Class #9

Stress glut $\underline{\underline{S}} = \underline{\underline{T}}^{Hooke} - \underline{\underline{T}}^{true}$

Equivalent body force $\underline{f} = -\nabla\cdot\underline{\underline{S}}$ in $\oplus$

Equivalent surface force $\underline{t} = \hat{n}\cdot\underline{\underline{T}}$ on $\partial\oplus$

source volume V

Total force $\underline{F}$ and torque $\underline{N}$ are zero (latter because $\underline{\underline{S}} = \underline{\underline{S}}^T$)

Acceleration response

$$\underline{a}(\underline{x},t) = \sum_k (a_k \cos\omega_k t + b_k \sin\omega_k t)\,\underline{s}_k(\underline{x})$$

$$\begin{Bmatrix} a_k \\ b_k \end{Bmatrix} = \underbrace{\int_{t_0}^{t_f}\int_V}_{} \partial_t \underline{\underline{S}} : \underline{\underline{\varepsilon}}_k \begin{Bmatrix} \cos\omega_k t \\ \sin\omega_k t \end{Bmatrix} dV\,dt$$

integrate only over source volume V and interval $t_0 \le t \le t_f$

Ideal fault

tang. slip $\underline{\Delta s}(\underline{x},t) = \underline{s}(\underline{x}^+,t) - \underline{s}(\underline{x}^-,t)$

$\hat{\nu}\cdot\underline{\Delta s} = 0$

moment tensor density : $m_{ij} = C_{ijkl}\,\nu_j\,\Delta s_k$

$$\underline{\underline{m}} = \underline{\underline{C}} : \hat{\nu}\,\underline{\Delta s}$$

units $\quad \frac{\text{moment}}{\text{area}} = \frac{\text{mass}}{\text{time}^2}$

Then $\quad \underline{\underline{S}} = \underline{\underline{m}}\,\delta_\Sigma$, confined to fault surface, depends only on slip $\underline{\Delta s}$ and orientation $\hat{\nu}$

$[\,\underline{\underline{c}}\,]_-^+$ across Σ

Equivalent body & surface force densities

$$\underline{f} = -\underline{\underline{m}} \cdot \nabla\delta_\Sigma \quad \text{in } \oplus$$

$$\underline{t} = (\hat{n} \cdot \underline{\underline{m}})\,\delta_\Sigma \quad \text{on } \partial\oplus$$

Completely general: dynamic time-dependent faulting in an anisotropic $\oplus$

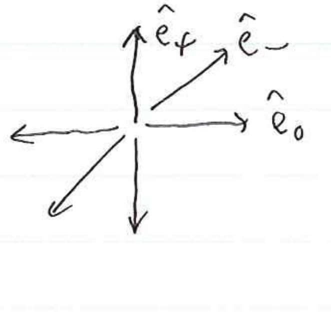
Symmetry $\quad \underline{\underline{m}} = \underline{\underline{m}}^T \Rightarrow$ diagonalizable

$$\underline{\underline{m}} = m_+ \hat{e}_+\hat{e}_+ + m_0 \hat{e}_0\hat{e}_0 + m_- \hat{e}_-\hat{e}_-$$

Equivalent body force density

$$f_j = -m_{ij}\,\partial_i\,\delta_\Sigma$$

Three <u>linear</u> <u>vector</u> <u>dipoles</u>



$m_{+,0,-} > 0$: outward

$m_{+,0,-} < 0$: inward

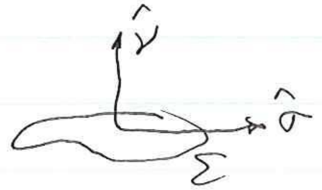
Isotropic $\oplus$: $\quad c_{ijkl} = \overbrace{\left(\kappa - \tfrac{2}{3}\right)\delta_{ij}\delta_{kl}}^{\text{no contr. since } \nu_k \Delta s_k = 0} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

