Normal mode excitation on a SNREI Earth

We begin with the general formula for the acceleration response:

$$a(x,t) = \sum_k M : \varepsilon_k(x_s)\, s_k(x) \cos \omega_k t \; e^{-\omega_k t / 2Q_k}$$

$$\uparrow \; e^{-\alpha_k t}$$

or equivalently in the Fourier transform domain:

$$a(x,\omega) = \int_0^\infty a(x,t)\, e^{-i\omega t}\, dt$$

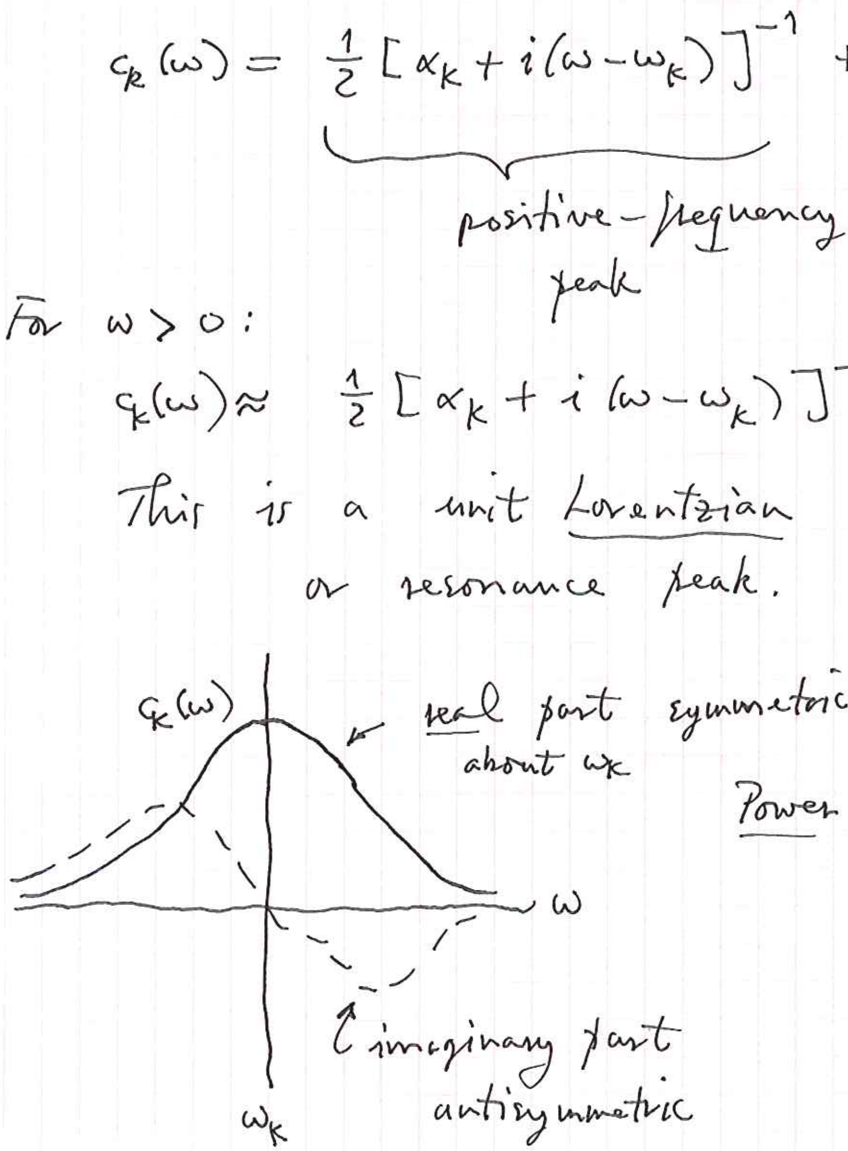
$$a(x,\omega) = \sum_k A_k(x)\, c_k(\omega) \qquad \text{where}$$

$$c_k(\omega) = \underbrace{\tfrac{1}{2}\left[\alpha_k + i(\omega - \omega_k)\right]^{-1}}_{\text{positive-frequency peak}} + \underbrace{\tfrac{1}{2}\left[\alpha_k + i(-\omega - \omega_k)\right]^{-1}}_{\text{negative-freq peak}}$$

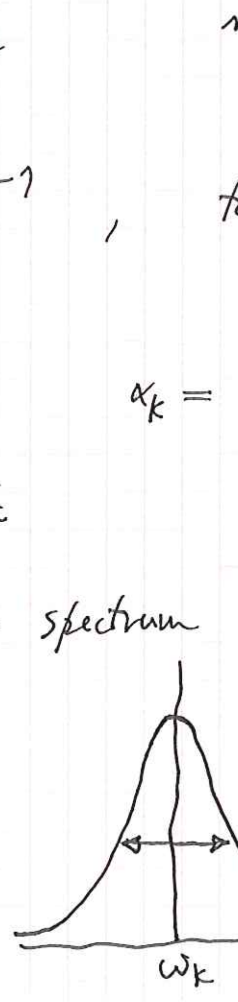
For $\omega > 0$:

$$c_k(\omega) \approx \tfrac{1}{2}\left[\alpha_k + i(\omega - \omega_k)\right]^{-1}, \quad \text{to a good approximation}$$

This is a unit Lorentzian or resonance peak. $\alpha_k = \dfrac{\omega_k}{2Q_k}$ is the decay rate



Power spectrum $|c_k(\omega)|^2$:



half-power width:

$$\frac{\delta\omega_k}{\omega_k} = \frac{1}{Q_k}$$

The response looks like a sum of decaying cosines in the time domain, or like a sum of resonance peaks in the frequency domain.

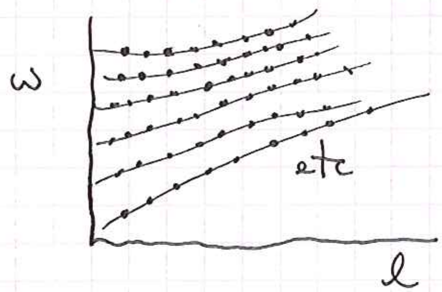
The amplitude of each peak is:

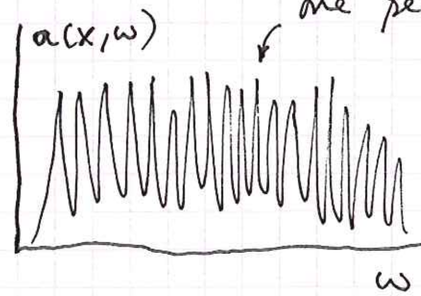
$$A_k(\omega) = M : \varepsilon_k(x_s)\, s_k(x)$$

depends on quake mechanism — and on quake location — and on receiver location

$A_k(\omega)$ is real in this approximation — (the origin time is assumed known)

A spectrum of a single seismogram "collapses" all the dots • in the ω–l diagram over onto the frequency axis.





One is faced with a severe problem of mode identification. Some peaks are well-isolated and can be identified simply on the basis of their frequency, but many superposed peaks can't be resolved due to the line broadening caused by attenuation and finite record lengths.

