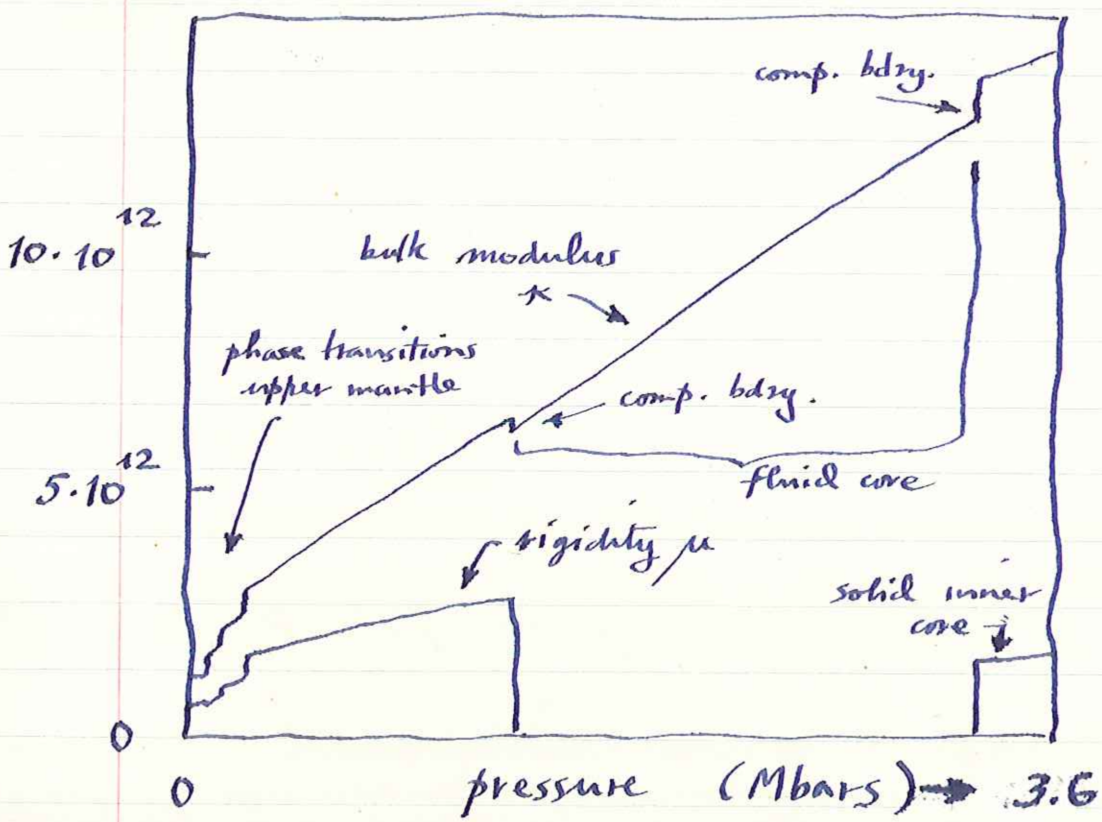
ratio of shear force to shear deformation just as K is ratio compressive force to ~~[illegible]~~ compressive deformation

$$\boxed{F/A = 2\mu\theta}$$

A <u>fluid</u> by definition has <u>zero rigidity</u>.

	μ (dyne/cm²)	
granite	20	$2 \cdot 10^{11}$
basalt	40	$4 \cdot 10^{11}$
peridotite ~~dunite~~	80	$8 \cdot 10^{11}$
	↑ GPa	

These are "typical" values of geological materials at <u>1 atmosphere</u>.
A given material becomes more rigid and more incompressible (and of course denser) when the <u>initial</u> pressure acting on it is increased.
This accounts for most but not all of the <u>increase with depth</u> of ρ, κ and μ <u>in the mantle</u>.



1 Mbar $= 10^6$ bars

1 bar $= 10^6$ dyne/cm²

1 Pa = 10 dyne/cm²

A major goal of seismology, esp. global seismology, is the determination of variation of $\rho(r)$, $\kappa(r)$ and $\mu(r)$ with depth in ⊕. Will discuss how this is done in detail.

Isotropic materials are those which "look the same" or more accurately respond the same" in all directions. Minerals are typically anisotropic but rocks are aggregates of randomly oriented minerals and thus are usually reasonably isotropic. Evidence for crustal and upper mantle anisotropy exists but as first approximation we'll assume isotropic. A perfectly elastic ⊕ has only 2 independent elastic parameters, 1 for compression and one for shear.

There are other related, not independent parameters, e.g. Lamé's parameters λ and μ, the former defined by

$$\boxed{\lambda = \kappa - \frac{2}{3}\mu}$$

units again dyne/cm^2

Poisson's ratio ν : dimensionless

stretch a rod

$$\boxed{\nu \equiv \frac{\text{decrease in either lateral dimension}}{\text{increase in length}}}$$

Can show that $\boxed{\nu = \dfrac{\kappa - \frac{2}{3}\mu}{2(\kappa + \frac{1}{3}\mu)}}$

fluid : $\nu = 1/2$

For most common materials $\nu \sim 1/4$,
corresponds to $\lambda = \mu$ or $\kappa = 5/3\,\mu$
Note from above tables this true for granite, basalt, etc.

Seismic wave propagation

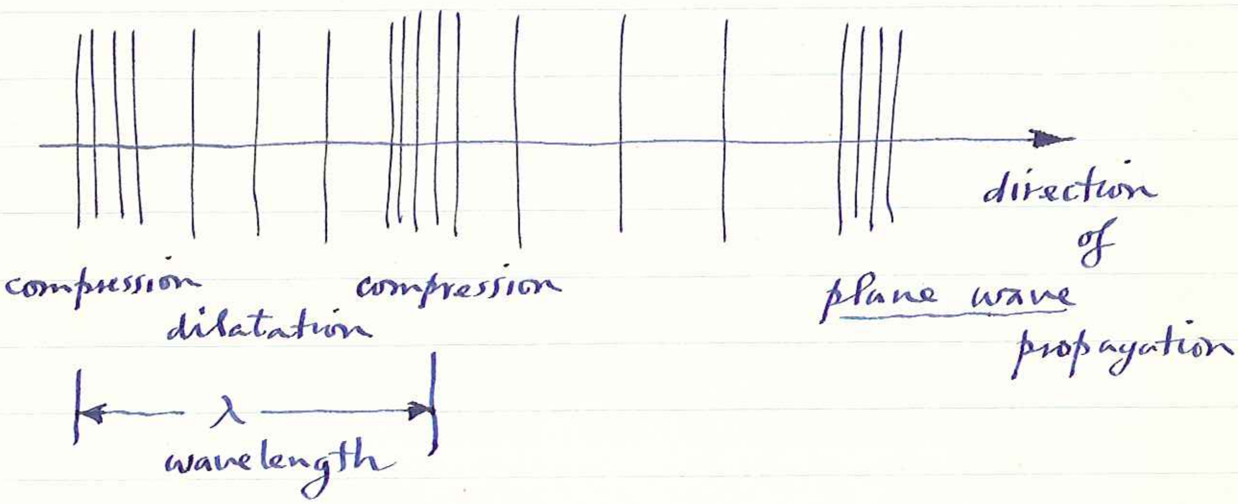
In an isotropic and homogeneous material : $\kappa(\underline{r}) = \kappa$

$$\mu(\underline{r}) = \mu$$

$$\rho(\underline{r}) = \rho$$

two types of seismic waves can propagate.

1. <u>P waves</u> (P stands for <u>prim</u>ary historically, as these are the first to arrive after an earthquake, can also stand mnemonically for pressure)



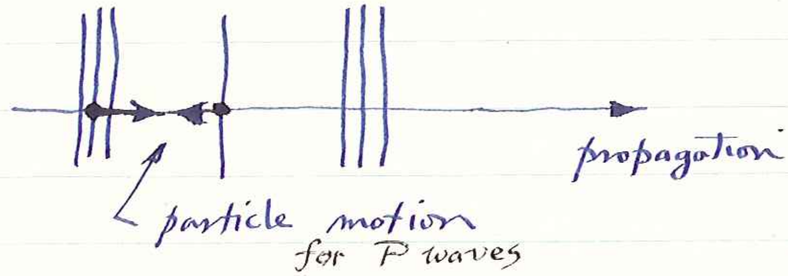
These are waves of compression and dilatation, although shearing motions are involved also. These are basically <u>sound waves</u>

The speed of P waves is

$$\boxed{\alpha = v_p = \left(\frac{K + \frac{4}{3}\mu}{\rho} \right)^{1/2}}$$

Depends on both K and μ as both compression and shear are involved. These can propagate in a <u>fluid</u> ($\mu = 0$) e.g. in air or H_2O (sound waves)