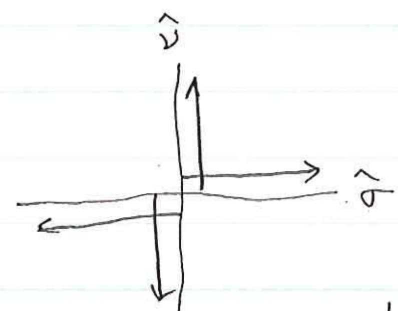
$$m_{ij} = \mu \Delta s \, (\nu_i \sigma_j + \sigma_i \nu_j)$$

where $\underline{\Delta s} = \Delta s \, \hat{\sigma}$

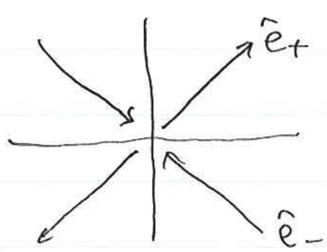
$$\boxed{\underline{\underline{m}} = \mu \Delta s \, (\hat{\nu}\hat{\sigma} + \hat{\sigma}\hat{\nu})}$$

Equivalent body force a <u>double couple</u>



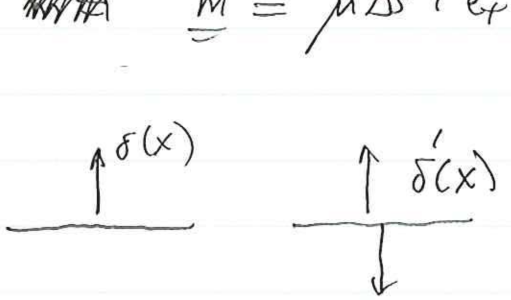


or



obviously no net force or torque

$$\underline{\underline{m}} = \mu \Delta s \, (\hat{e}_+ \hat{e}_+ - \hat{e}_- \hat{e}_-) \qquad \hat{e}_\pm = \tfrac{1}{\sqrt{2}}(\hat{\nu} \pm \hat{\sigma})$$



Valid for a <u>curved</u> or corrugated fault

Mode response to faulting

$$\begin{Bmatrix} a_k \\ b_k \end{Bmatrix} = \int_{t_0}^{t_f} \int_{\Sigma} \partial_t \underline{\underline{m}} : \underline{\underline{\varepsilon}}_k \begin{Bmatrix} \cos \omega_k t \\ \sin \omega_k t \end{Bmatrix} dV\, dt$$

integrate only ~~over~~ over portion of Σ where slip is actively occurring.

<u>Point source</u> approx<u>imation</u> : moment tensor

long wavelength >> source dimensions
long period >> source duration

$$\underline{\underline{\varepsilon}}_k (\underline{x}) \begin{Bmatrix} \cos \omega_k t \\ \sin \omega_k t \end{Bmatrix} \approx \underline{\underline{\varepsilon}}_k (\underline{x}_s) \begin{Bmatrix} \cos \omega_k t_s \\ \sin \omega_k t_s \end{Bmatrix}$$

x_s, t_s hypocenter location in point source approximation

$$\begin{Bmatrix} a_k \\ b_k \end{Bmatrix} = \underline{\underline{M}} : \underline{\underline{\varepsilon}}_k (\underline{x}_s) \begin{Bmatrix} \cos \omega_k t_s \\ \sin \omega_k t_s \end{Bmatrix}$$

where $M = \int_{t_0}^{t_f} \int_V \partial_t \underline{\underline{S}}\, dV\, dt$, <u>moment tensor</u>

$$= \int_V \underline{\underline{S}}^{\text{final state}}\, dV$$

volume integral of the final static stress glut.

$$\cos a \cos b + \sin a \sin b = \cos(a-b)$$

Responce

$$\underline{a}(\underline{x},t) = \sum_k \underbrace{\underline{\underline{M}} : \underline{\underline{\varepsilon}}_k(\underline{x}_s)}_{\text{ampl.}} \underbrace{\underline{s}_k(\underline{x})}_{\text{shape}} \underbrace{\cos \omega_k (t-t_s)}_{\substack{\text{begins at} \\ t=t_s \\ \text{origin time}}}$$