To determine the response of a SNREI Earth it is convenient to use <u>epicentral coordinates</u> (i.e. consider the source $\mathbf{x}_s$ to be at the north pole)

Recall that the SNREI modes are of the form

$$\mathbf{s}_k = {}_nW_\ell(r)\,[-\hat{\mathbf{r}} \times \nabla_1 Y_\ell^m]\,, \quad \text{toroidal}$$

$$\mathbf{s}_k = {}_nU_\ell(r)\,\hat{\mathbf{r}}\, Y_\ell^m + {}_nV_\ell(r)\,\nabla_1 Y_\ell^m\,, \quad \text{spheroidal}$$

The "generic" index k stands for all of

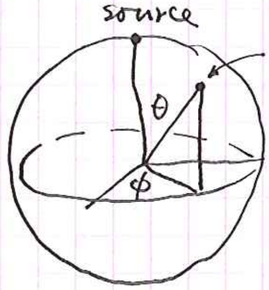
$\{S, T\}$, n, ℓ, m.

spheroidal toroidal radial overtone number

We must evaluate the number $\mathbf{M} : \boldsymbol{\varepsilon}_k(\mathbf{x}_s)$

where $\boldsymbol{\varepsilon}_k = \frac{1}{2}\,[\nabla \mathbf{s}_k + (\nabla \mathbf{s}_k)^T]$

source



receiver at $\mathbf{x} = (r, \theta, \phi)$

In the limit as $\theta \to 0$, the spherical harmonics look like

$$Y_\ell^m \sim b_m\, \theta^m \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix}$$

go to zero like θ^m — only Y_ℓ^0 is non-zero at the pole

where $b_m = \dfrac{(-1)^m}{2^m\, m!}\sqrt{\dfrac{2\ell+1}{4\pi}}\left[\dfrac{\ell+m!}{\ell-m!}\right]^{1/2}$

To find $\boldsymbol{\varepsilon}_k$ we must differentiate Y_ℓ^m <u>twice</u>. <u>As a result only $-2 \le m \le 2$ are excited</u>.

The exact expression for $\mathscr{U}(\hat{\mathbf{r}}, \sigma)$ on a spherical Earth has been given by Gilbert & Dziewonski (1975). Let us use $\tilde{\Sigma}_i(\Phi)$, $i = 1-5$, to denote the quantities

$$\tilde{\Sigma}_1(\Phi) = (\lambda/2\pi)^{1/2}\,[M_{zz}\,\partial_r U_s + \tfrac{1}{2}(M_{xx} + M_{yy}) r_s^{-1}(2U_s - (\lambda^2 - \tfrac{1}{4})\,V_s)]$$
$$\tilde{\Sigma}_2(\Phi) = (\lambda/2\pi)^{1/2}(\lambda^2 - \tfrac{1}{4})^{1/2}(\partial_r V_s + r_s^{-1}(U_s - V_s))\,[M_{yz}\sin\Phi + M_{xz}\cos\Phi]$$
$$\tilde{\Sigma}_3(\Phi) = (\lambda/2\pi)^{1/2}(\lambda^2 - \tfrac{1}{4})^{1/2}(\lambda^2 - \tfrac{9}{4})^{1/2} r_s^{-1} V_s\,[M_{xy}\sin 2\Phi + \tfrac{1}{2}(M_{xx} - M_{yy})\cos 2\Phi] \qquad (53)$$
$$\tilde{\Sigma}_4(\Phi) = (\lambda/2\pi)^{1/2}(\lambda^2 - \tfrac{1}{4})^{1/2}(\partial_r W_s - r_s^{-1} W_s)\,[-M_{xz}\sin\Phi + M_{yz}\cos\Phi]$$
$$\tilde{\Sigma}_5(\Phi) = (\lambda/2\pi)^{1/2}(\lambda^2 - \tfrac{1}{4})^{1/2}(\lambda^2 - \tfrac{9}{4})^{1/2} r_s^{-1} W_s\,[-\tfrac{1}{2}(M_{xx} - M_{yy})\sin 2\Phi + M_{xy}\cos 2\Phi].$$

In addition, let us define $X_l^m(\Delta)$ by

$$Y_l^m(\Delta, \Phi) = X_l^m(\Delta)\exp(im\Phi). \qquad (55)$$

The exact expression for $\mathscr{U}(\hat{\mathbf{r}}, \sigma)$ is then, for a spheroidal multiplet ${}_nS_l$,

$$\mathscr{U}(\hat{\mathbf{r}}, \sigma) = [\tilde{\Sigma}_1(\Phi)X_l^0(\Delta) - \tilde{\Sigma}_2(\Phi)X_l^1(\Delta) + \tilde{\Sigma}_3(\Phi)X_l^2(\Delta)]c_0(\sigma), \qquad (56)$$

and, for a toroidal multiplet ${}_nT_l$,

$$\mathscr{U}(\hat{\mathbf{r}}, \sigma) = [\tilde{\Sigma}_4(\Phi)X_l^1(\Delta) - \tilde{\Sigma}_5(\Phi)X_l^2(\Delta)]c_0(\sigma). \qquad (57)$$

Both equations (56) and (57) are valid, on a spherical Earth, for all epicentral distances Δ.

Translation: $c_0(\sigma)$ is $c_k(\omega)$ — the unit resonance peak.

$X_\ell^0(\Delta)$, $X_\ell^1(\Delta)$, $X_\ell^2(\Delta)$ are the ~~[illegible]~~ latitudinal parts of Y_ℓ^m.

Δ ~~[illegible]~~ is the epicentral distance

$\lambda = \ell + 1/2$ so that $\lambda^2 - \frac{1}{4} = \ell(\ell+1)$ & $\lambda^2 - \frac{9}{4} = (\ell-1)(\ell+2)$

Δ, Φ are the same as θ, ϕ

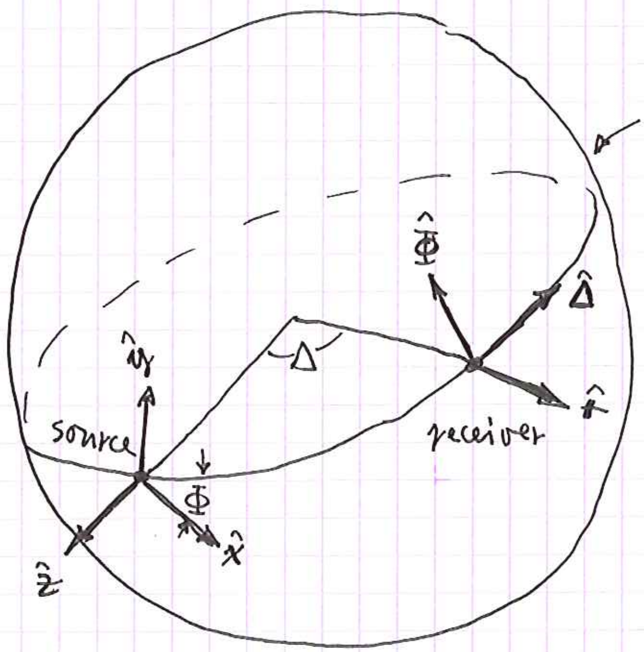
The vector polarization and amplitude of the acceleration are given by:

$$A_k(x) = W(r)\,[-\hat{r} \times \nabla_1 \mathcal{Y}(\theta,\phi)] \quad : \text{ toroidal}$$

$$A_k(x) = [\,U(r)\,\hat{r} + V(r)\,\nabla_1\,]\,\mathcal{Y}(\theta,\phi) \quad : \text{ spheroidal}$$

