Shrinking Ice Sheets, Rising Sea Level

Today and in the Last InterGlacial

Frederik J Simons

Princeton University

Robert E. Kopp — *Rutgers University*

Chris T. Harig — *University of Arizona*

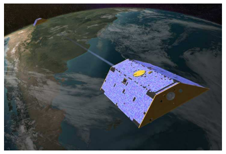
Adam C. Maloof **Michael O. Oppenheimer**

Princeton University

Jerry X. Mitrovica

Harvard University

Outline

- The GRACE mission, in a nutshell 

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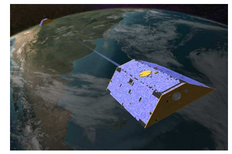

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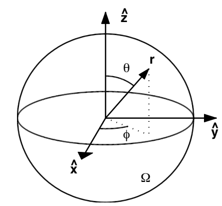

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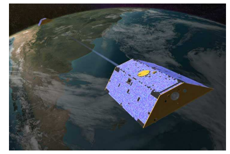

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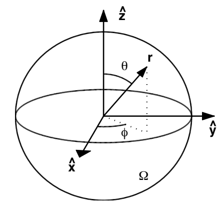

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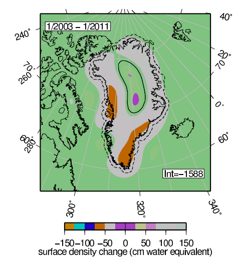

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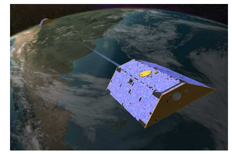

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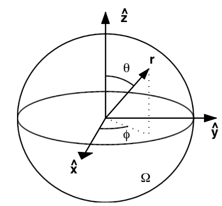

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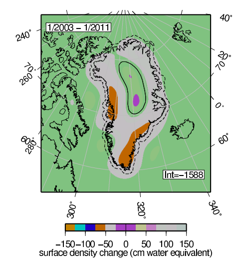

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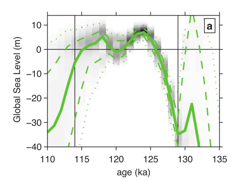
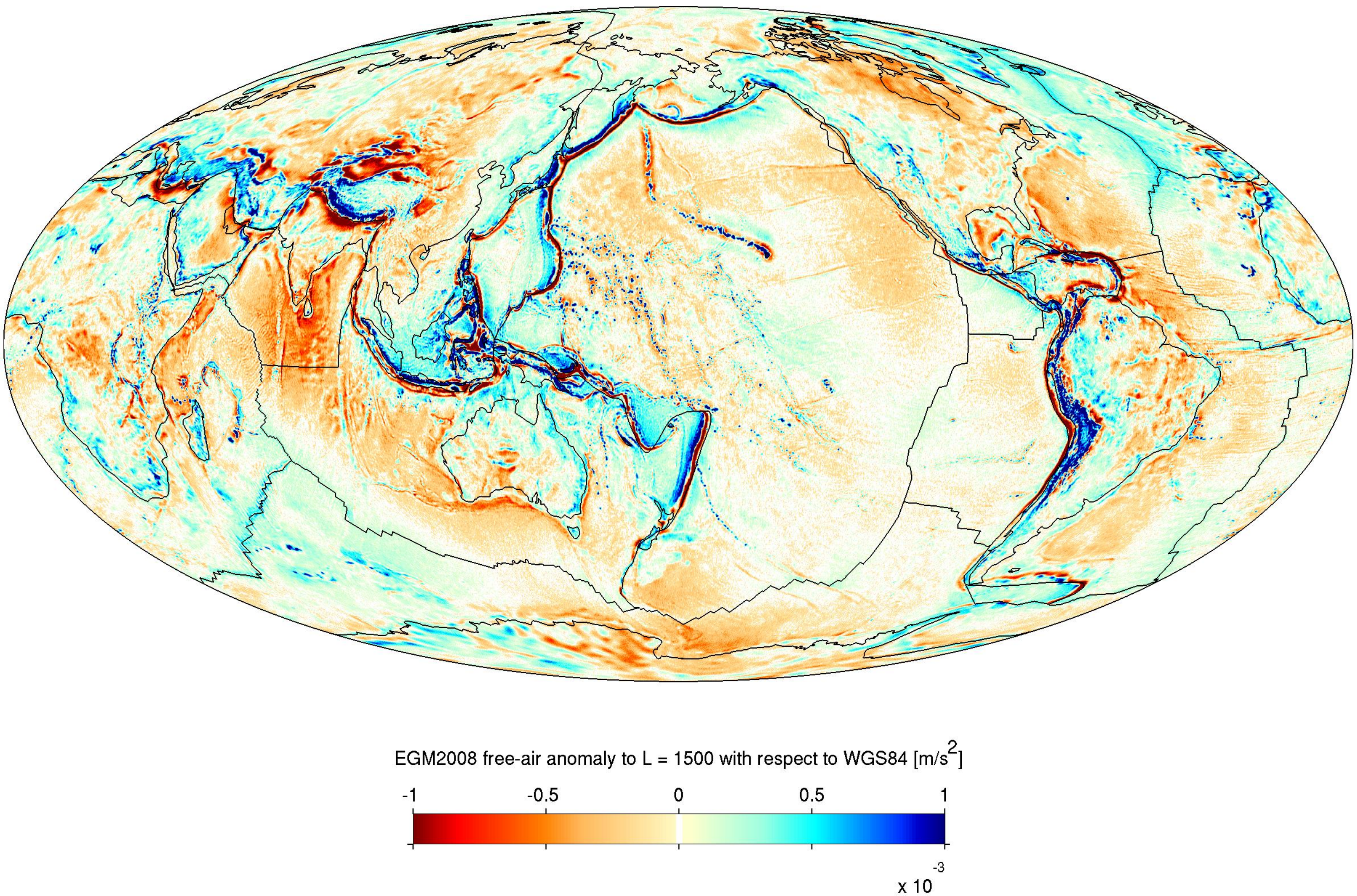
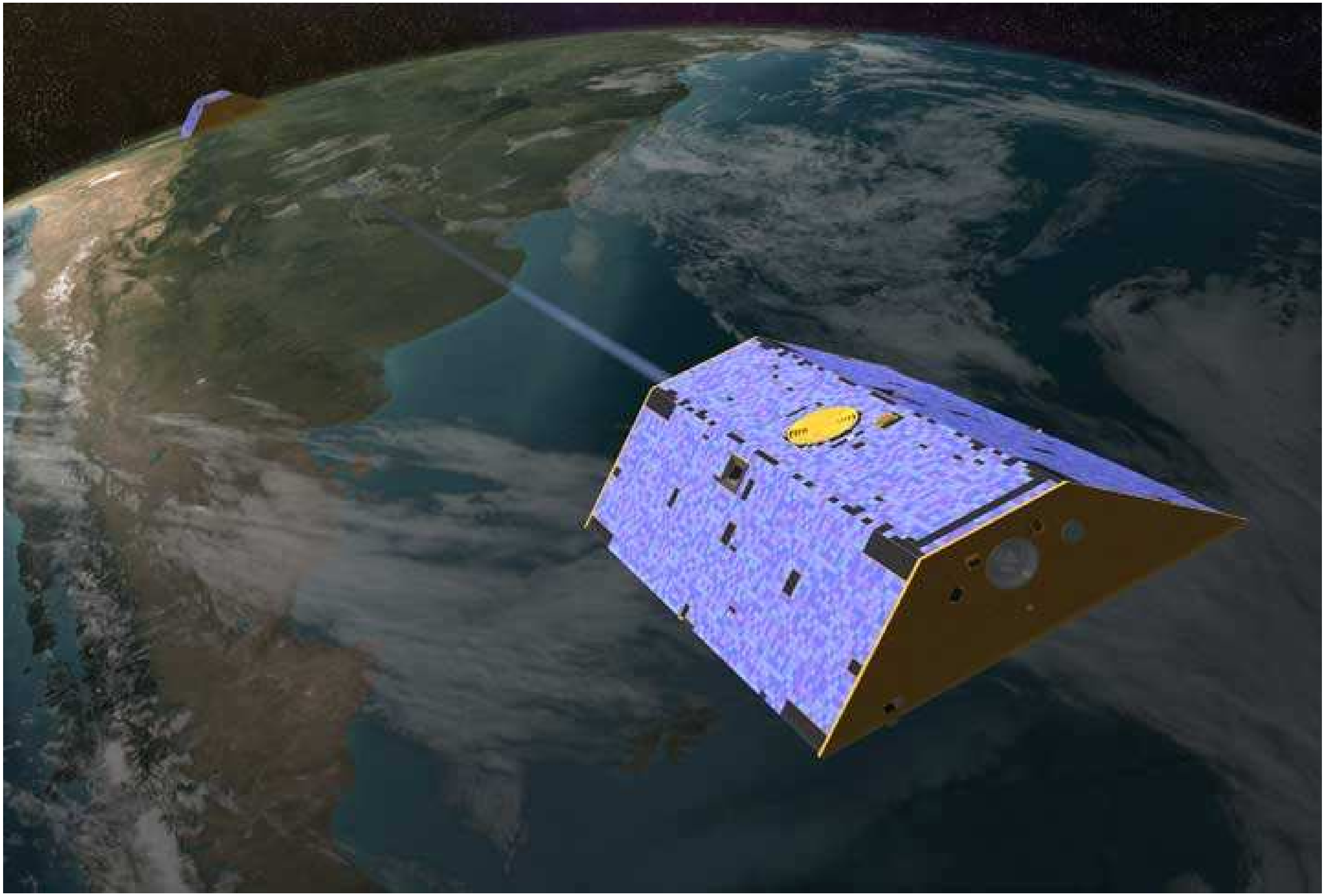