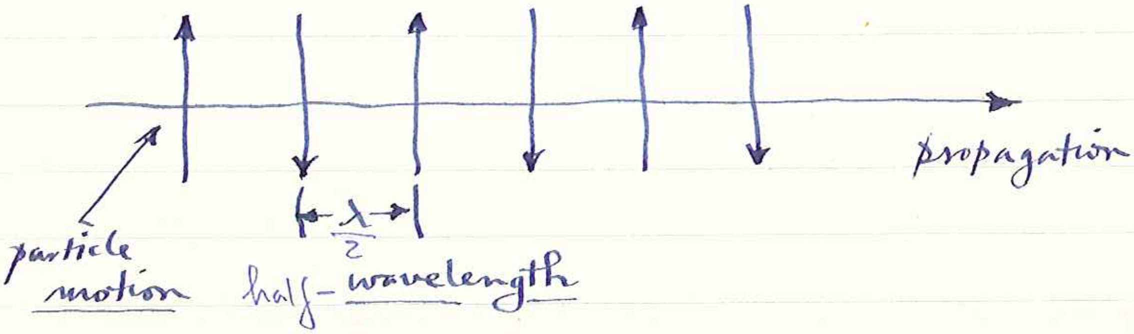
Particle motions in a shear wave are parallel to the direction of propagation



2. S waves (stands for secondary, also mnemonically for shear), pure shear, speed depends only on rigidity μ

$$\boxed{\beta = v_s = (\mu/\rho)^{1/2}}$$

Particle motions are transverse or $\perp$ to the direction of propagation



The orientation of the particle motion is referred to as the polarization.

Note that both κ and μ must by defn. be positive:

volume must decrease when pressure applied

block must shear in direction shown for force F

Thus α must be greater than β for any (stable) material. For a Poisson solid $\lambda = \mu$ or $\nu = 1/4$

$\boxed{\alpha \approx 1.7\beta}$

$\alpha = \sqrt{3}\,\beta = 1.732\,\beta$. This a good rule of thumb, P waves about 1.7 times faster than shear waves.

	α (km/s)	β (km/s)
granite	6.0	3.6
basalt	7.0	3.8 – 3.9
peridotite ~~dunite~~	7.9 – 8.1	4.6

this, again, at 1 atm.

Kinematic properties of plane waves:

The variation with time and space of, say, particle displacements in a wave (or pressure in a P wave) is of the form (for monochromatic wave)

$$\boxed{A \cos(\omega t - kx)}$$

A – amplitude

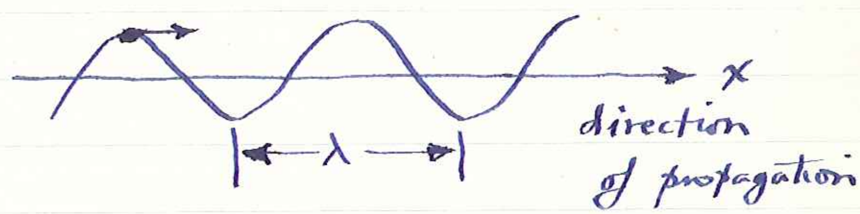
k – wavenumber (radians / km)

ω – angular frequency of wave (radians / sec)

Snapshot at time t:

period $T = \frac{2\pi}{\omega}$

wavelength $\lambda = \frac{2\pi}{k}$



The speed of wave crests (• in picture) called the phase speed is

$$\boxed{v = \omega / k = \frac{\lambda}{T}}$$

v – either α for P waves or β for S waves

The <u>wavelength</u> $\boxed{\lambda = 2\pi/k \quad \text{in km.}}$

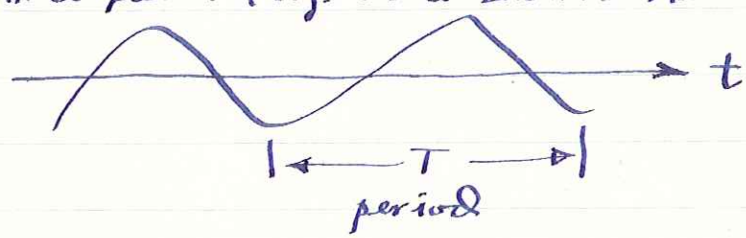
Can also write in form

$$A \cos \omega \underbrace{(t - x/v)}$$

this takes the same value at points $x = vt$, this the speed of wave crests

At a fixed x, the <u>variation in time</u> also sinusoidal, frequency ω.

variation with t
at fixed posn. (e.g. at a seismic station)



$$\boxed{\text{period } T = 2\pi/\omega}$$

Can also write speed in terms of λ and T, viz

$$\boxed{c = \lambda/T}$$

peridotite (upper mantle)

Consider granite, e.g. $\alpha =$ ~~6.0~~ 8.0 km/sec

freq (Hz)	period (sec)	wavelength λ_P	application
~~10~~	~~0.1~~	~~600 m~~	~~prospecting 10 Hz and up~~
1	1	8 ~~6~~ km	short period WWSSN
0.1	10	80 ~~60~~ km	long period WWSSN

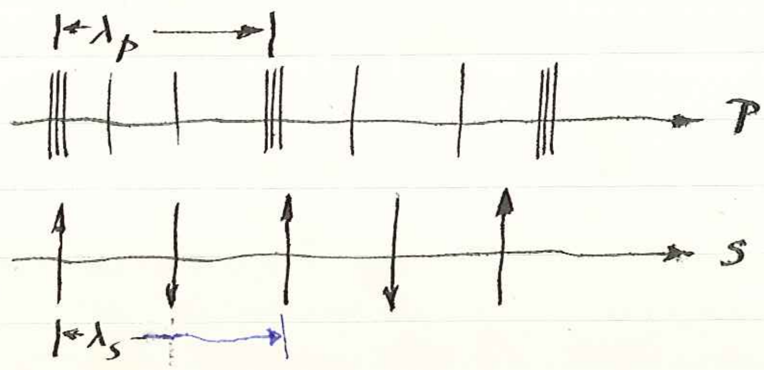
In the seismology labs will be looking at waves in the range 1–10 sec with λ in the range 6 – 60 km. Then we'll look at longer wavelength surface waves.

For a given period T or frequency ω S waves are shorter wavelength than P waves. We'll look at both in the lab.

$$\lambda = cT$$

$$\lambda_p / \lambda_s = \alpha / \beta \sim 1.7$$

S waves are ~1.7 times shorter wavelength



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Anelasticity and damping of seismic waves

No real material is perfectly elastic. If a block were perfectly elastic it would require no net work to flex it back and forth with a harmonically varying force

Any real block will dissipate energy due to internal friction. On a microscopic level might be, e.g. grain boundary friction or bowing of pinned dislocations. The dissipated energy appears as heat in the block. When the block is deformed there is stored elastic energy in it. The intrinsic Q of a material is defined by

$$\boxed{Q = 4\pi \frac{\text{average stored energy}}{\text{energy dissipated per cycle}}}$$

Thus $4\pi Q^{-1}$ is the percentage of the <u>average</u> energy dissipated per cycle. <u>Dimensionless</u>.

In the above experiment the deformation is <u>pure shear</u> and the Q is called the <u>shear Q</u>, <u>usually denoted by Q_μ</u>.

In principle energy can also be dissipated in <u>pure compression</u>

There are thus for an isotropic material two intrinsic Q's

$$\boxed{\begin{array}{lcl} Q_\mu & : & \text{shear} \\ Q_K & : & \text{bulk} \end{array}}$$

There is some (not completely convincing) evidence that Q_K in the mantle is finite, namely the Q's of the radial modes which have very little shear.

<u>Reason</u>: bulk $\frac{dV}{V} \Rightarrow$ shears on a grain-size scale.

But for a first approx. it suffices to assume that:

$$\boxed{Q_K(\underline{r}) = \infty \quad \text{throughout } \oplus}$$

The shear Q_μ can vary in the $\oplus$ (must be ∞ in the fluid core where there is no shear), depends strongly on temp. T, dislocation density, impurities, etc.

Thus 4 parameters seismology seeks to determine and which characterize the middle box completely:

$$\text{source} \longrightarrow \boxed{\oplus\text{'s response}} \longrightarrow \text{instrument}$$

$\rho(\underline{r})$ density
$\kappa(\underline{r})$ bulk modulus
$\mu(\underline{r})$ rigidity
$Q_\mu(\underline{r})$ shear Q

these three can be replaced by others, common to use instead

$\rho(\underline{r})$, $\alpha(\underline{r})$, $\beta(\underline{r})$
$\left(\alpha^2 = \dfrac{\kappa + 4/3\,\mu}{\rho}, \quad \beta^2 = \dfrac{\mu}{\rho}\right)$

From a rather abstract point of view this is all seismology can ever tell us about the ⊕, the middle box, the variation of these four parameters as fcn. of posn. within the ⊕. The principal variation is with depth but there are also <u>lateral differences</u> which are of interest, e.g. that between continents and oceans, subducted slabs, etc.

<u>Attenuation of seismic waves</u>: a consequence of finite Q.