$\mathcal{Y}(\theta,\phi)$ is a kind of scalar potential

The moment-tensor components are defined in a general coordinate system:



source-receiver great circle

$\hat{r}$: vertical or radial polarization

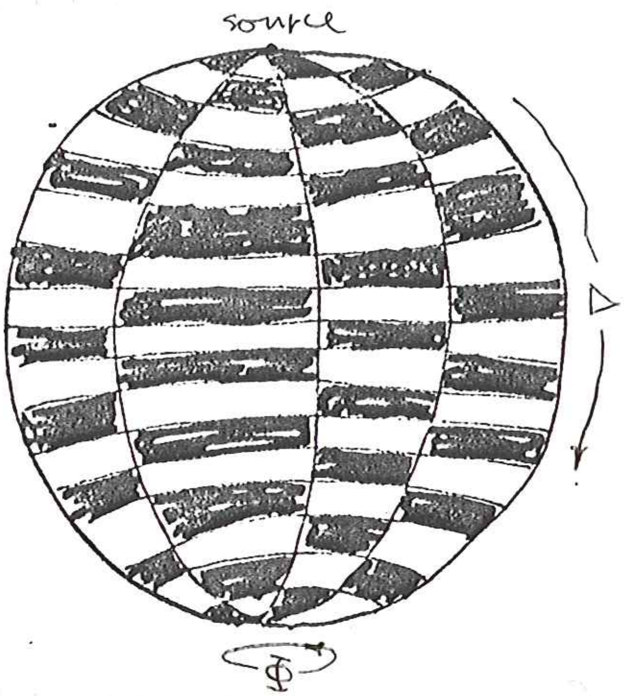
$\hat{\Delta}$: longitudinal polarization

$\hat{\Phi}$: transverse polarization

$\hat{x}$ is typically taken due east and $\hat{y}$ due north

Φ - the takeoff angle at the source is measured counterclockwise from $\hat{x}$.

The pattern of displacement associated with a given mode on the surface of the Earth looks like:



There are l nodal lines as a function of epicentral distance Δ but only 4 (or fewer — depending on the source mechanism) as a function of azimuth Φ. The phase of the oscillation is the same except for sign at every point on the Earth.

Only $-2 \le m \le 2$ excited
$\Rightarrow$ only 4 or fewer nodal lines in Φ.

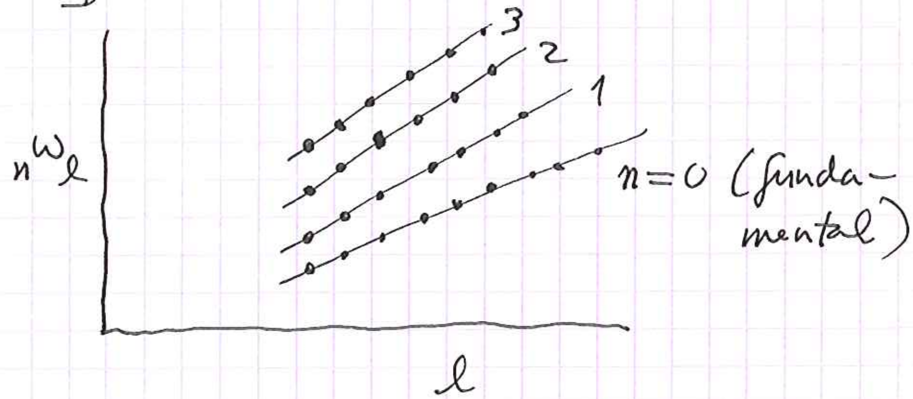
The method used to identify & isolate modes is stacking. Call the above pattern $A_k(\theta,\phi)$. Given many seismograms $a(\theta_i, \phi_i, \omega)$ one forms the stack:

$$\sum_{\substack{i \\ \text{stations}}} a(\theta_i, \phi_i, \omega)\, A_k(\theta_i, \phi_i)$$

This will reinforce the desired mode & tend to cancel all others — by this means more than 1000 modes have been identified and ω_k measured.

Decomposition into travelling surface waves

The surface-wave-equivalent modes are those having $n \ll l$:



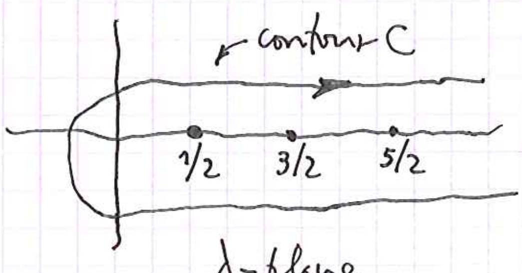
We consider these a branch at a time. We fix $n = 0, 1, 2, \cdots$ and consider the sum for $l \gg 1$.

$$a(x,\omega) = \sum_l \underbrace{A_l(r,\theta,\phi)}_{\text{excitation amplitude}} \underbrace{c_l(\omega)}_{\text{unit resonance peak}}$$

For $l \gg 1$ and $m \ll l$ we employ the asymptotic expansion:

$$X_l^m(\Delta) \sim \frac{1}{\pi}(\sin\Delta)^{-1/2} \cos\left[\left(l+\tfrac{1}{2}\right)\Delta + \frac{m\pi}{2} - \frac{\pi}{4}\right]$$

The classical technique for converting the sum over l into an integral over wavenumber k is the Watson transformation



If $f(\lambda)$ is analytic then

$$\sum_{l=0}^{\infty} f\left(l+\tfrac{1}{2}\right) = \frac{1}{2}\int_C f(\lambda)\, e^{-i\pi\lambda} \sec\pi\lambda \, d\lambda$$

Proof: $\sec \pi\lambda = \frac{1}{\cos \pi\lambda}$ has simple poles at

$$\lambda = 1/2,\ 3/2,\ 5/2,\ \cdots$$

Evaluate $\int_C$ by the residue theorem:

$$-2\pi i \sum \text{residues} = -\pi i \sum_\ell f(\ell + \tfrac{1}{2}) \operatorname{Res}_{\ell+1/2} \frac{e^{-i\pi\lambda}}{\cos \pi\lambda}$$

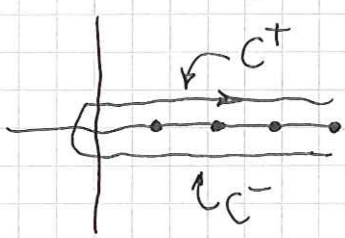
↑ because C goes around poles in cw direction

$$\operatorname{Res}_{\ell+1/2} \frac{e^{-i\pi\lambda}}{\cos\pi\lambda} = \frac{e^{-i\pi(\ell+1/2)}}{-\pi \sin(\ell+\frac{1}{2})\pi}$$

$$= i/\pi$$

So $-2\pi i \sum \text{Res} = f(\ell + \frac{1}{2})$. QED

More useful in our problem is a variant of this result known as the Poisson sum formula. The two results are equivalent despite numerous allegations to the contrary in the geophysical literature



On C^+ we can write

$$\sec\pi\lambda = 2\sum_{k=0}^{\infty} (-1)^k e^{i(2k+1)\pi\lambda} \quad *$$

Proof: $\sec\pi\lambda = \frac{2}{e^{i\pi\lambda} + e^{-i\pi\lambda}} = \frac{2e^{i\pi\lambda}}{1+e^{2i\pi\lambda}}$

$$= 2\sum_{k=0}^{\infty} (-1)^k e^{i\pi\lambda} e^{2\pi i k\lambda}$$

* Converges on C^+ where $\operatorname{Im}\lambda > 0$ but not on C^- where $\operatorname{Im}\lambda < 0$.