Earth's gravity field is highly variable...



$[\pm 1] \times 10^{-3}$

...and it changes over time



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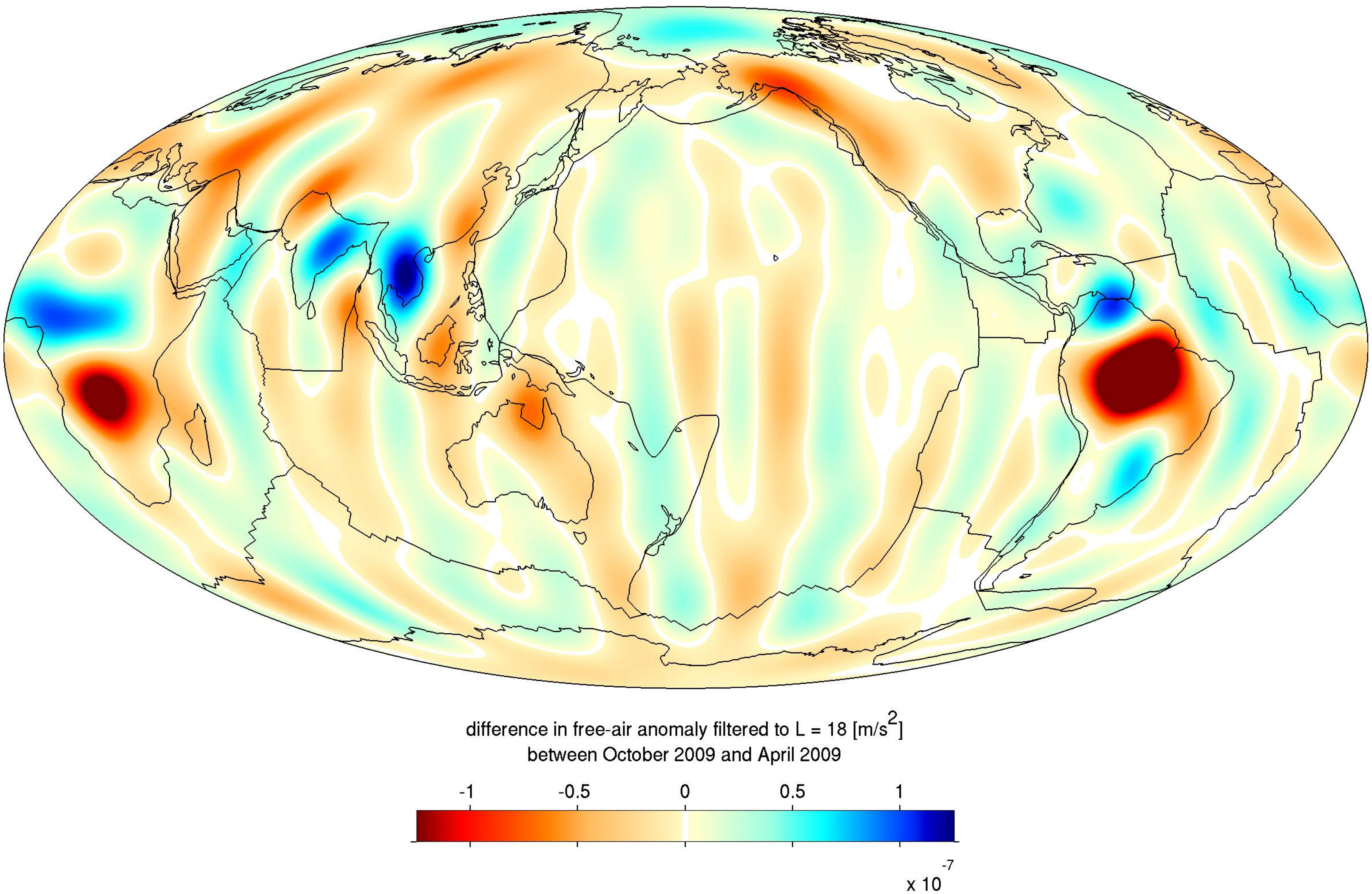
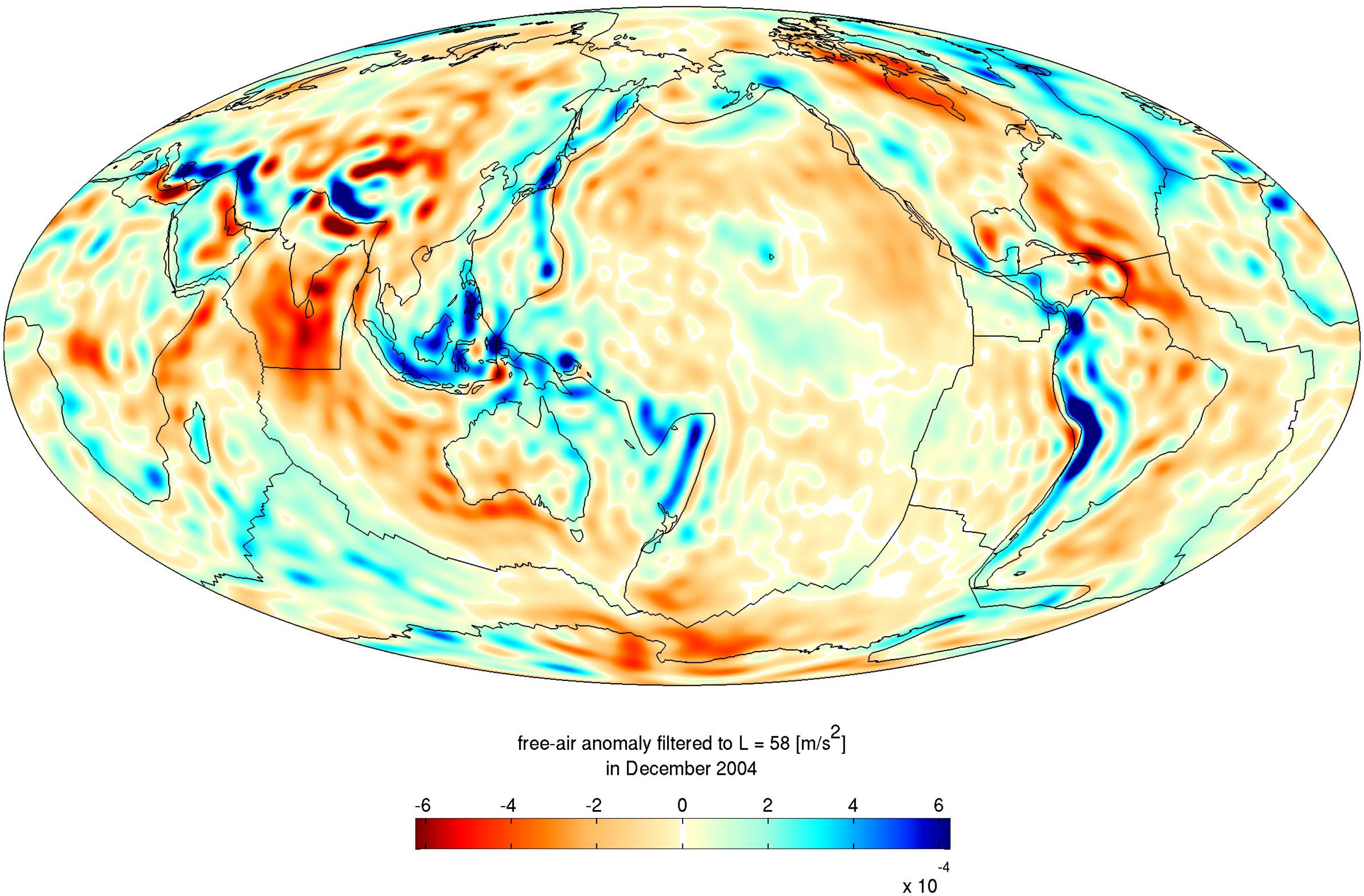
- **The question is, of course:**
  with what *spatial*, *temporal*, and *spectral* resolution?