In a non-attenuating medium a plane wave propagates with no diminution of its amplitude

$$\underbrace{A}_{\text{amplitude}} \cos(\omega t - kx)$$

A trick for determining the amplitude decay which we shall not prove but merely assert is the following:

1. write signal in the form

$$\mathrm{Re}\left[A e^{i(\omega t - kx)} \right]$$

$$= \mathrm{Re}\left[A e^{i\omega(t - x/v)} \right]$$

2. replace the real moduli κ and μ by

$$\boxed{\kappa \to \kappa(1 + i/Q_\kappa)}$$

$$\boxed{\mu \to \mu(1 + i/Q_\mu)}$$

i.e. $1/Q_\mu$ is the <u>imaginary part</u> of the shear modulus

Consider first an <u>S wave</u>, velocity $\beta = (\mu/\rho)^{1/2}$.

It is essentially always a good approximation to assume that $Q_\mu \gg 1$, fails only in very wet sediments, etc., but a good approx. for any igneous rock, for mantle, etc.

$$\beta \to \left[\frac{\mu(1 + i/Q_\mu)}{\rho} \right]^{1/2}$$

$$= \beta(1 + i/Q_\mu)^{1/2}$$

$$\approx \beta(1 + i/2Q_\mu)$$

$$\boxed{\beta \to \beta(1 + i/2Q_\mu)}$$

Thus $\quad t - \dfrac{x}{\beta} \to t - \dfrac{x}{\beta(1 + i/2Q_\mu)}$

$$\approx t - \frac{x}{\beta} + i\frac{x}{\beta 2Q_\mu}$$

$$\mathrm{Re}\left[A e^{i\omega(t - x/\beta)} \right] \to$$

$$A e^{-\frac{\omega x}{2\beta Q_\mu}} \cos(\omega t - kx)$$

decay of amplitude of wave with distance, can also write as

since $\dfrac{\omega}{\beta} = k = \dfrac{2\pi}{\lambda}$

$$\boxed{A \exp\left(-\frac{\pi}{Q_\mu}\frac{x}{\lambda}\right) \cos(\omega t - kx)}$$

decays by $1/e$ after a propagation distance $x = (Q_\mu/\pi)\lambda$, i.e. Q_μ/π wavelengths

There is some evidence that Q_μ goes up above 1 Hz; important for detection of "decoupled" explosions.

A typical value for Q_μ in the mantle is $Q_\mu \sim 300$. A 10 s wave with an S wave velocity of 6 km/sec in mantle and a λ of 60 km will thus travel about 6000 km $\sim$ radius of $\oplus$ before attenuating by $1/e$. A 1 s wave will only go 600 km, higher frequency waves are attenuated more quickly.

Now consider a P wave (with $Q_K = \infty$):

$$\alpha \to \left[\frac{K + \frac{4}{3}\mu(1 + i/Q_\mu)}{\rho} \right]^{1/2}$$

$$= \left[\frac{(K + 4/3\,\mu)\left(1 + \frac{4/3\,\mu}{K + \frac{4}{3}\mu}\,\frac{i}{Q_\mu}\right)}{\rho} \right]^{1/2}$$

$$\approx \alpha \left(1 + i\,\frac{4/3\,\mu}{2(K + \frac{4}{3}\mu)}\,Q_\mu^{-1}\right)$$

$$\boxed{\alpha \to \alpha\left[1 + i/2Q_\mu\,(3\alpha^2/4\beta^2)\right]}$$

Q_μ also called Q_β in study of body wave attenuation.

One defines for P waves

$$\boxed{Q_\alpha = (3\alpha^2/4\beta^2)\,Q_\beta}$$

ratio of Q for P waves to Q for S waves if $Q_K = \infty$

Then:

S waves: $$A \exp\left(-\frac{\omega x}{2\beta Q_\beta}\right)\cos(\omega t - kx)$$
$$= A \exp\left(-\frac{\pi x}{Q_\beta \lambda}\right)\cos(\omega t - kx)$$

P waves: $$A \exp\left(-\frac{\omega x}{2\alpha Q_\alpha}\right)\cos(\omega t - kx)$$
$$= A \exp\left(-\frac{\pi x}{Q_\alpha \lambda}\right)\cos(\omega t - kx)$$

For a Poisson solid $\alpha^2 = 3\beta^2$ and

$$\boxed{Q_\alpha = \frac{9}{4} Q_\beta = 2.25\, Q_\beta}$$

Q_α is bigger than Q_β because deformation in an S wave is pure shear while that in an S wave only partly shear.

In a given distance x how much more rapidly does an S wave of a given frequency decay?
Two reasons for difference:

1. $Q_\alpha > Q_\beta$
2. λ for S waves shorter because they are slower and $\therefore \exists$ more stress cycles in a given path.

If we write <u>amplitude decay</u> as

$Ae^{-\gamma x}$, then:

$$\gamma_P = \frac{\omega}{2\alpha Q_\alpha}, \quad \gamma_S = \frac{\omega}{2\beta Q_\beta}$$

same

$$\boxed{\gamma_S / \gamma_P = \frac{\alpha Q_\alpha}{\beta Q_\beta} = \frac{3}{4}\frac{\alpha^3}{\beta^3}}$$

For a Poisson solid $\gamma_S / \gamma_P \sim 3.9$
S waves should decay $\sim$ 4 times as fast as P waves over a given path.

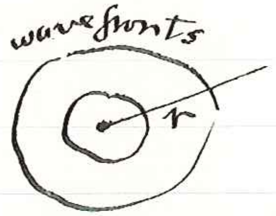
Seismic body waves:

The subject we shall study in greater detail is global seismology, used to infer global properties of $\oplus$ and nature of natural sources, earthquakes.

Large nuclear explosions have also been employed in travel time studies.

The most useful way of analyzing and comprehending seismic motion in range $\lesssim 40\,s$ is as propagating body waves.

Consider first a point source earthquake in an ∞ homogeneous medium. Energy spreads out from the hypocenter



both P and S waves will (in gen'l) be generated, spread out at speeds α and β, respectively

The wave equation for the P waves can be shown to be

$$\boxed{\begin{array}{l} \nabla^2 \theta(\underline{r}, t) = \dfrac{1}{\alpha^2}\, \partial_t^2\, \theta(\underline{r}, t) \\ \qquad\qquad \text{where } \quad \theta \equiv \nabla \cdot \underline{s}(\underline{r}, t), \\ \qquad\qquad \text{the dilatation} \\ \star \quad \text{Same eqn for pressure } p(\underline{r}, t) \end{array}}$$

This valid everywhere except at the source $r = 0$. Recall this in <u>homog.</u> medium :

$$\alpha = \left(\frac{\kappa + 4/3\mu}{\rho} \right)^{1/2} = \text{const}.$$

General solution of $\star$ is

$$\theta(\underline{r}, t) = \frac{1}{r} f(r - \alpha t) + \frac{1}{r} g(r + \alpha t),$$

f and g arbitrary fcns

$g(r + \alpha t)$ ~~outward~~ inward traveling, non-physical

Check : for a function independent of θ, ϕ we have

$$\boxed{\nabla^2 = \frac{1}{r^2}\, \partial_r\, (r^2\, \partial_r)} \quad \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)$$

$$\nabla^2 \left[\frac{1}{r} f(r - \alpha t) \right] =$$

$$\frac{1}{r^2} \frac{\partial}{\partial r}(rf)' = \frac{1}{r^2} \frac{\partial}{\partial r} (rf' - f)$$

$'$ and ∂_r are same

Here $f' =$ derivative of f

$$= \frac{1}{r} f'' + \frac{1}{r^2} f' - \frac{1}{r^2} f' = \frac{1}{r} f''$$

equal, checks

And $\frac{1}{\alpha^2} \partial_t^2 \left[\frac{1}{r} f(r - \alpha t) \right] = \frac{1}{r} f''$

Thus the general solution is

$$\boxed{\theta(\underline{r}, t) = \frac{1}{r} f(r - \alpha t)}$$

travels outward with velocity α

amplitude falls off like $\frac{1}{r}$

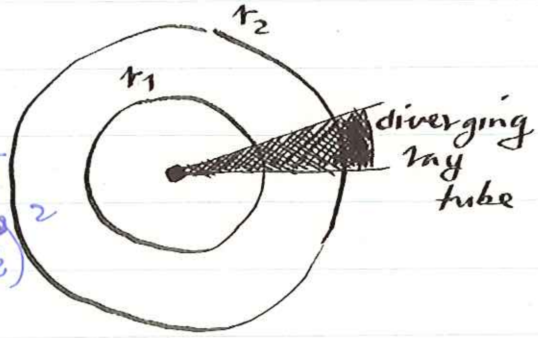
The $\frac{1}{r}$ falloff called <u>geometrical attenuation</u>.

Simple physical interpretation. The energy of wave motion is

$$\boxed{\text{energy} \sim (\text{amplitude})^2}$$

Thus energy $\sim \frac{1}{r^2}$.

