Banana-Doughnut Kernels, Adjoint Methods & Seismic Tomography: The Legacy of F.A. Dahlen

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We discuss the theory behind seismic tomography based on 3D Earth models, numerical simulations of 3D wave propagation, and adjoint methods. Our approach involves computing the Fréchet derivatives for tomographic inversions via the interaction between a forward wavefield, propagating from the source to the receivers, and an 'adjoint' wavefield, propagating from the receivers back to the source. Both wavefields are computed using a spectral-element method (SEM) and a heterogeneous wave-speed model. We specify an objective or misfit function that defines a suitable measure of misfit between data and synthetics. For a given receiver, the differences between the data and the synthetics are time reversed and used as the source of the adjoint wavefield. For each earthquake, the interaction between the regular and adjoint wavefields is used to construct finite-frequency sensitivity kernels, which we call 'event' kernels. These kernels may be thought of as weighted sums of the measurement-specific 'banana-doughnut' kernels first introduced by F.A. Dahlen, with weights determined by the measurements. The overall sensitivity is simply the sum of event kernels, which defines the 'misfit' kernel, i.e., the Fréchet derivative. A conjugate gradient algorithm is used to iteratively improve the model while reducing the misfit function. We illustrate the characteristics of these 3D finite-frequency kernels based upon adjoint simulations for a variety of global arrivals, e.g., Pdiff, P'P', S650S, and SKS, and we demonstrate how the approach may be used to investigate body- and surface-wave anisotropy.

In 'adjoint tomography', any time segment in which the data and synthetics match reasonably well is suitable for measurement, and this implies a much greater number of phases per seismogram can be used compared to classical tomography, in which the sensitivity of the measurements is determined analytically for specific arrivals, e.g., P or S. We use an automated picking algorithm based upon short-term/long-term averages and strict phase and amplitude anomaly criteria to determine arrivals and time windows suitable for measurement. For shallow global events the algorithm typically identifies of the order of 1000 windows suitable for measurement, whereas for a deep event the number can reach 4000. For southern California earthquakes the number of phases is of the order of 100 for a magnitude 4.0 event and up to 450 for a magnitude 5.0 event.