The hydrological signal is big and large



$L = 18; [\pm 1]\times 10^{-7}$

What lurks in the high-frequency “noise”? – 1



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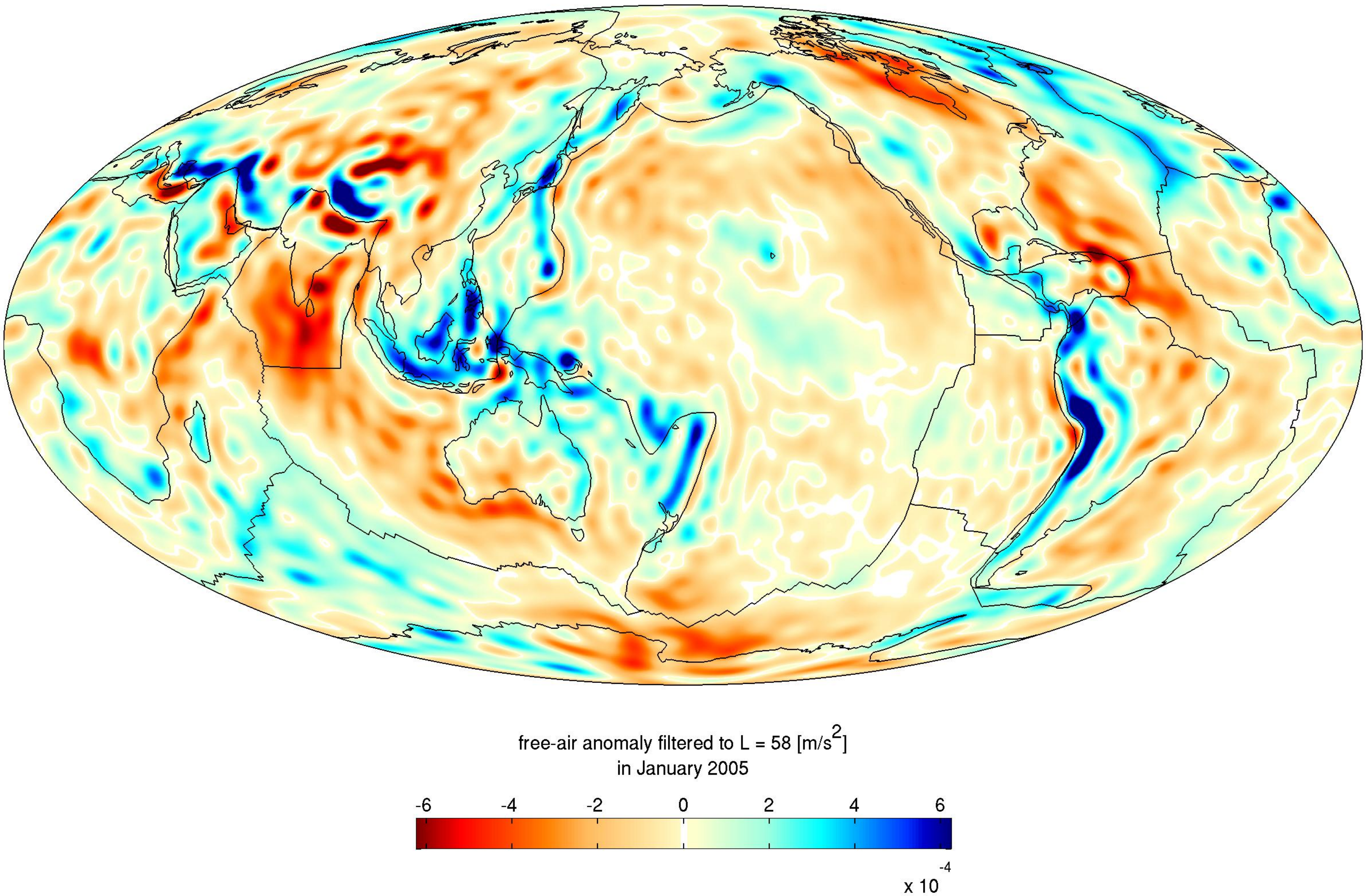
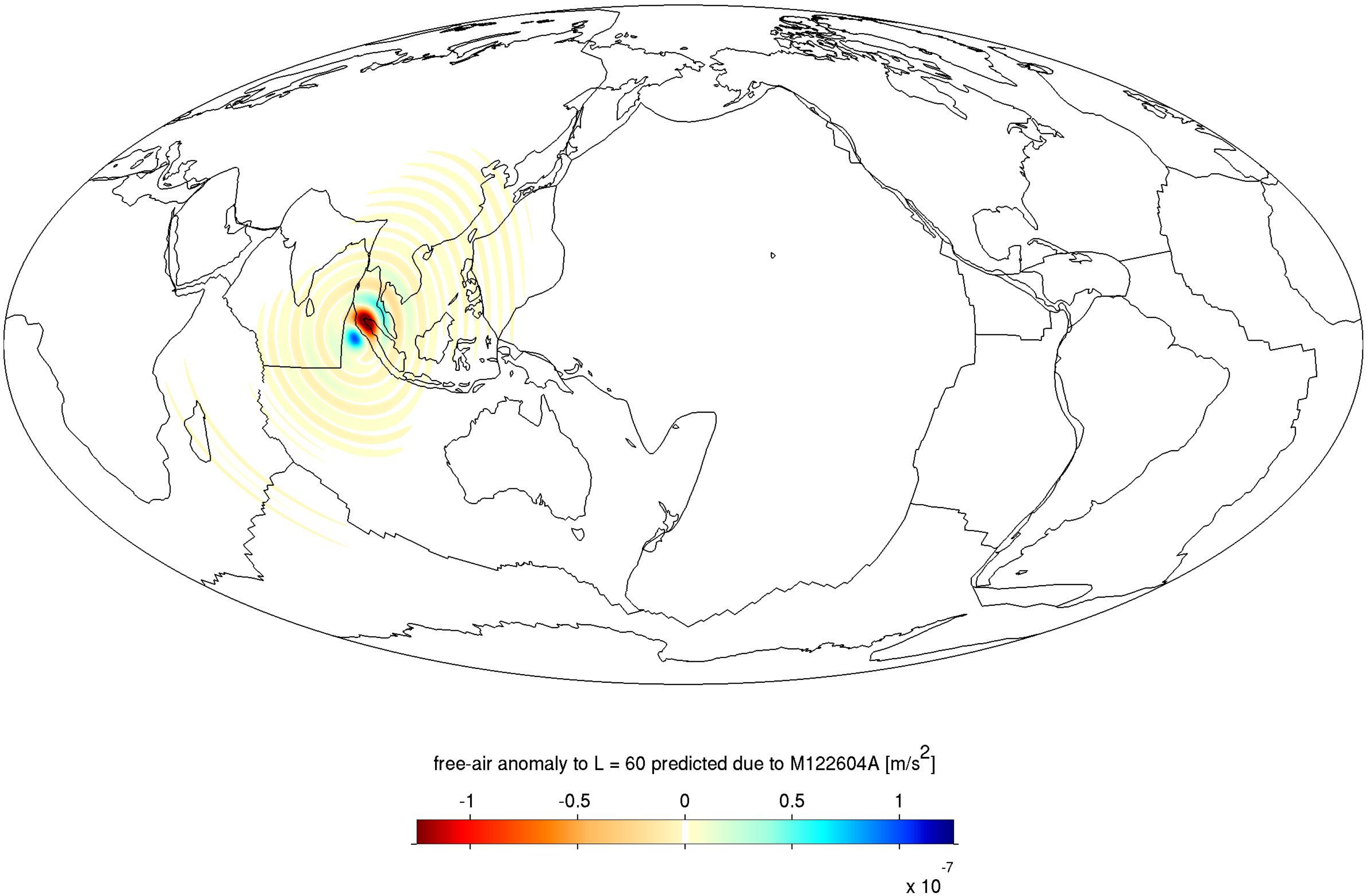
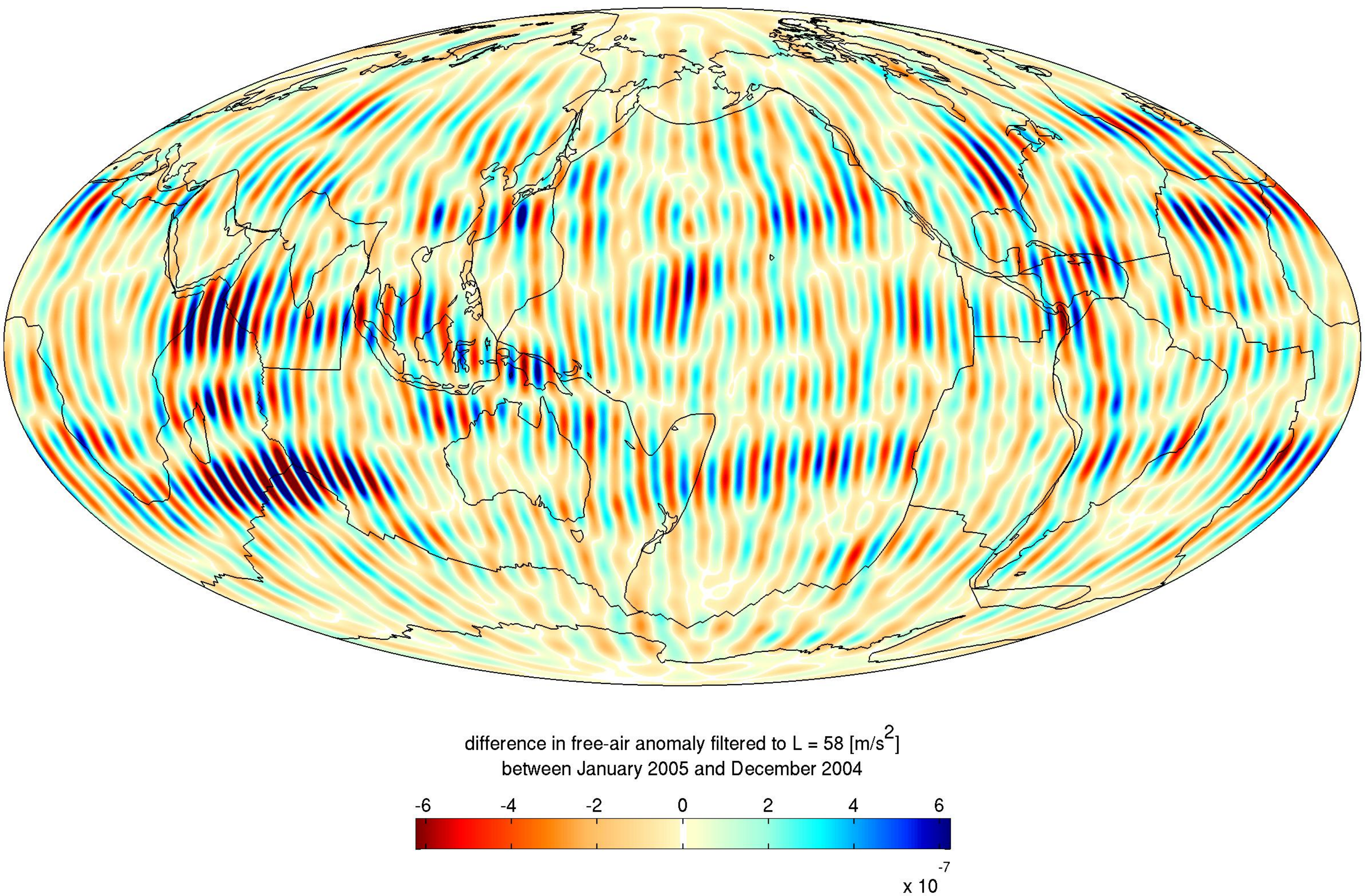