But by a similar argument:

$$\sec \pi\lambda = -2 \sum_{k=-\infty}^{-1} (-1)^k e^{i(2k+1)\pi\lambda}$$ and this converges on C^-

Combining $\int_C = \int_{C^+} + \int_{C^-}$ we get

$$\sum_{\ell=0}^{\infty} f(\ell+\tfrac{1}{2}) = \sum_{k=-\infty}^{\infty} (-1)^k \int_0^\infty f(\lambda)\, e^{2i\pi k\lambda}\, d\lambda,$$ the <u>Poisson sum formula</u>

We use this result with

$$f(\ell+\tfrac{1}{2}) = A_{\ell-1/2}(r,\theta,\phi)\, c_{\ell-1/2}(\omega)$$

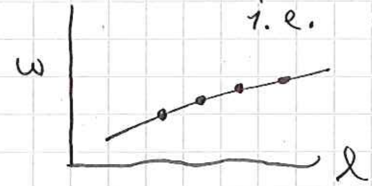
Substitute in & <u>rearrange terms</u>. Use the symmetry that U_ℓ, V_ℓ, W_ℓ, ω_ℓ, α_ℓ are all invariant under the transformation $\ell+1/2 \to -\ell-1/2$.

Obtain the result

$$a(r,\theta,\phi,\omega) = \sum_{p=1}^{\infty} \int_{-\infty}^{\infty} [\, a_p^+(r,\theta,\phi,\lambda)\, e^{i\lambda\Delta_p}$$

$$+\ a_p^-(r,\theta,\phi)\, e^{-i\lambda\Delta_p}\,]\, c(\lambda,\omega)\, d\lambda$$

where
$$c(\lambda,\omega) = \tfrac{1}{2}\, [\, \kappa(\lambda) + i(\omega-\omega(\lambda))\,]^{-1}$$



i.e. analytic continuation ("connect the dots"

$$\Delta_p = \begin{cases} \Delta + (p-1)\pi & p \text{ odd} \\ p\pi - \Delta & p \text{ even} \end{cases}$$

where, for Rayleigh waves,

$$\mathbf{a}_p^{\pm}(\lambda, \mathbf{r}) = \frac{1}{2\pi}(\sin\theta)^{-1/2}[\hat{\mathbf{r}}U(r) \pm i\hat{\boldsymbol{\theta}}\lambda V(r)]\,\mathscr{R}_1(\phi)\exp[(p-1)\pi i/2], \qquad p \text{ odd}$$

$$\mathbf{a}_p^{\pm}(\lambda, \mathbf{r}) = \frac{1}{2\pi}(\sin\theta)^{-1/2}[\hat{\mathbf{r}}U(r) \mp i\hat{\boldsymbol{\theta}}\lambda V(r)]\,\mathscr{R}_2(\phi)\exp(p\pi i/2), \qquad p \text{ even},$$

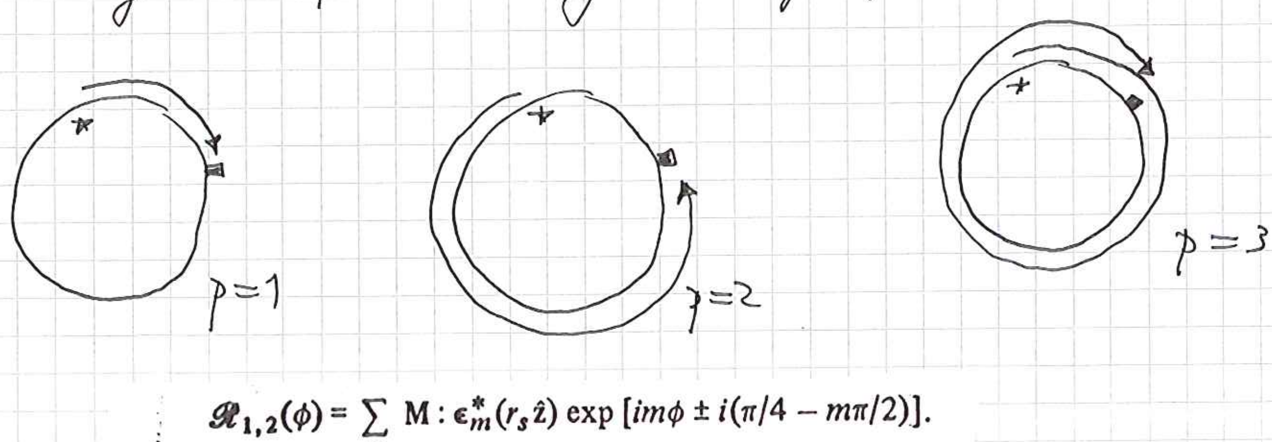
and for Love waves,

$$\mathbf{a}_p^{\pm}(\lambda, \mathbf{r}) = \frac{1}{2\pi}(\sin\theta)^{-1/2}[\pm i\hat{\boldsymbol{\phi}}\lambda W(r)]\,\mathscr{R}_1(\phi)\exp[(p-1)\pi i/2], \qquad p \text{ odd}$$

$$\mathbf{a}_p^{\pm}(\lambda, \mathbf{r}) = \frac{1}{2\pi}(\sin\theta)^{-1/2}[\mp i\hat{\boldsymbol{\phi}}\lambda W(r)]\,\mathscr{R}_2(\phi)\exp(p\pi i/2), \qquad p \text{ even}.$$

The $\int_{-\infty}^{\infty} d\lambda$ comes from $\int_0^{\infty} d\lambda$ in the Poisson sum formula, together with the symmetry, and the $\sum_p$ comes from the $\sum_k$ with the terms rearranged.

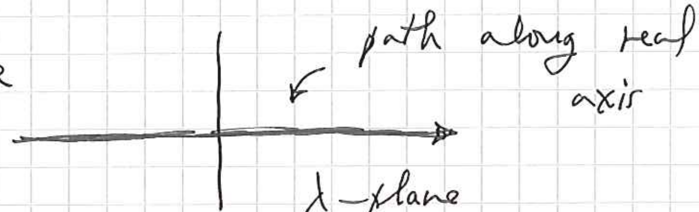
The quantity Δ_p is the distance travelled by the p^{th} arriving wavegroup



$$\mathscr{R}_{1,2}(\phi) = \sum_m \mathbf{M} : \boldsymbol{\epsilon}_m^*(r_s\hat{\mathbf{z}})\exp[im\phi \pm i(\pi/4 - m\pi/2)].$$

The quantity $\lambda = \frac{k}{a}$ is the wavenumber on the unit sphere ($ka = l + 1/2$)

The integral looks like



The factors $a_p^{\pm} e^{\pm i \lambda \Delta_p}$ are analytic.

The only singularities of the integrand are those associated with $c(\lambda, \omega)$.