Linear in moment tensor $\underline{\underline{M}}$

Read quote from Gilbert

Moment tensor for an ideal earthquake fault

$$\underline{\underline{M}} = \int_\Sigma \underline{\underline{m}}^{\text{static final}} \, dA \qquad \left(\int_\oplus \underline{\underline{m}}^{\text{final}} \delta_\Sigma \, dV \right)$$

Isotropic $\oplus$:

$$\boxed{\underline{\underline{M}} = \int_\Sigma \mu \Delta s^{\text{final}} (\hat{\nu}\hat{\sigma} + \hat{\sigma}\hat{\nu}) \, dA}$$

Glut rate in point source approx

$$\partial_t \underline{\underline{S}} = \underline{\underline{M}} \, \delta(\underline{x}-\underline{x}_s) \, \delta(t-t_s)$$

Body force $\quad \underline{f} = -\underline{\underline{M}} \cdot \nabla \delta(\underline{x}-\underline{x}_s) \, \delta(t-t_s)$

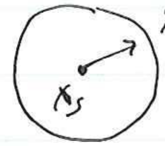
3 linear vector dipoles

Planar fault, uni-directional slip :

$$\underline{\underline{M}} = M_0 (\hat{\nu}\hat{\sigma} + \hat{\sigma}\hat{\nu}) \qquad M_0 = \int_\Sigma \mu \Delta s^f \, dA$$

classic double couple $\qquad$ moment Aki (1966) skip this

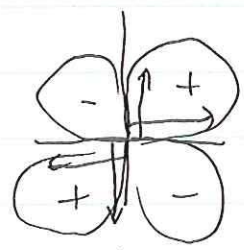
P-wave radiation pattern :



$\hat{r} \cdot \underline{\underline{M}} \cdot \hat{r}$

focal sphere

beachball



quadrupole pattern

talk about later

CMT = centroid - moment tensor

Solve for $\underline{\underline{M}}$ as well as an updated location $\underline{x}_s + \underline{\Delta x}$, $t_s + \Delta t$

$$\underline{a}(\underline{x}, t) = \sum_k \underline{\underline{M}} : \underline{\underline{\varepsilon}}_k(\underline{x}_s)\, \underline{s}_k(\underline{x}) \cos \omega_k (t - t_s)$$

$$+ \sum_k \underline{\Delta x}\, \underline{\underline{M}} : \underline{\underline{\nabla \varepsilon}}_k(\underline{x}_s)\, s_k(\underline{x}) \cos \omega_k (t - t_s)$$

$$+ \sum_k \omega_k \Delta t\, \underline{\underline{M}} : \underline{\underline{\varepsilon}}_k(\underline{x}_s)\, \underline{s}_k(\underline{x}) \sin \omega_k (t - t_s)$$

Linear inverse problem for $\underline{\underline{M}}$, $\underline{\Delta x}$, Δt

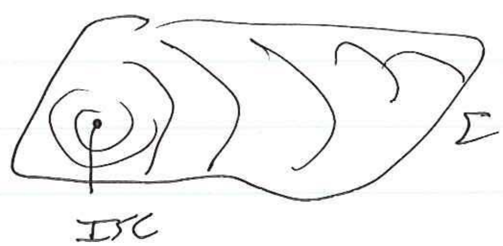
Can be shown (see D&T 5.4.2) that for a planar fault, uni-directional slip

$$\underline{\Delta x} = \frac{1}{M_0} \int_\Sigma (\underline{x} - \underline{x}_s) \mu \, \Delta s^f \, dA$$

on fault

$$\Delta t = \frac{1}{M_0} \int_{t_0}^{t_f} \int_\Sigma (t - t_s) \mu \, \partial_t \Delta s \, dA \, dt$$