Earthquakes are small (even large ones)



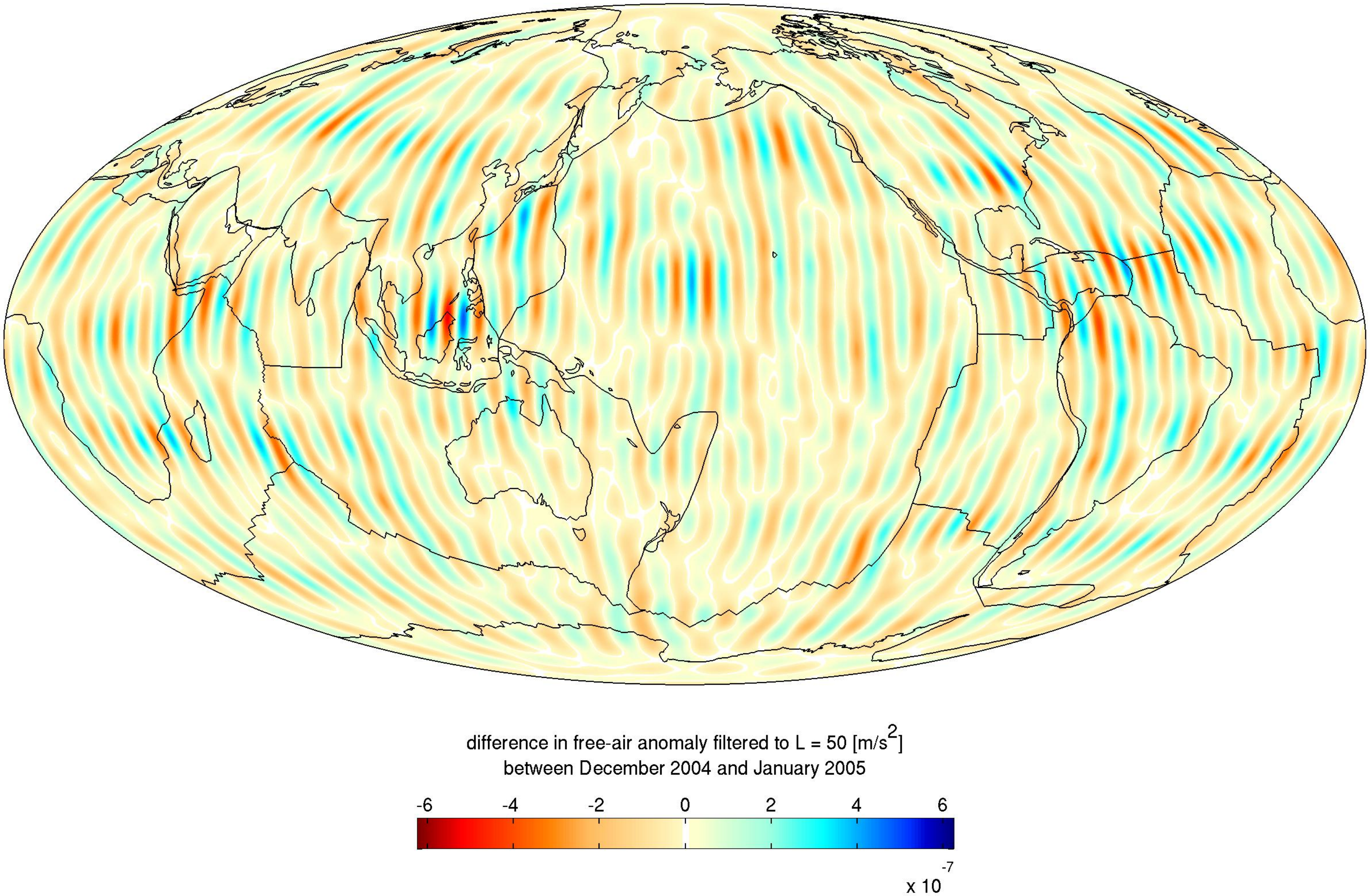
$L = 60; [\pm 1] \times 10^{-7}$

Difference Jan 2005 – Dec 2004



$L = 58; [\pm 6] \times 10^{-7}$

Filtered difference Jan 2005 – Dec 2004