It has a pole at:

$$\alpha_n(\lambda) + i(\omega - \omega_n(\lambda)) = 0$$

Write $$\omega_n(\lambda) \approx \omega_n(\lambda_n) + (\lambda - \lambda_n) \frac{d\omega_n}{d\lambda}(\lambda_n) + \cdots$$

$$= \omega_n(\lambda_n) + (\lambda - \lambda_n)\, U_n(\lambda_n)$$

group velocity in rad/sec

Likewise to the same accuracy:

$$\alpha_n(\lambda) = \alpha_n(\lambda_n) + \cdots$$

To first-order the pole is at

$$\lambda = \lambda_n(\omega) - \frac{i\alpha_n(\omega)}{U_n(\omega)}$$

$\lambda_n(\omega)$ is the wavenumber on the unit sphere of waves of frequency ω and $\alpha_n(\omega)$ is their temporal decay rate

The wavenumber and group velocity on $r=a$ are given in terms of those on the <u>unit sphere</u> by

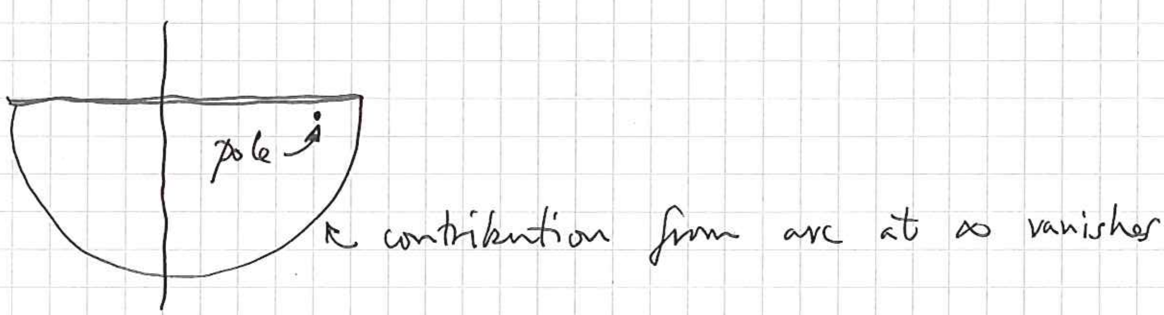
$$k_n(\omega) = \frac{\lambda_n(\omega)}{a}$$

$$U_n(\omega) = a\,\mathcal{U}_n(\omega)$$

We must evaluate the integral

$$\frac{1}{2i\mathcal{U}_n} \int_{-\infty}^{\infty} \frac{a_p^{\pm}\, e^{\pm i\lambda\Delta_p}}{\lambda - (\lambda_n - i\alpha_n/\mathcal{U}_n)}\, d\lambda$$

For the $e^{i\lambda\Delta_p}$ integral we close the contour in the upper half plane (and get zero). For the $e^{-i\lambda\Delta_p}$ integral we close in the lower half-plane.



This gives the final result shown on the next page...

We find finally for the pth arriving group of Rayleigh waves, for frequencies $\omega > 0$, the result

$$
\begin{aligned}
a_p(\mathbf{r},\omega) &\sim +\tfrac{1}{2}[\hat{\mathbf{r}}U(r) - i\hat{\boldsymbol{\theta}}\lambda_0 V(r)](\sin\theta)^{-1/2}u_0^{-1}\exp(-\alpha_0\Delta_p/u_0) \\
&\quad\times \mathscr{R}_1(\phi)\exp[-i\lambda_0\Delta_p + i(p-1)\pi/2], \quad p \text{ odd} \\
a_p(\mathbf{r},\omega) &\sim +\tfrac{1}{2}[\hat{\mathbf{r}}U(r) + i\hat{\boldsymbol{\theta}}\lambda_0 V(r)](\sin\theta)^{-1/2}u_0^{-1}\exp(-\alpha_0\Delta_p/u_0) \\
&\quad\times \mathscr{R}_2(\phi)\exp[-i\lambda_0\Delta_p + ip\pi/2], \quad p \text{ even.}
\end{aligned}
\tag{174}
$$

For the spectrum of the pth arriving group of Love waves, for frequencies $\omega > 0$, we obtain

$$
\begin{aligned}
a_p(\mathbf{r},\omega) &\sim +\tfrac{1}{2}[i\hat{\boldsymbol{\phi}}\lambda_0 W(r)](\sin\theta)^{-1/2}u_0^{-1}\exp(-\alpha_0\Delta_p/u_0) \\
&\quad\times \mathscr{R}_1(\phi)\exp[-i\lambda_0\Delta_p + i(p-1)\pi/2], \quad p \text{ odd} \\
a_p(\mathbf{r},\omega) &\sim +\tfrac{1}{2}[-i\hat{\boldsymbol{\phi}}\lambda_0 W(r)](\sin\theta)^{-1/2}u_0^{-1}\exp(-\alpha_0\Delta_p/u_0) \\
&\quad\times \mathscr{R}_2(\phi)\exp[-i\lambda_0\Delta_p + ip\pi/2], \quad p \text{ even.}
\end{aligned}
\tag{175}
$$

Both equations (174) and (175) are valid, except near the poles, for waves which satisfy $\lambda_0(\omega) \gg 1$. The region of validity on Ω is $\epsilon < \theta < \pi - \epsilon$, where $\epsilon \ll \lambda_0^{-1}(\omega)$. The normalization of the functions $U(r)$, $V(r)$ and $W(r)$ is, in accordance with equation (4),

$$\int_0^a \rho_0(r)[U^2(r) + l(l+1)V^2(r)]r^2\,dr = 1, \tag{176}$$

$$\int_0^a \rho_0(r)[l(l+1)W^2(r)]r^2\,dr = 1. \tag{177}$$

We find, upon neglecting terms of order $\lambda_0^{-1}(\omega)$, for Rayleigh waves,

$$
\begin{aligned}
\mathscr{R}_{1,2}(\phi) &\sim (\lambda_0/2\pi)^{1/2}[M_{rr}\partial_r U(r_s) + \tfrac{1}{2}(M_{\theta\theta} + M_{\phi\phi})r_s^{-1}(2U(r_s) - \lambda_0^2 V(r_s))]\exp(\pm i\pi/4) \\
&\quad - (\lambda_0/2\pi)^{1/2}\lambda_0[\partial_r V(r_s) + r_s^{-1}(U(r_s) - V(r_s))]\,[M_{r\phi}\sin\phi + M_{r\theta}\cos\phi] \\
&\quad \times \exp(\mp i\pi/4) - (\lambda_0/2\pi)^{1/2}\lambda_0^2[r_s^{-1}V(r_s)]\,[M_{\theta\phi}\sin 2\phi + \tfrac{1}{2}(M_{\theta\theta} - M_{\phi\phi}) \\
&\quad \times \cos 2\phi]\exp(\pm i\pi/4),
\end{aligned}
\tag{178}
$$

and for Love waves,

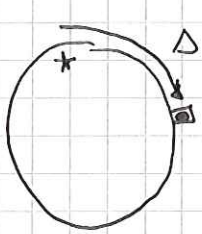
$$
\begin{aligned}
\mathscr{R}_{1,2}(\phi) &\sim (\lambda_0/2\pi)^{1/2}\lambda_0[\partial_r W(r_s) - r_s^{-1}W(r_s)]\,[-M_{r\theta}\sin\phi + M_{r\phi}\cos\phi]\exp(\mp i\pi/4) \\
&\quad + (\lambda_0/2\pi)^{1/2}\lambda_0^2[r_s^{-1}W(r_s)]\,[-\tfrac{1}{2}(M_{\theta\theta} - M_{\phi\phi})\sin 2\phi + M_{\theta\phi}\cos 2\phi]\exp(\pm i\pi/4).
\end{aligned}
\tag{179}
$$

Example: let's consider the first-arriving group of Love waves — it's spectrum is:

write $\lambda = ka$ and $\alpha = \omega/2Q$ and U in km/sec.

$$\boxed{a(r,\Delta,\phi,\omega) = -\hat{\phi}\,(ka)\,W(r)\,(\sin\Delta)^{-1/2}\;U^{-1}(\omega)\,e^{-\omega a\Delta/2QU}\left[-\tfrac{1}{2}i\,R_1(\phi)\right]e^{-ika\Delta}}$$