Geometrical attenuation thus just a consequence of energy conservation. Consider two spherical surfaces, radii r_1 and r_2. The only source of radiated energy at the hypocenter.



energy × area = const

$$E_1 (4\pi r_1^2) = E_2 (4\pi r_2^2)$$

$$\frac{E_2}{E_1} = \left(\frac{r_1}{r_2}\right)^2$$

Thus the total energy passing thru the 2 spheres must be the same. Ratio of areas is $(r_2/r_1)^2$. Thus ratio of seismic energy / unit area in the wavefronts must be $(r_1/r_2)^2$, or

$$\boxed{\text{energy / unit area} \sim \frac{1}{r^2}}$$

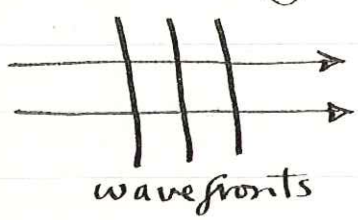
Same law for intensity of light emitted by a lightbulb, for example. We now know essentially everything about wave propagation in homogeneous media:

1. two types of waves, P and S, velocities α and β.
2. waves generated by point sources suffer geometrical

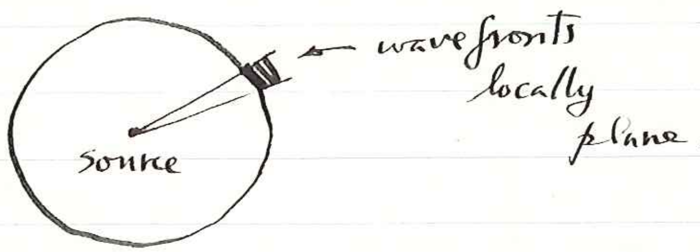
attenuation, amplitude $\sim \frac{1}{r}$,
energy $\sim \frac{1}{r^2}$.

3. anelasticity causes further anelastic attenuation $\sim e^{-\alpha r}$, should call this γ not α
where $\overset{\gamma_S}{\alpha_S} / \overset{\gamma_P}{\alpha_P} = \frac{3}{4}(\alpha/\beta)^3 \sim 3.9$.
The region between r_1 and r_2 absorbs this energy. Actually, $\gamma = \frac{\omega}{2vQ}$.

We previously discussed anelastic attenuation in context of plane waves

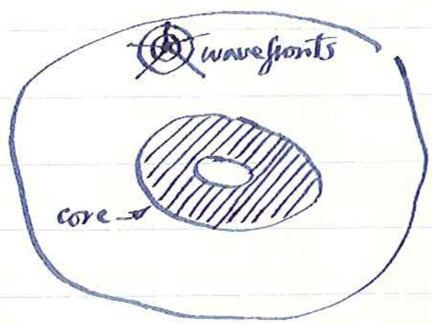


Point sources generate spherical waves but several wavelengths from the source these appear to be locally plane and our reasoning about anelastic attenuation is valid.

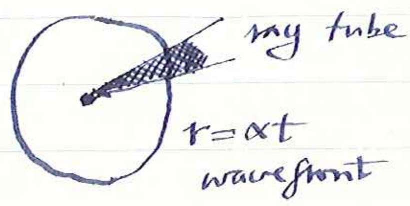


Now consider an earthquake in the Earth, not homogeneous, α and β functions of radius.

Near the source can think of as essentially locally homogeneous. Thus wavefronts ~ spherical: But then what happens? A very good approximation is provided by the theory of <u>geometrical optics</u> or <u>seismic ray theory</u>.



Near the source can think of energy as propagating along straight <u>rays</u> with velocity either α or β. After time t disturbance confined to sphere of radius αt or βt



The amplitude falls off like

$$\boxed{\text{amplitude} \sim (\text{energy})^{1/2} \sim \frac{1}{r} \sim \frac{1}{\sqrt{\text{area of ray tube}}}}$$

Geometrical optics provides the generalization of this to inhomogeneous media.

<u>Conditions for validity</u> of geometrical optics are: scale length L of changes in $\kappa(\underline{r})$, $\mu(\underline{r})$ or $\rho(\underline{r})$ must be much greater than λ = wavelength of waves

$$\boxed{\lambda \ll L}$$

Under these conditions propagation still along rays, that along a given ray essentially independent of other rays.

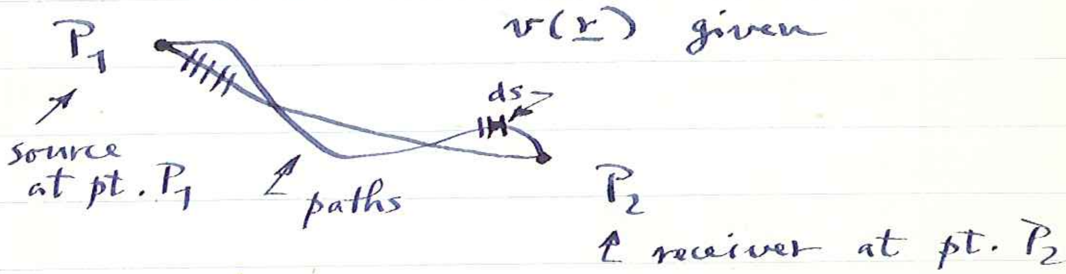
"Typical" seismic wavelength $\alpha \sim 8$ km/s upper mantle, long-period body wave $T \sim 10$ s, $\lambda \sim \alpha T \sim 80$ km.

<u>Ray approximation</u> will be good almost everywhere since changes in ρ, κ, μ are small on this scale, except for a few sharp <u>discontinuities</u>, e.g. Moho and core-mantle boundary, <u>to which will return later</u>. An extension of geometrical optics allows us to deal simply with these also.

The two <u>laws</u> of <u>ray theory</u> are:

1. <u>Fermat's principle</u> of stationary time: of all possible paths between 2 fixed points the actual ray paths are those for which the travel time achieves a stationary value (usually but not always a minimum)

To be more specific, say $v(\underline{r})$ is given everywhere in space and say that $\lambda \ll L$.



Now parameterize paths by <u>arclength</u> s along path. Then velocity of wavefronts along a possible path is $v(s)$. The <u>time</u> to travel a path is

$$\boxed{T = \int_{P_1}^{P_2} \frac{ds}{v(s)}}$$

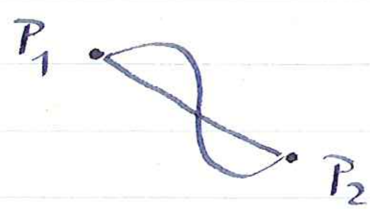
in a sense, $\frac{\delta T}{\delta \text{path}} = 0$.

$$\boxed{\text{\underline{Fermat's} principle : } \delta T = 0 \text{ for a small change in the path.}}$$

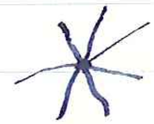
Note : this agrees with what we know for a <u>homogeneous</u> medium, $v(\underline{r}) = v = \text{const.}$

$$T = \int_{P_1}^{P_2} \frac{ds}{v} = \frac{1}{v} \underbrace{\int_{P_1}^{P_2} ds}_{\text{arclength}}$$

$$T = v \times \text{arclength}$$



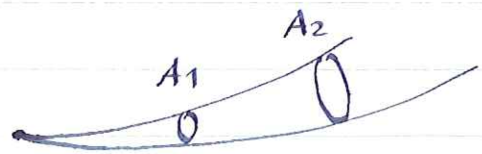
Path with <u>shortest</u> arclength is <u>straight line</u> : rays are straight lines in homog. media, radiate straight outward from source



In this case the stationary value is a <u>minimum time path</u>.

2. Geometrical attenuation law:
This law governs the wave amplitude along a ray in the absence of attenuation.

Fermat's principle determines the paths and the propagation times along them.



A_i = cross-sectional area of a ray tube or bundle of rays

The geometrical attenuation law states that:

$$\boxed{\frac{\text{amplitude at 2}}{\text{amplitude at 1}} = \left(\frac{A_1}{A_2}\right)^{1/2}}$$

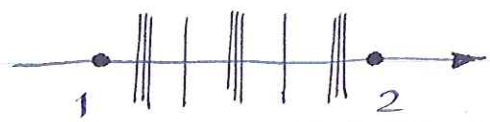
Energy ~ 1/area
Amplitude ~ $1/\sqrt{\text{area}}$

This agrees with and generalizes $1/r$ falloff in homog. media.