Centroid may differ from ISC location = beginning of rupture



Number of unknowns in a CMT inversion : 10

5.4.3 Deviatoric & double-couple sources

Can always decompose into isotropic + deviatoric

$$\underline{\underline{M}} = \frac{1}{3} (\mathrm{tr}\, \underline{\underline{M}}) \underline{\underline{I}} + \underline{\underline{M}}^{dev}$$

ideal fault source in isotropic ⊕ : $\mathrm{tr}\, \underline{\underline{M}} = 0$

$$\underline{\underline{M}} = \int_\Sigma \mu \, \Delta s_f \, (\hat{\nu} \hat{\sigma} + \hat{\sigma} \hat{\nu}) \, dA$$

curved fault

Common to impose linear constraint $\mathrm{tr}\, \underline{\underline{M}} = 0$
no volume change ⊕ source

Some evidence for $\mathrm{tr}\,\underline{\underline{M}} = 0$ mechanisms in volcanic settings

CMT unknowns → 9

May also wish to compare with best double couple

$\det \underline{\underline{M}}$ for a double couple

Non-linear : hard to impose

Instead CMT finds best-fitting double couple

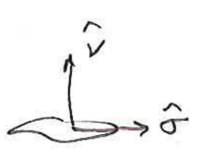
$$(\underline{\underline{M}}_{bfdc} - \underline{\underline{M}}) : (\underline{\underline{M}}_{bfdc} - \underline{\underline{M}}) = \min$$

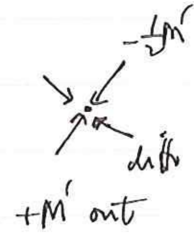
Solution found by diagonalization

$$\underline{\underline{M}} = \begin{pmatrix} M & & \\ & -M - M' & \\ & & M' \end{pmatrix}$$ where $|M| \geq |M'|$

note $\mathrm{tr}\,\underline{\underline{M}} = 0$

$$= \underbrace{\begin{pmatrix} M + \frac{1}{2}M' & & \\ & -M - \frac{1}{2}M' & \\ & & 0 \end{pmatrix}}_{bfdc} + \underbrace{\begin{pmatrix} -\frac{1}{2}M' & & \\ & -\frac{1}{2}M' & \\ & & M' \end{pmatrix}}_{clvd}$$





CMT also gives so-called major and minor double couple

$$= \underbrace{\begin{pmatrix} M & & \\ & -M & \\ & & 0 \end{pmatrix}}_{\text{major}} + \underbrace{\begin{pmatrix} 0 & & \\ & -M' & \\ & & M' \end{pmatrix}}_{\text{minor}}$$

4% of events in CMT are significantly non-double-couple

Reasons: (1) noise in estimate

(2) anisotropy

$$M = \int_\Sigma \underline{\underline{c}} : \hat{\nu} \, \Delta s_f \, dA$$

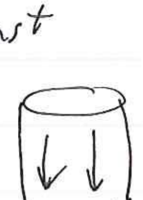
probably small

(3) curvature of fault surface

$$\underline{\underline{M}} = \int_\Sigma \mu \, \Delta s_f \, (\hat{\nu}\hat{\sigma} + \hat{\sigma}\hat{\nu})$$

Candidate geometries limited:

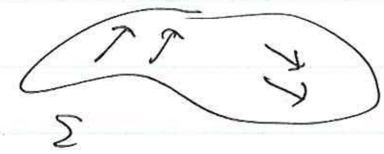
e.g. $\hat{\sigma} = \text{const}$



need not be whole can

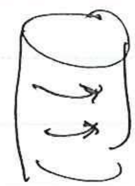
$$\underline{\underline{M}} = \int_\Sigma \mu \, \Delta s_f \, \hat{\nu} \, dA \, \hat{\sigma} + \hat{\sigma} \int_\Sigma \mu \, \Delta s_f \, \hat{\nu} \, dA$$