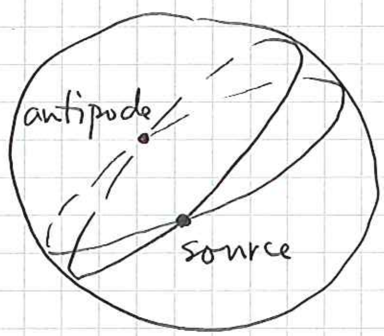
$a\Delta$ is the distance travelled in kilometers



Particle motion is transverse

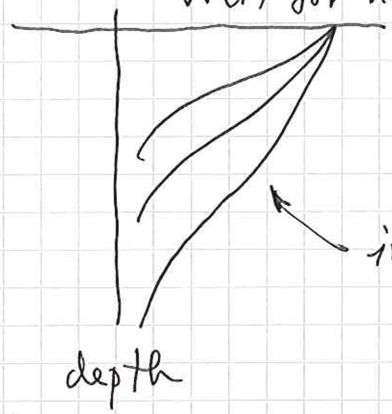
The factor $(\sin\Delta)^{-1/2}$ is the geometrical spreading in 2-d on a sphere. Signal weakest at $\Delta = 90°$ with a focus at the antipode.

raytube width $\sim \sin\Delta$



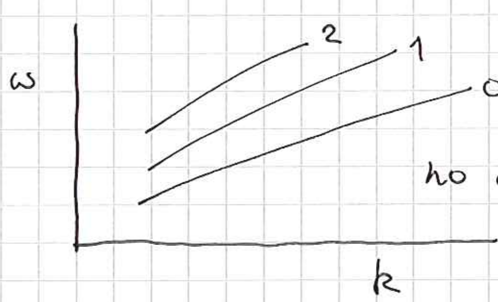
The depth dependence is that of the Love wave eigenfunction $W(r)$

$W(r)$ for $n=0$ (fundamental mode)



increasingly trapped to ⊕'s surface with increasing frequency ω and wavenumber k.

The term $e^{-ika\Delta}$ is the phase delay due to propagation. Can write in form $e^{-i\omega a\Delta/c(\omega)}$ where $c(\omega) = \dfrac{\omega}{k(\omega)}$ is the phase velocity of the wave. We now think of the ω–l diagram as an ω–k diagram



$c = \dfrac{\omega}{k}$

no dots — continuous curves

The factor $U^{-1}(\omega)$ partly, but not entirely, accounts for the dominance of frequencies corresponding to group velocity maxima — they are preferentially excited.

The term $e^{-\omega a\Delta/2QU}$ represents the physical attenuation of the wave due to anelasticity. The quality factor $Q^{-1}(\omega)$ is related to Q_K^{-1} and Q_μ^{-1} in the Earth as we have seen.

Note the appearance of the group velocity

$e^{-\frac{\omega a \Delta}{2QU}}$ not $e^{-\frac{\omega a \Delta}{2Qc}}$

This is because energy propagates with the group velocity.

Equation (7.92) can be easily understood if we measure the attenuation of dispersed waves using the stationary-phase approximation (7.18). At a given x and t, the frequency ω given by $x/t = U(\omega)$ dominates the record. Since the wave with frequency ω has existed in the medium over the time period $t = x/(U(\omega))$, it must have been attenuated by a factor

$$\exp\left[\frac{-\omega t}{2\ temporal\ Q(\omega)}\right] = \exp\left[\frac{-\omega x}{2U(\omega)\ temporal\ Q(\omega)}\right]. \qquad (7.93)$$

Since, by definition, this is equal to

$$\exp\left[\frac{-\omega x}{2c(\omega)\ spatial\ Q(\omega)}\right],$$

we obtain (7.92).

Finally the term $-\frac{i}{2}\mathcal{R}_1(\phi)$ represents the Love-wave radiation pattern:

depends on strain at source

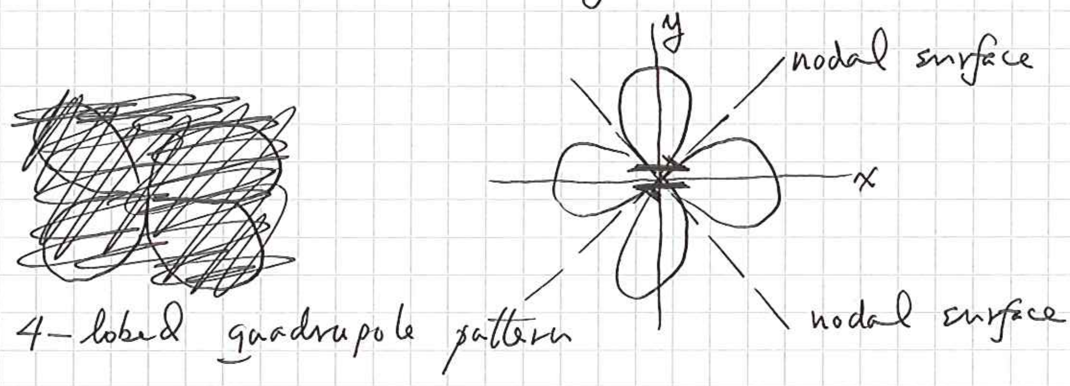
$$-\frac{i}{2}\mathcal{R}_1(\phi) = \frac{1}{2}\left(\frac{ka}{2\pi}\right)^{1/2} ka\left[\partial_r W_s - r_s^{-1} W_s\right]\left[M_{xz}\sin\phi - M_{yz}\cos\phi\right]e^{i\pi/4}$$

$$+\frac{1}{2}\left(\frac{ka}{2\pi}\right)^{1/2} k^2a^2\left[r_s^{-1}W_s\right]\left[-\frac{1}{2}(M_{xx}-M_{yy})\sin 2\phi + M_{xy}\cos 2\phi\right]e^{-i\pi/4}$$

The dependence only on $\sin\phi$, $\cos\phi$, $\sin 2\phi$, $\cos 2\phi$ stems from the $-2 \le m \le 2$ dependence

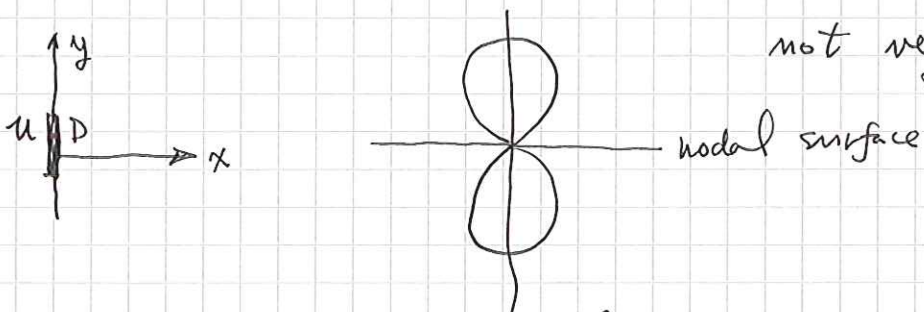
The phase shifts $e^{\pm i\pi/4}$ "leaving" the source are important if one wishes to use phase information to invert for the mechanism M. This is, however, difficult because the phase delay due to propagation must be known and it is path-dependent. A common expedient is to use only amplitude information (esp. for smaller events)

Example – strike-slip fault ($M_{xy} \neq 0$)



Example – vertical dip-slip (normal) fault ($M_{xz} \neq 0$)

not very intuitive



Note in addition that such a fault at the surface does not excite Love (or Rayleigh) waves (since $\partial_r W_s - r^{-1} W_s = 0$ at surface)

Combinations of the above two sources yield patterns like  or , etc.