The additional attenuation due to anelasticity can also be easily calculated in the ray approximation.

skip this

For a straight ray in a homogeneous medium we have seen it is



frequency (rad/s) of wave

distance

$$\frac{\text{amplitude at 2}}{\text{amplitude at 1}} = e^{-\frac{\omega x}{2 v Q}}$$

either Q_α or Q_β

either α or β

The obvious generalization of this if both $v(\underline{r})$ and $Q(\underline{r})$ are variable is

$$\frac{\text{amplitude at 2}}{\text{amplitude at 1}} = e^{-\omega \int_{P_1}^{P_2} \frac{ds}{2 v Q}}$$

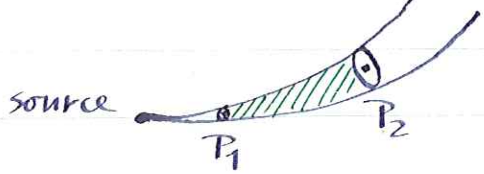
It is conventional to call the integral above

$Q(\underline{r})$ fcn. of posn., e.g. LVZ is a low Q zone as well

$$\boxed{t^* \equiv \int_{P_1}^{P_2} \frac{ds}{2 v Q}}$$

If $Q(\underline{r}) = Q$, constant, then $t^* = T/2Q$ where T = travel time.

The *total* expression for the variation in amplitude along a ray is thus



this energy is absorbed within the shaded region

$$\boxed{\frac{\text{ampl. at } 2}{\text{ampl. at } 1} = \left(\frac{\text{area } A_1}{\text{area } A_2}\right)^{1/2} e^{-\omega t^*}}$$

frequency-independent geometrical attenuation

anelastic attenuation affects high frequencies much more strongly, t^* typically a few seconds.

e.g. if $T \sim 30$ min (teleseismic S) and $Q_\beta \sim 300$, $t^* \sim 3$ secs.

S waves:

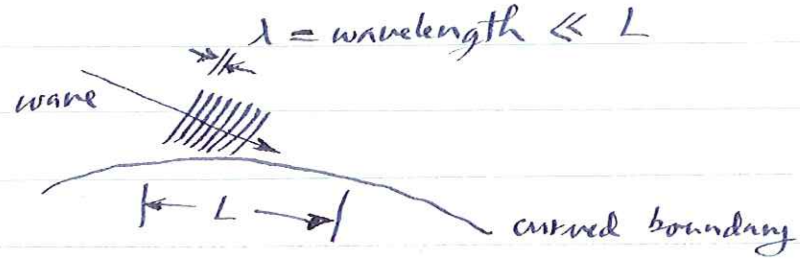
T(sec)	$e^{-\omega t^*}$
60	0.73
10	0.15
1	6.6×10^{-9}

very sharp cutoff.

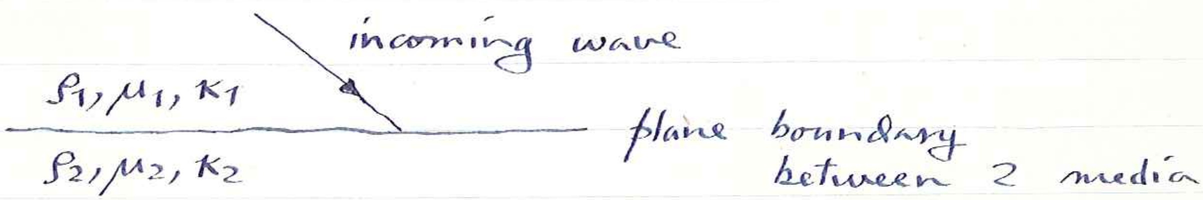
Discontinuities : what happens when a wave encounters a sudden discontinuity, e.g. the core-mantle boundary?

An extension of ray theory is applicable in this case subject to the same proviso that $\lambda \ll L$ where now $L \approx$ radius of curvature of boundary.

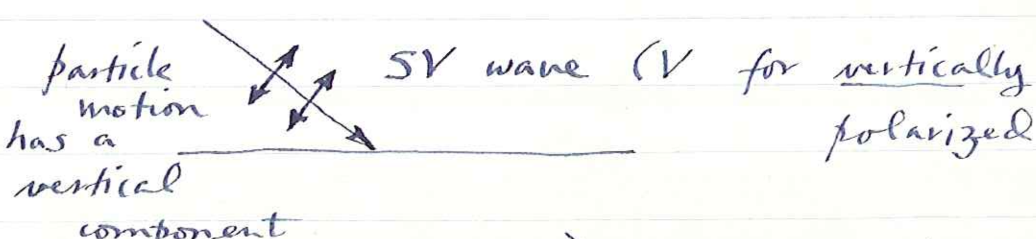
Far from the source, wavefronts appear locally plane. If on scale of waves the bdry appears to be essentially locally plane as well,

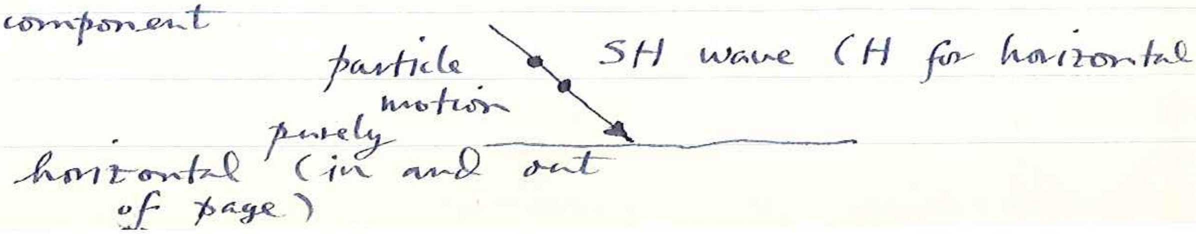


Can then model as <u>plane wave</u> impinging on a plane boundary

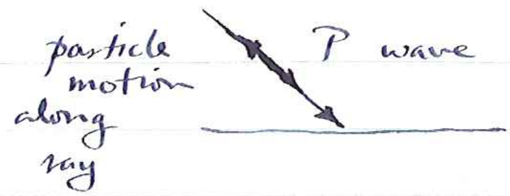


Now becomes important to consider <u>polarization</u> of <u>S wave</u>.
<u>Nomenclature</u>:

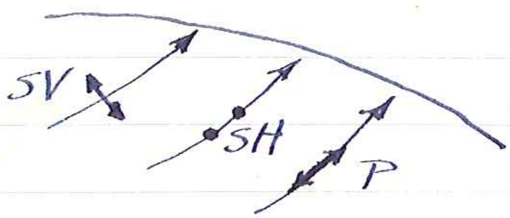




Can also have incident P wave



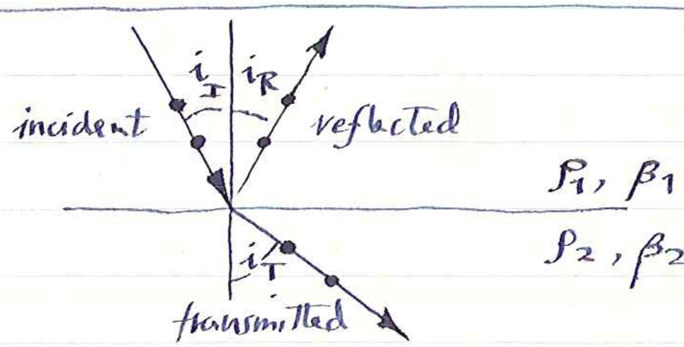
Can also impinge on bdry from below:



⊕ surface, e.g.

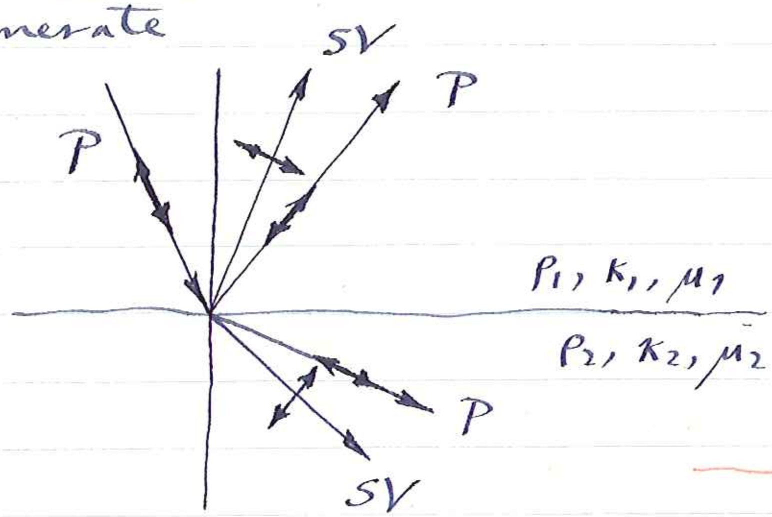
A wave incident on a bdry can, in general, give rise to both a transmitted and reflected wave

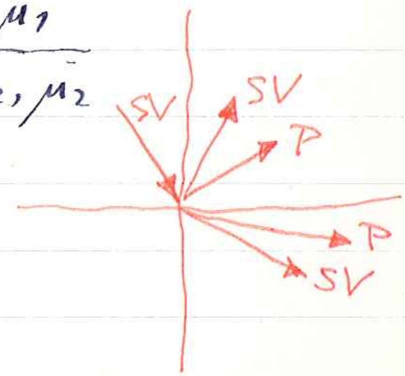
Incident SH can only generate reflected and transmitted SH.