$L = 50; [\pm 6] \times 10^{-7}$

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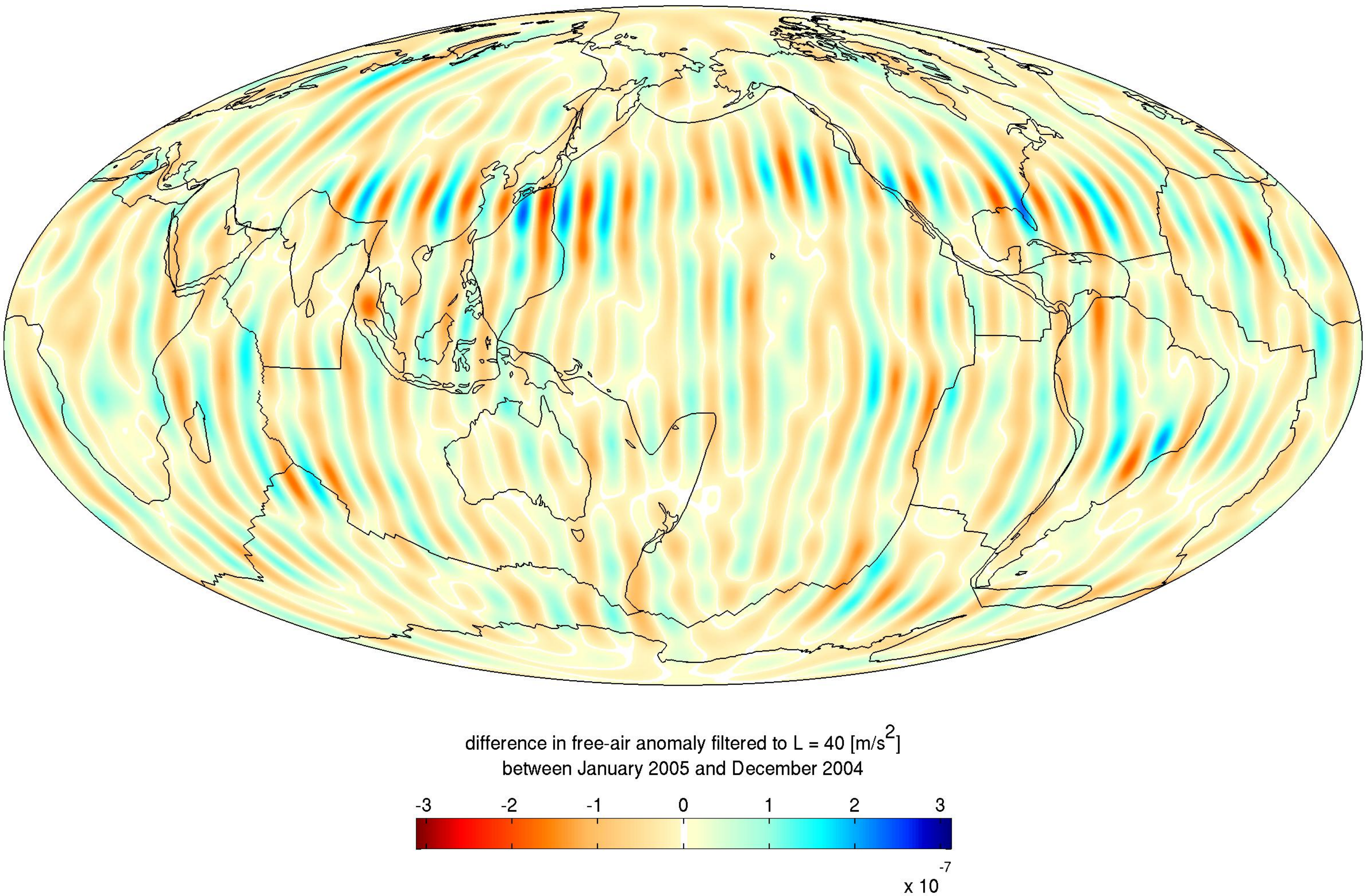

$L = 40; [\pm 3] \times 10^{-7}$

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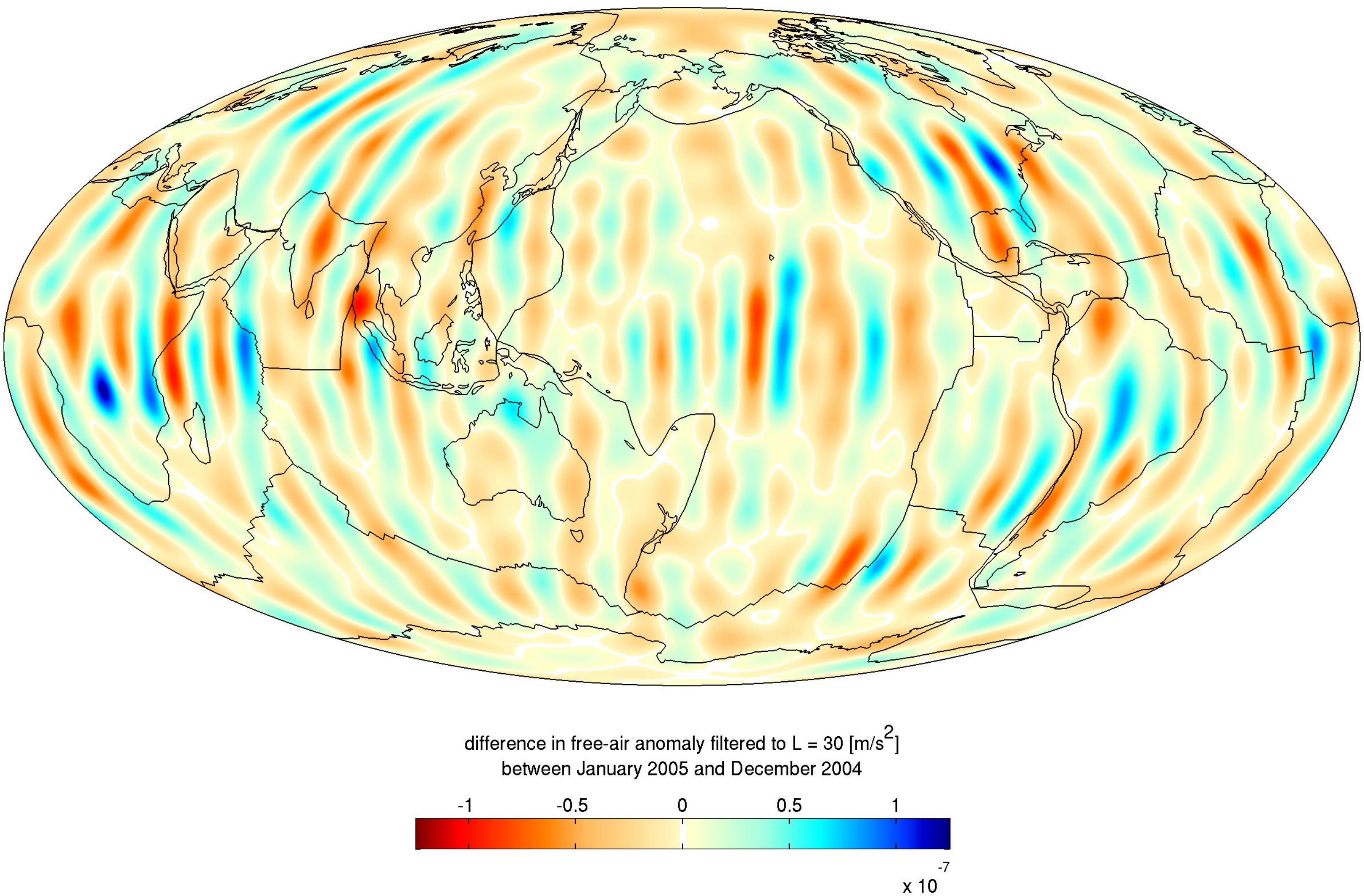