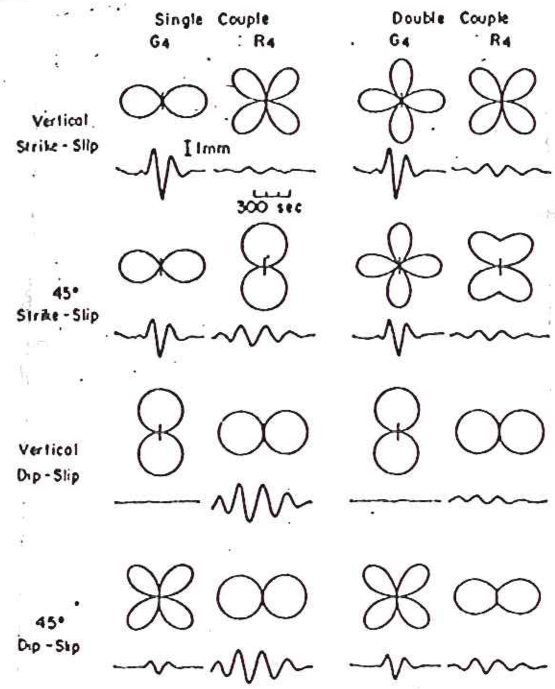


Fig. 7. Radiation pattern (maximum trace amplitude) and relative excitation of Love and Rayleigh waves for eight fundamental source geometries. The wave forms in the loop direction are shown.

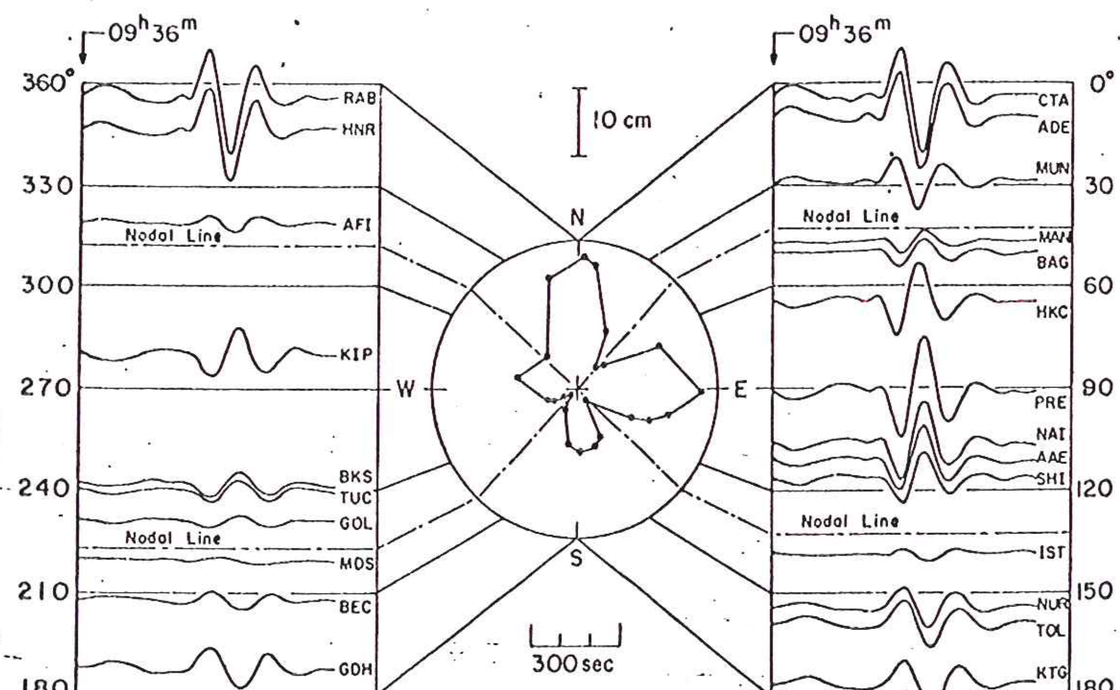


Fig. 12. Synthetic seismograms of Love waves ($G4$). The moment is 7×10^{28} dyne-cm. (For other parameters, see the caption for Figure 11).

LONG-PERIOD MECHANISM OF THE EUREKA, CA, EARTHQUAKE

THORNE LAY, JEFFREY W. GIVEN, AND HIROO KANAMORI

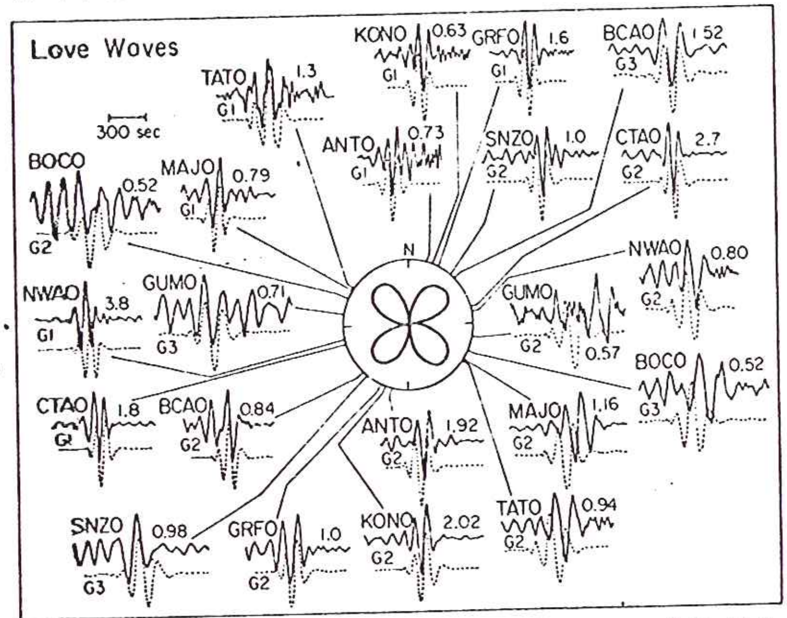
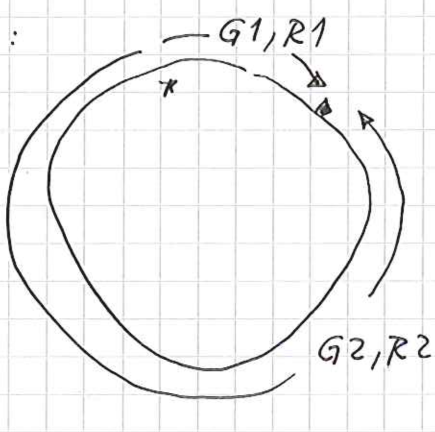


FIG. 4. The Love wave radiation pattern and comparison between synthetic (dotted lines) and observed seismograms. The fault parameters used in the synthetic calculation are listed in Table 9, and a source process time of 30 sec was assumed. The observed seismograms and synthetics for GUMO G_2, GUMO G_3, BOCO G_2, BOCO G_3, TATO G_2, TATO G_1, MAJO G_2, and MAJO G_1 were filtered with a Gaussian band-pass filter between 120 and 1500 sec. The rest of the data and synthetics were band-pass filtered between 80 and 1500 sec. The number above each observed record is the moment (in units of 10^{27} dyne-cm) that would be obtained using that record alone.