This picture is appropriate if β_2 is greater than β_1

P and SV waves are however <u>coupled</u> at interfaces, thus, e.g. an <u>incident P wave</u> can generate



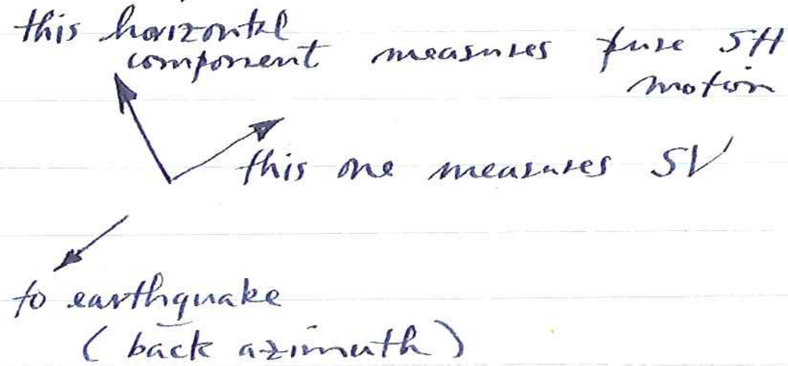


Likewise incident SV can generate P as well as SV. Analysis of this situation to find the <u>amplitudes</u> of the four generated waves is slightly complicated.

Note: SH particle motion is purely <u>horizontal</u>, cannot be detected by vertical motion instrument.

Often in studying S waves investigators limit themselves to SH waves to avoid SV → P conversion problems.

This is easy to do. Consider map view at station



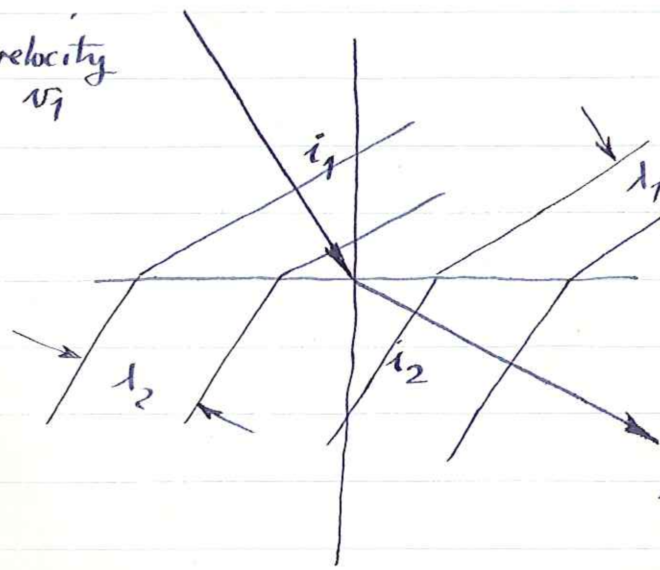
Usually instruments oriented NS and EW. Must be digitized and <u>rotated</u> to separate SH.

We shall find the P-SV and SH separation prevails in other branches of seismology also

body waves	surface waves	modes
SH	Love	toroidal ${}_nT_\ell$
P-SV	Rayleigh	spheroidal ${}_nS_\ell$

To find the <u>amplitudes</u> of reflected and transmitted waves is slightly complicated but we can easily find the ray paths and travel times by an extension of geometrical optics.

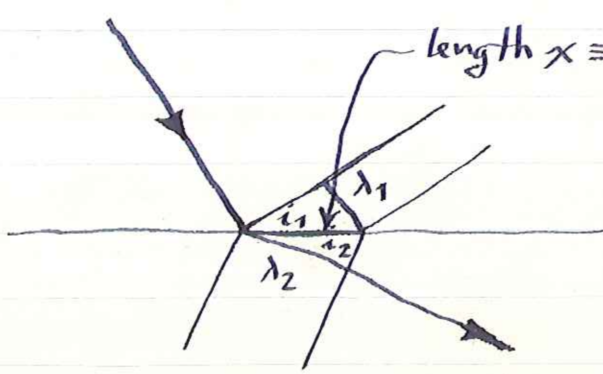
Consider a wave of any type impinging and one of any other type leaving, e.g.



Incoming wave period T, wavelength λ_1, outgoing same period T (can't change), wavelength λ_2

Thus

$$\boxed{T = \frac{\lambda_1}{v_1} = \frac{\lambda_2}{v_2}}$$



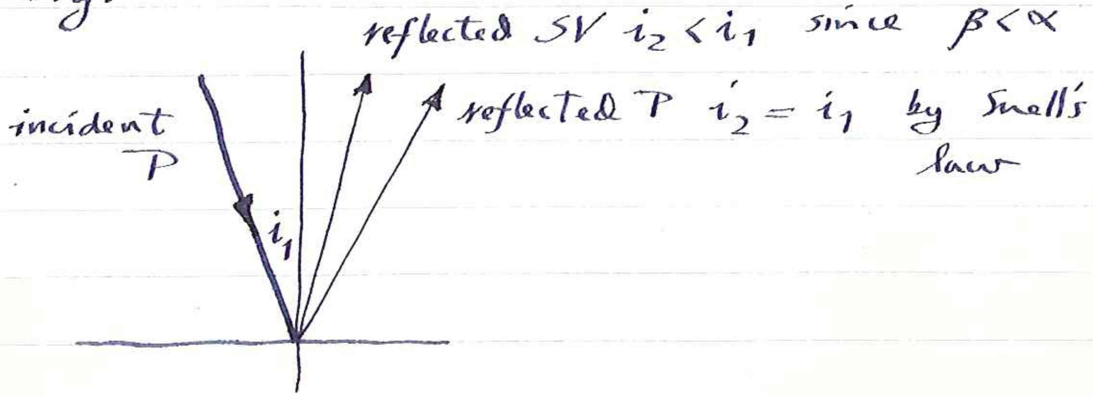
length $x \equiv$ apparent horizontal wavelength

But we also have

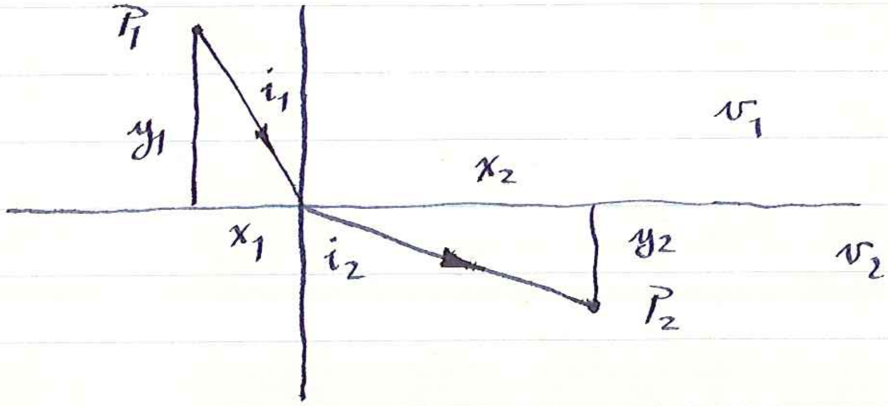
$\lambda_1 = x \sin i_1$, $\lambda_2 = x \sin i_2$

Thus $$\boxed{\frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2}}$$ Snell's law

Governs both reflected and refracted waves, including P–SV conversions, e.g.



Alternatively Snell's law can be derived from Fermat's principle. Consider the refraction case



We know in both media 1 and 2 that

ray paths must be straight lines. Consider all possible connected straight line paths between P_1 and P_2.

$$T = \frac{y_1}{v_1 \cos i_1} + \frac{y_2}{v_2 \cos i_2}$$

Also $\quad x_1 + x_2 = C =$ constant , or

$$\star \quad y_1 \tan i_1 + y_2 \tan i_2 = \text{const}$$

y_1 & y_2 are fixed

By Fermat's principle

$$\delta T = \frac{y_1}{v_1} \frac{\sin i_1}{\cos^2 i_1} \delta i_1 + \frac{y_2}{v_2} \frac{\sin i_2}{\cos^2 i_2} \delta i_2 = 0$$

Fermat

But from $\star$

$$\frac{y_1}{\cos^2 i_1} \delta i_1 + \frac{y_2}{\cos^2 i_2} \delta i_2 = 0$$

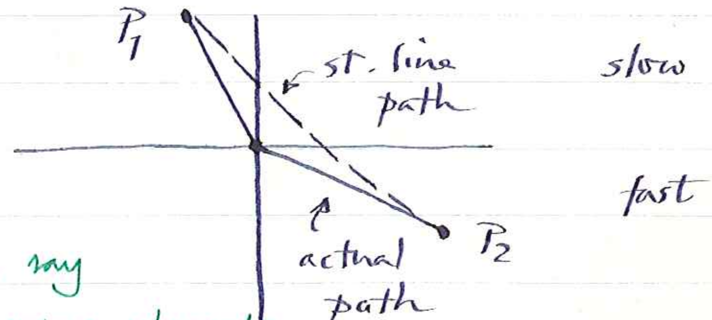
$$\delta i_1 = -\frac{y_2}{y_1} \frac{\cos^2 i_1}{\cos^2 i_2} \delta i_2$$

$$\delta T = \left[-\frac{y_2}{\cos^2 i_2} \frac{\sin i_1}{v_1} + \frac{y_2}{\cos^2 i_2} \frac{\sin i_2}{v_2} \right] \delta i_2 = 0$$

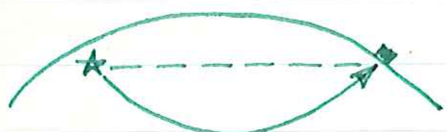
Must vanish for arbitrary δi_2. Thus

$$\boxed{\frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2}}$$ <u>Snell's law</u>

If we compute the second variation $\delta^2 T$ we find $\delta^2 T > 0$, T is a <u>minimum</u> time path, the ray goes down to get out of the slower medium 1 more quickly.