$L = 30; [\pm 1] \times 10^{-7}$

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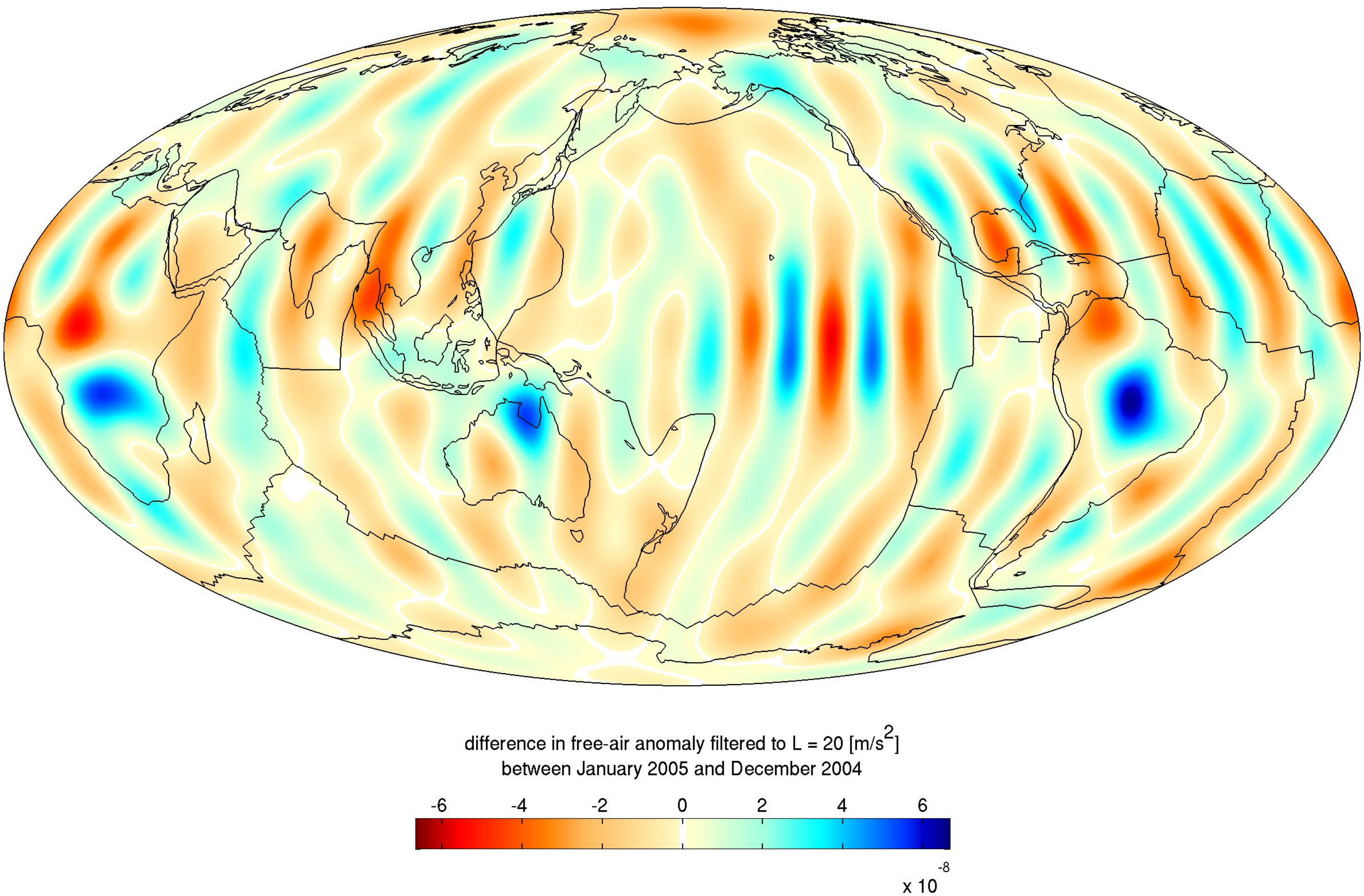

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  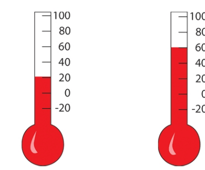

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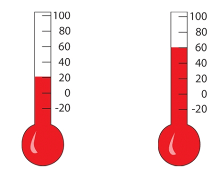
- Let's *focus* on the **climate signal**: longer-term, multi-annual trends. 

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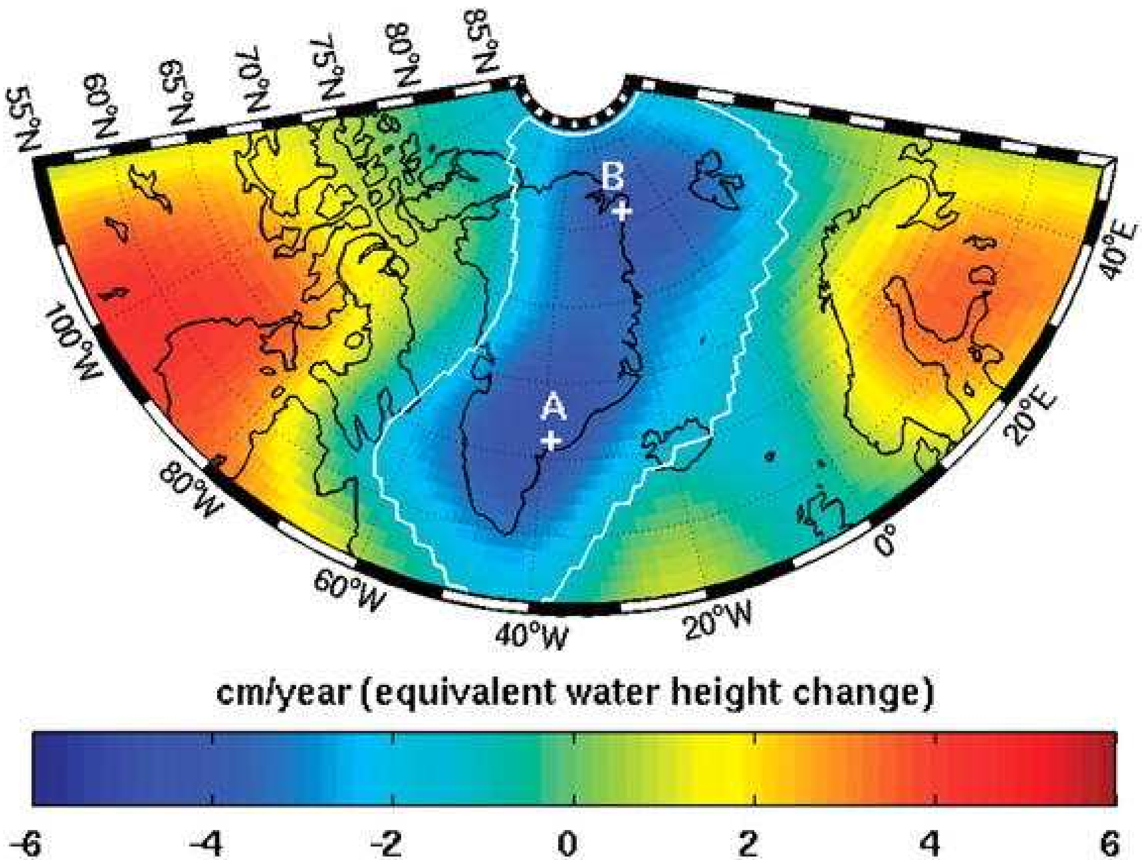
  Aware of the huge challenges to beat elevated noise levels at small spatial footprints, the community has developed a multitude of **filtering** methods to **enhance signal-to-noise ratios** and, in particular, to eliminate the prominent effect of the satellite orbits on the behavior of the solutions (**destriping**).