For higher orbits $p > 1$ the waves continue to be delayed $e^{-ika\Delta_p}$ and attenuated $e^{-\omega a \Delta_p / 2UQ}$

Notation:



G1, G2 etc. for Love waves (G for Gutenburg)
R1, R2 etc. for Rayleigh waves

Can see up to R11–R12 after a large event (see e.g. the cover of Aki & Richards)

The terms $e^{ip\pi/2}$ and $e^{i(p-1)\pi/2}$ represent the polar phase shift due to the passage through the focal points (degenerate caustics) at the source and antipode.

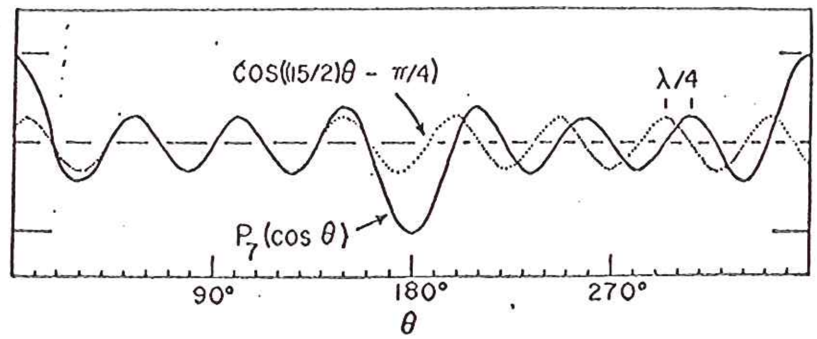


FIG. 1. A comparison of the phase of P_7 with that of its asymptotic representative.

Now, consider Rayleigh waves — for $p = 1$ we have

$$a_1(r,\theta,\phi,\omega) = +\tfrac{1}{2}\left[\hat{\mathbf{r}}\, U(r) - ika\,\hat{\mathbf{\Delta}}\, V(r)\right]$$
$$(\sin\Delta)^{-1/2}\, U^{-1}(\omega)\, e^{-\omega a\Delta/2Q(\omega)\, U(\omega)}\, R_1(\phi,\omega)\, e^{-ik(\omega)\Delta}$$

The interpretation of all the factors is the same — now the motion is radial plus longitudinal.

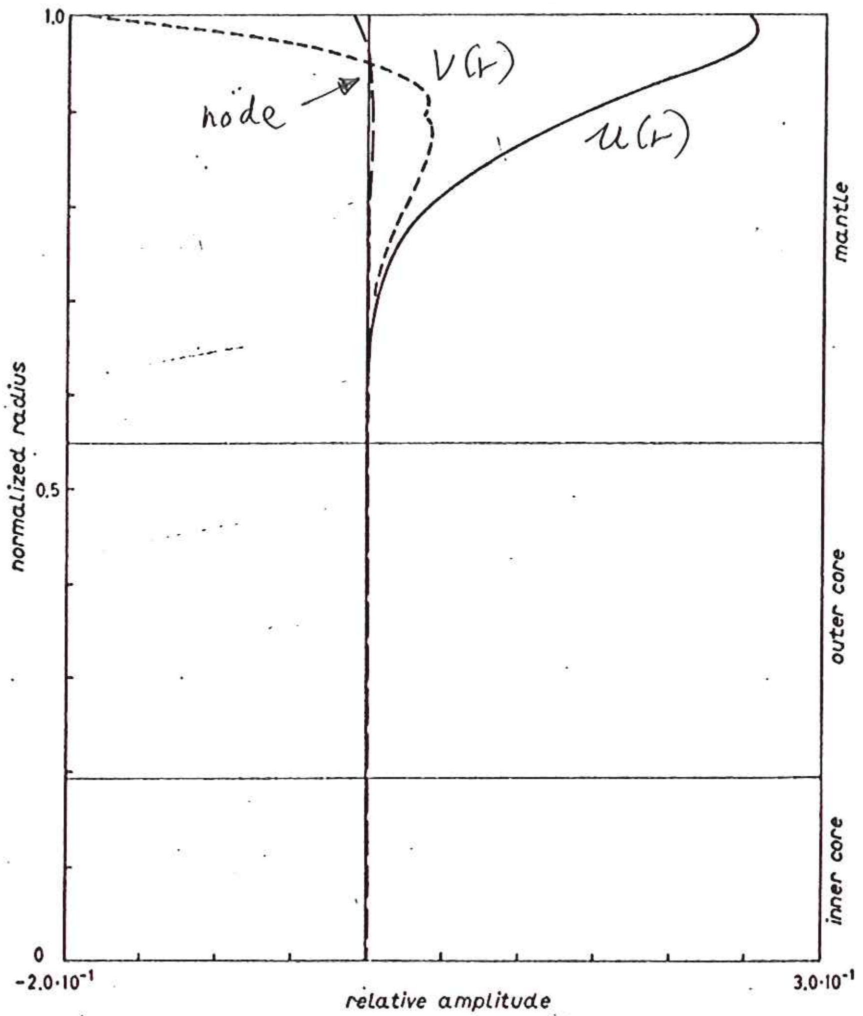
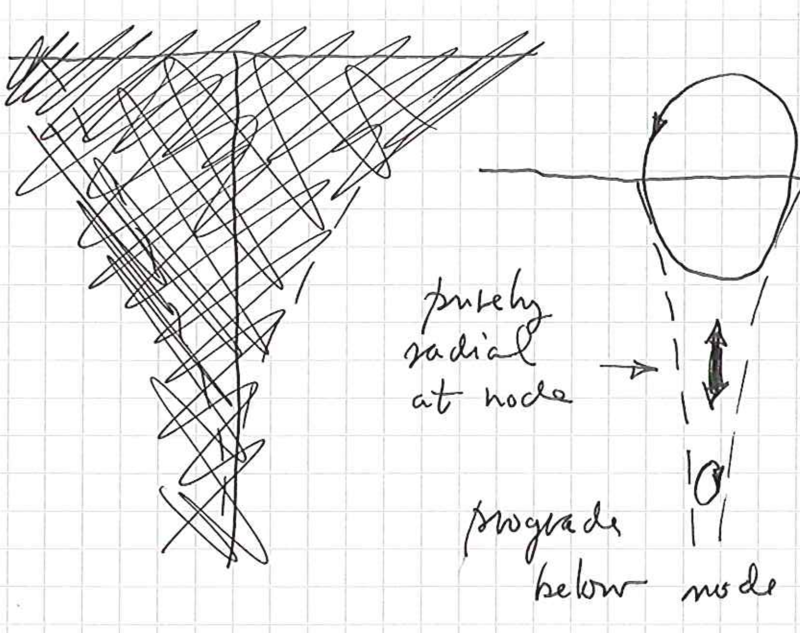


Fig. 10. – The mode ${}_0S_{19}$: displacement scolars U (——) and V (- - -), and perturbation potential Φ (— — —).

Fundamental mode ${}_0S_{19}$ ($T \sim 300$ s)

At the surface $U(a)$ & $V(a)$ have opposite signs. The factor $-i\hat{\Delta}$ in $[\hat{r}U(r) - ika\hat{\Delta}V(r)]$ indicates that the $-\hat{\Delta}$ component lags the $\hat{r}$ component by 90°



well-known retrograde elliptical motion at the surface.