$\hat{\nu}$ constant



can do same

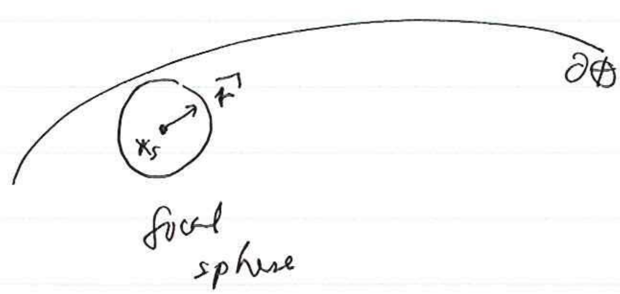
$\hat{\nu} \times \hat{r}$ constant



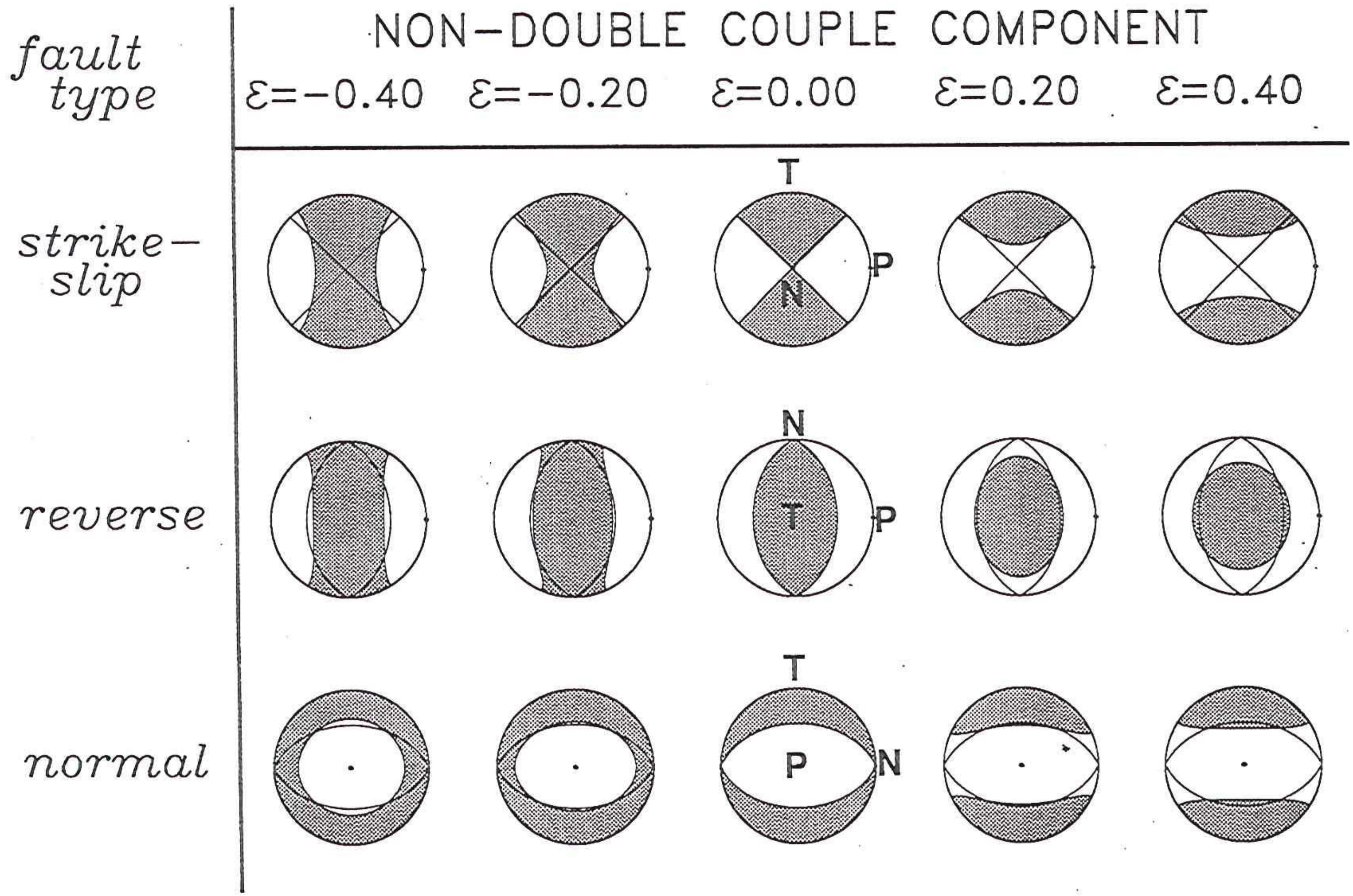
need not be whole can

Most interesting — example — Göran Ekström's volcanic ring faults.