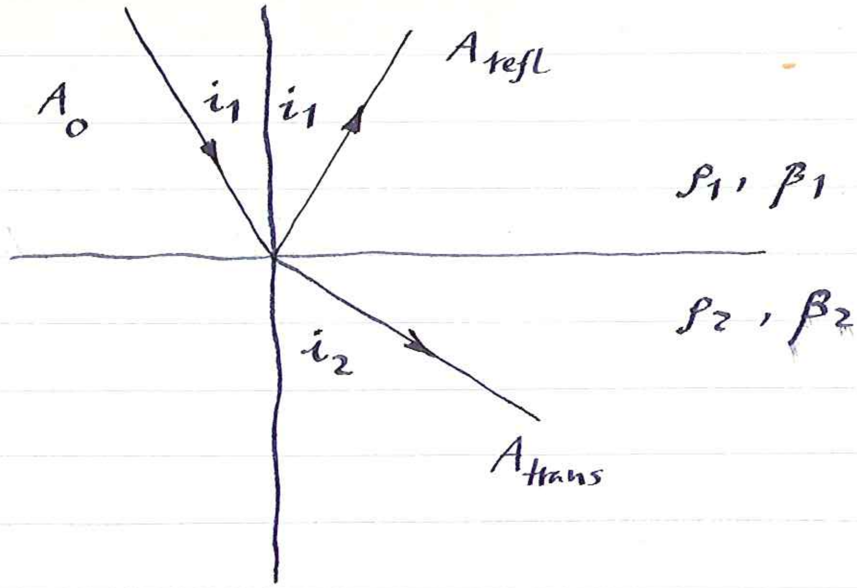


For the same reason ray paths in the mantle curve upwards



<u>Snell's law</u> enables us to find the <u>ray paths</u> and <u>arrival times</u> of reflected and refracted phases. The laws governing their <u>amplitudes</u> are more complicated, the transmission and reflection coefficients depend on incidence angle and on ρ_1, κ, μ_1 and ρ_2, κ_2, μ_2 across the boundary in a rather complicated way.

We shall give but two <u>examples</u>. The simplest case is <u>incident SH</u>.



both <u>solid</u> <u>media</u>

incident amplitude A_0, say
Then it can be shown that

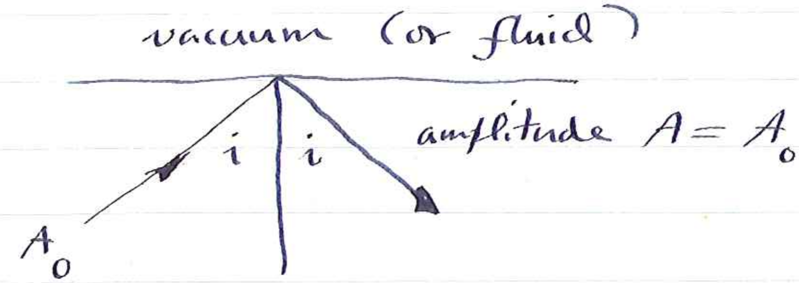
$$\boxed{\begin{aligned} A_{refl} &= \frac{\rho_1\beta_1 \cos i_1 - \rho_2\beta_2 \cos i_2}{\rho_1\beta_1 \cos i_1 + \rho_2\beta_2 \cos i_2} A_0 \\ A_{trans} &= \frac{2\rho_1\beta_1 \cos i_1}{\rho_1\beta_1 \cos i_1 + \rho_2\beta_2 \cos i_2} A_0 \end{aligned}}$$

Note → dependence on both ρ and β (actually only on product $\rho\beta$, the <u>impedance</u>)

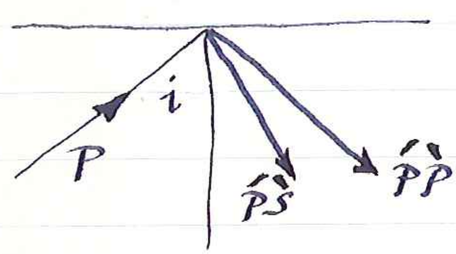
$\cancel{A_{refl} + A_{trans} = A}$

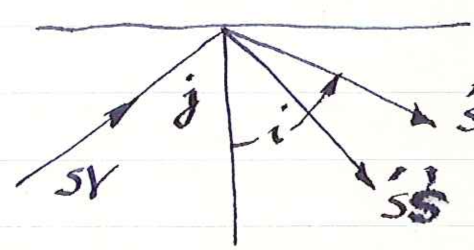
The second relatively simple special case is reflection off a <u>free</u> <u>surface</u>.

For SH, can be shown that there is complete reflection with no change of phase for every incident angle



For incident P or SV, get both reflected P and SV, thus $\exists$ 4 possible reflection coefficients





Note notation: i for P waves, j for S waves.

A plot of these four for $i < 90°$ is shown in Aki + Richards Fig. 5.6, plotted against horizontal slowness but there is a distorted angle scale at the top. Note: for incident SV i is the reflected angle of the P wave.

$i = 90^\circ$ is the critical angle for reflected P. For j even greater, all reflected energy is SV and $|\acute{S}\grave{S}| = 1$, see Figure 5.10.

For shallow angles of incidence $i \gtrsim 45^\circ$ or so, the reflected motion is in both cases almost all opposite in type from the incident motion, incident P is converted almost totally to reflected SV and vice-versa.

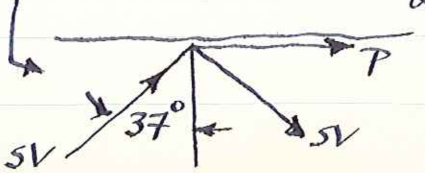
The plots are for the case $\alpha = 5$ km/s and $\beta = 3$ km/s, typical values for the Φ's crust. Beyond the critical angle SV is said to suffer total internal reflection. The $\acute{P}\grave{S}$ amplitude shown is that of the inhomogeneous wave.

The critical angle is given by Snell's law

$$\frac{\sin i}{\alpha} = \frac{\sin j}{\beta}, \quad \sin i = 1 \text{ for } i = 90^\circ$$

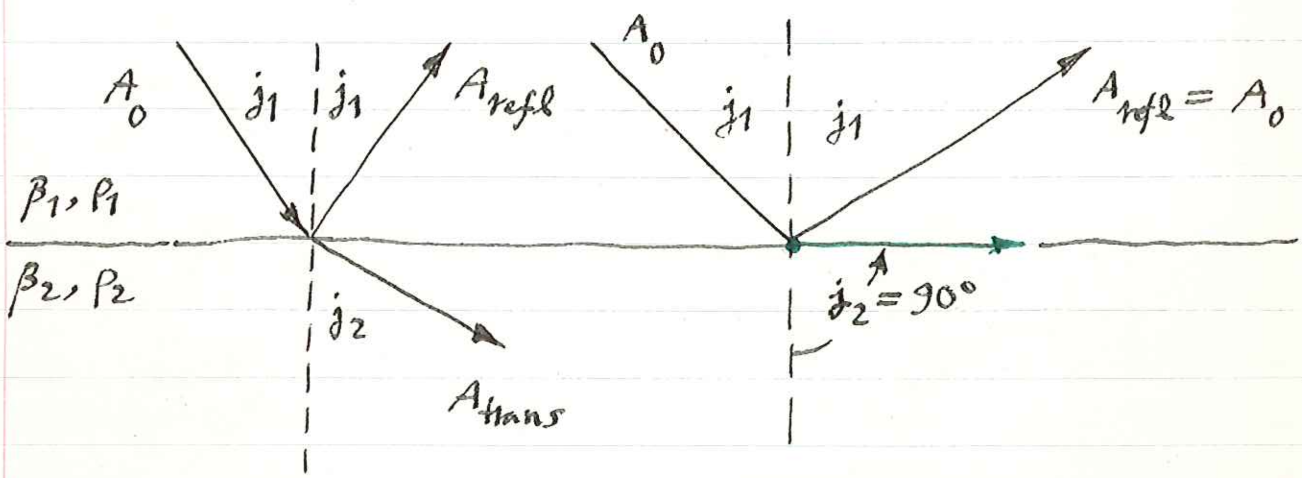
$$\boxed{j = \arcsin(\beta/\alpha)}$$

in example shown $\frac{\beta}{\alpha} = \frac{3}{5}$



SH is always totally internally reflected and SV is for $j > j_{crit}$.