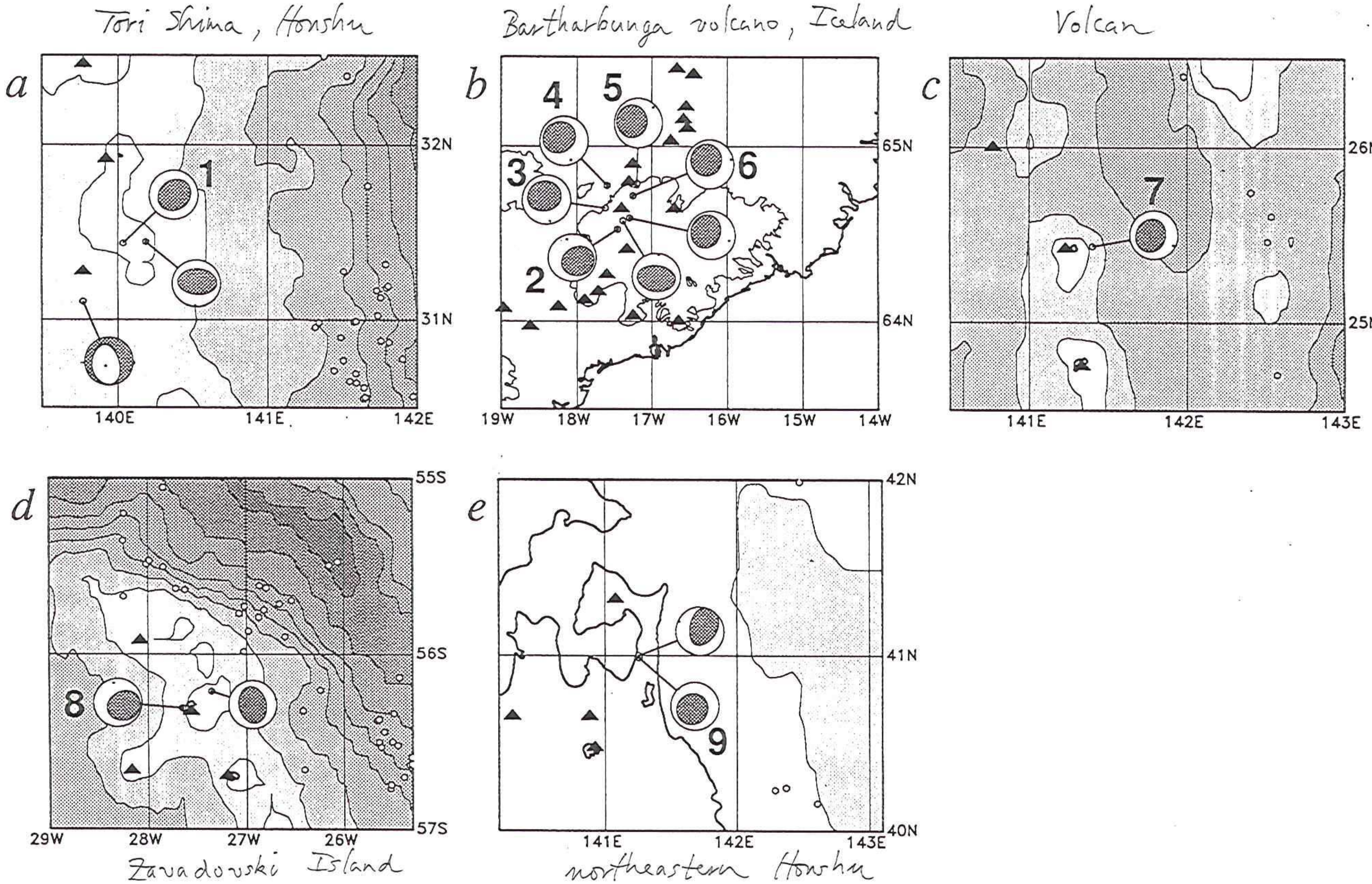
Beach balls $\hat{r} \cdot \underline{M} \cdot \hat{r}$

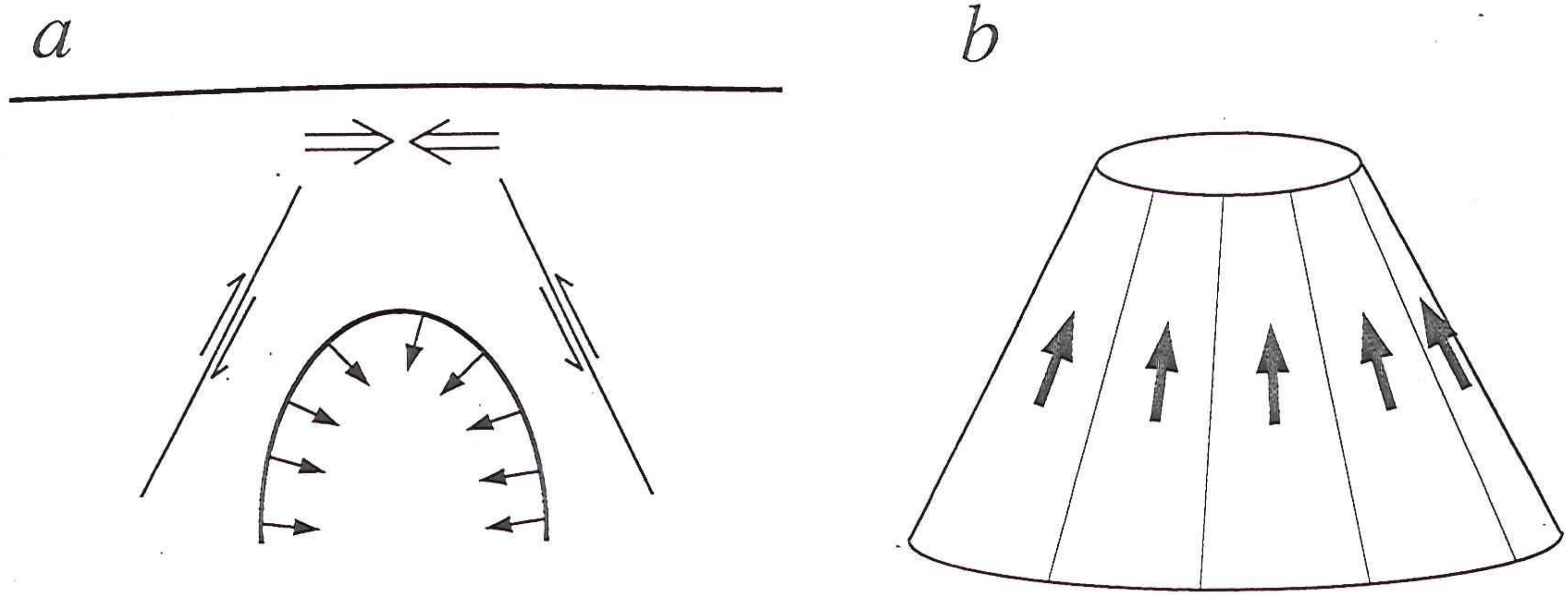


end class #9

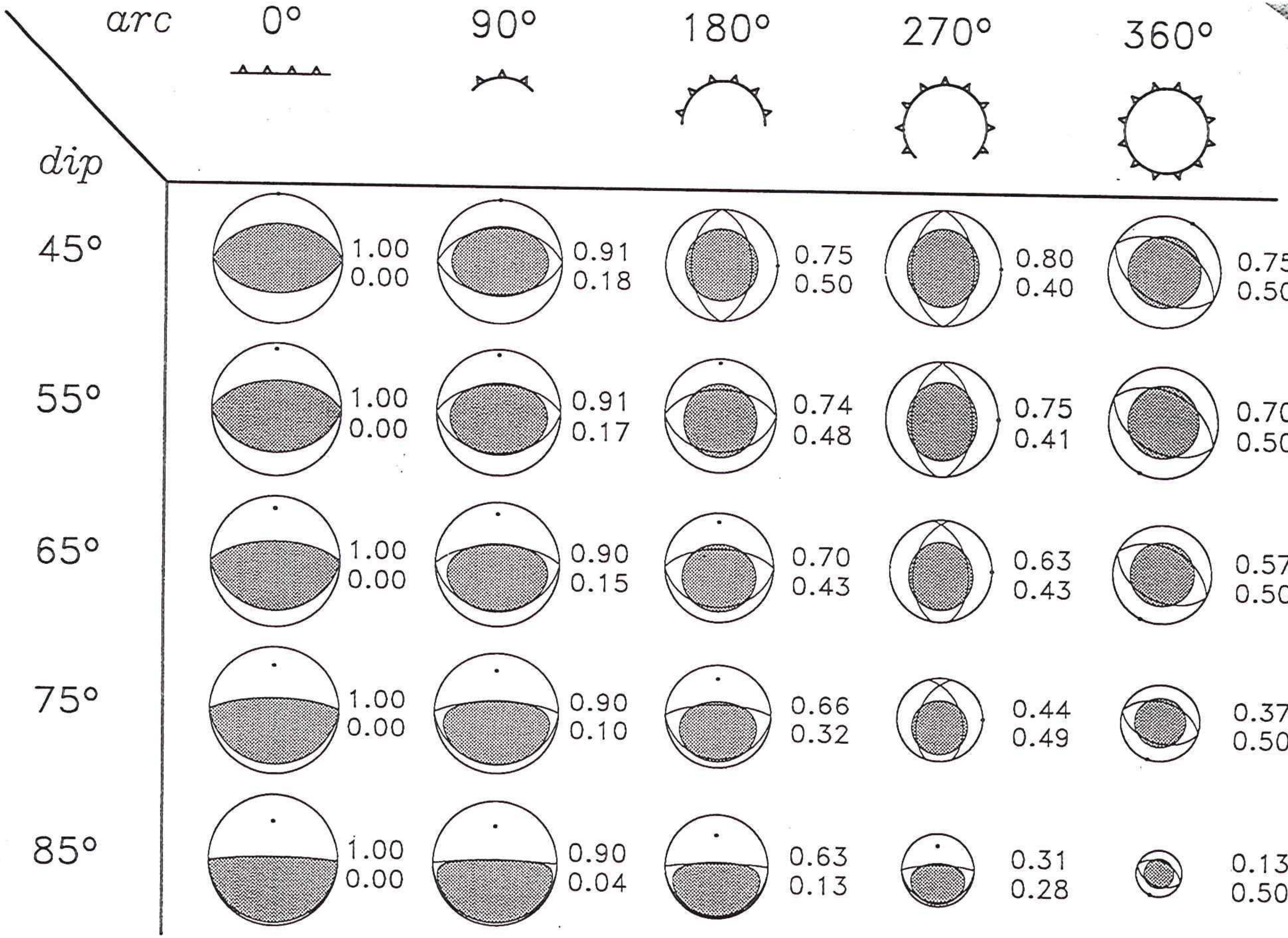


$$\varepsilon = \frac{-M_2}{\max(M_1, -M_3)} \qquad \text{where } M_1 \geq M_2 \geq M_3$$


Tori Shima, Honshu
a
1
32N
31N
140E
141E
142E
Bartharbunga volcano, Iceland
b
2
3
4
5
6
65N
64N
19W
18W
17W
16W
15W
14W
Volcan
c
7
26N
25N
141E
142E
143E
d
8
55S
56S
57S
29W
28W
27W
26W
Zavadovski Island
e
9
42N
41N
40N
141E
142E
143E
northeastern Honshu



volcanic ring fault



scalar moment
ε