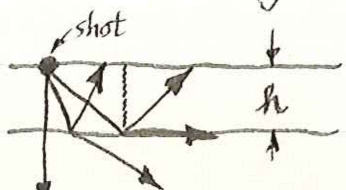
There is also a <u>critical angle</u> phenomenon in the reflection off a discontinuity between 2 materials, e.g. SH



j_1^{crit} occurs when $j_2 = 90°$ or when

$$\boxed{j_1^{crit} = \arcsin \frac{\beta_1}{\beta_2}}$$

Our previous formula for A_{refl} then shows that $A_{refl} = 1$.
<u>Wide-angle reflections</u> beyond the critical angle are <u>totally reflected</u>, thus typically a strong signal at great enough distance

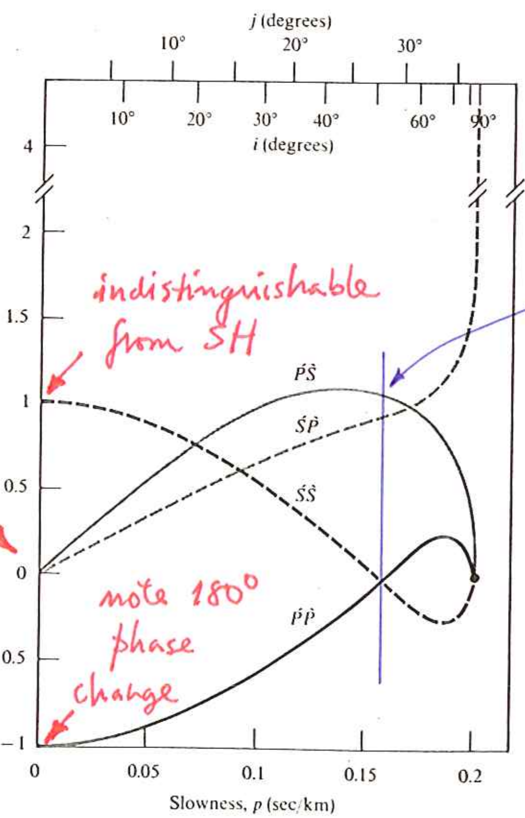


must be a distance $2h \tan i_{crit}$ away to see wide-angle reflections.

In this simple example of reflection from the free surface of a solid half-space, we have now obtained specific formulas for each component of the matrix

$$\begin{pmatrix} \acute{P}\grave{P} & \acute{S}\grave{P} \\ \acute{P}\grave{S} & \acute{S}\grave{S} \end{pmatrix}.$$

This matrix summarizes all possible reflection coefficients, and it is called a *scattering matrix*. Each of its components is plotted against slowness in Figure 5.6, and for this very simple interface, the components are found to vary quite strongly. Only the range $0 \le p \le 1/\alpha$ is shown. For slowness in the range 0.14 to 0.195 sec/km, note that the reflected motion is almost all opposite in type from the incident motion. That is, incident P is converted almost totally to reflected SV, and incident SV to reflected P. Far more complicated behavior can occur in other interface problems (solid/fluid, etc.), and seismologists are often forced to evaluate this behavior in great detail in order to interpret a particular piece of data. For convenience, therefore, we shall give the coefficient



no P–S conversion at normal incidence

complete P → SV conversion at $i \approx 52°$

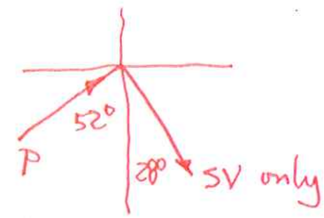
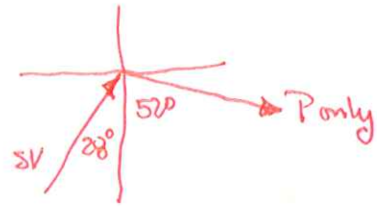


FIGURE 5.6

The four possible P-SV reflection coefficients (displacement amplitude ratios) for a free surface are shown against horizontal slowness p. See Figure 5.5. In this case, α = 5 km/sec and β = 3 km/sec, and we restrict p to lie in the range $0 \le p \le 1/\alpha$ so that incidence angle i is always real. For $i = 90°$, $\acute{S}\grave{P}$ is quite large (~4.1).

ditto



In Figure 5.10, we show $\acute{S}\grave{P}$ and $\acute{S}\grave{S}$ as functions of p in the range $0 \leq p \leq 1/\beta$, giving both amplitude and phase.

As another example of the need for using inhomogeneous waves, consider the reflection and transmission of SH-waves, as described in Figure 5.7 and equations (5.32). Inhomogeneous waves will be present in the upper medium if $1/\beta_1 < p$, and then this wave attenuates away from the interface, provided that we again choose

$$\frac{\cos j_1}{\beta_1} = +i\sqrt{p^2 - \frac{1}{\beta_1^2}} \qquad \text{(in the case } \omega > 0\text{)}, \tag{5.49}$$

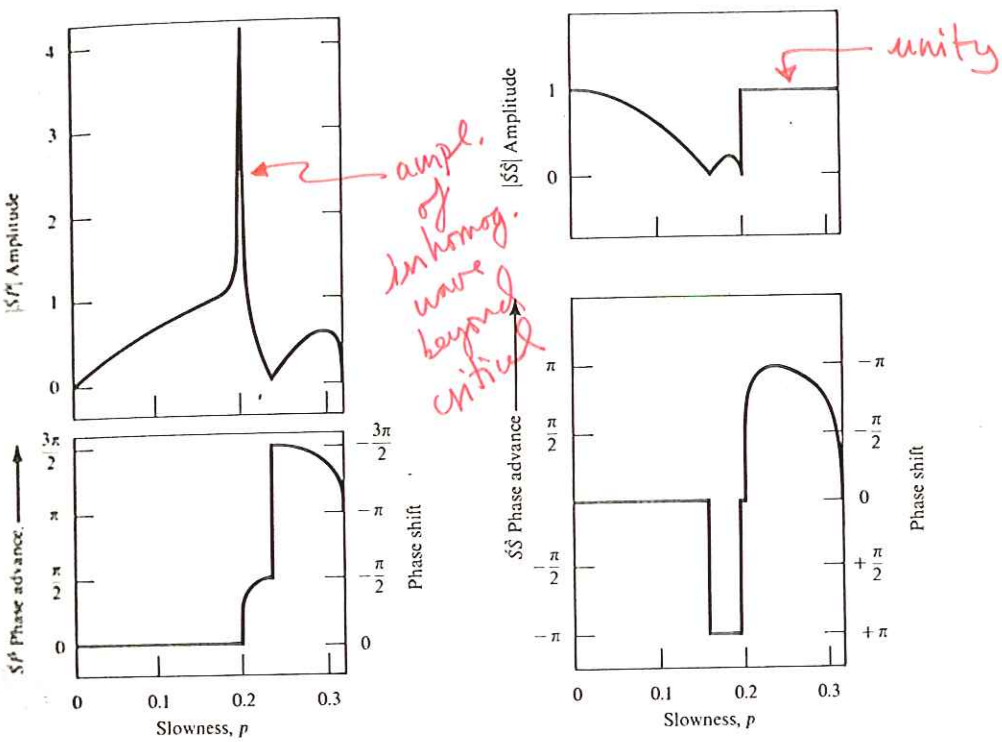


FIGURE **5.10**

The amplitude and phase of two reflection coefficients are shown as a function of horizontal slowness p. The coefficients are $\acute{S}\grave{P}$ and $\acute{S}\grave{S}$ for an SV-wave incident on the free surface, and we have taken $\alpha = 5$ km/sec, $\beta = 3$ km/sec, so that these coefficients have already been shown in Figure 5.6 for the range $0 \leq p \leq 1/\alpha$. Here we extend the range to $0 \leq p \leq 1/\beta$, so that an inhomogeneous P-wave is present for the range $1/\alpha < p \leq 1/\beta$. We have chosen to emphasize phase advance rather than phase shift, since the former is a quantity independent of the sign of frequency and independent of our Fourier sign convention. The phases actually plotted are those of $\acute{S}\grave{P}$ and $\acute{S}\grave{S}$ as determined by (5.30) and (5.31). Note that zeros in the coefficients are now associated with jumps of amount π in phase.