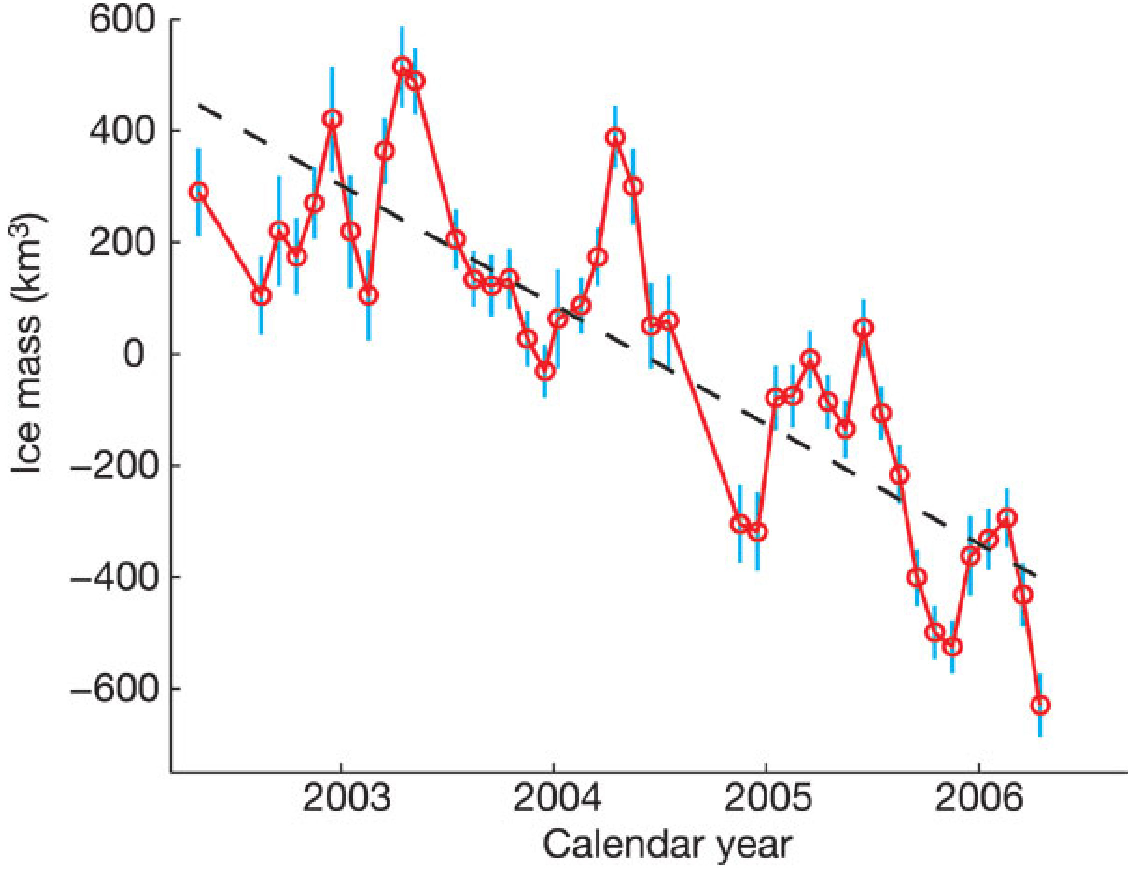
Greenland – 1 (Apr 2002 – Nov 2005)



Chen, Wilson & Tapley, *Science* (2006):

“Spatial leakage effects are also evident, because of filtering applied to suppress the noise in high-degree and high- order spherical harmonics.”

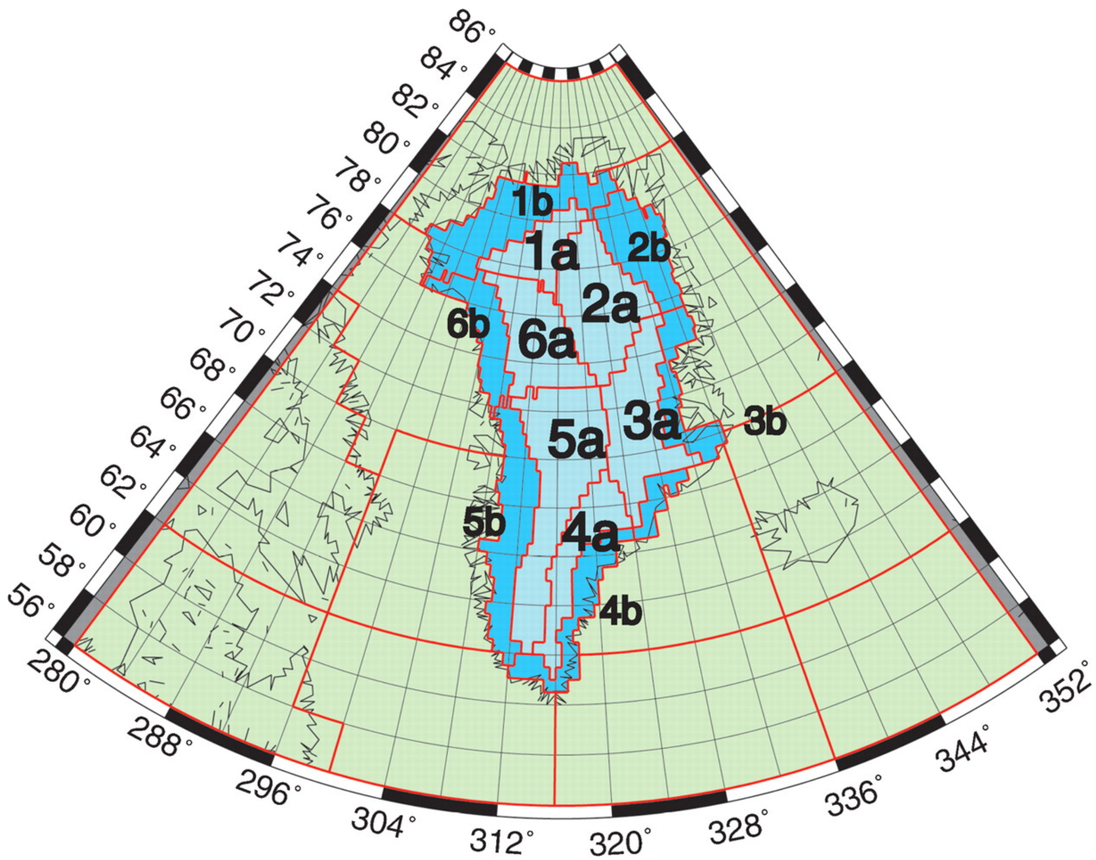
Greenland – 2 (Apr 2002 – Apr 2006)



Velicogna & Wahr, *Nature* (2006):

"Interpreting the trend as due entirely to a change in ice, and subtracting the leakage trend, we inferred an ice volume decrease of $240 \pm 12\ \mathrm{km}^3\mathrm{yr}^{-1}$."

Greenland – 3 (Jul 2003 – Jul 2005)



Luthcke *et al.*, *Science* (2006):

“Our overall Greenland mass trend of 101 ± 16 km^{3}yr^{-1} is a factor of 2 smaller than the recent GRACE*-based trend determined by Velicogna & Wahr (2006).”*

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- Authors *disagree* on matters as fundamental as the **choice of basis** to represent the solution. Pixels? Mascons? Spherical harmonics? How do these choices influence the results?

The problem – 1

The data **collected in** or **limited to** R are **signal** plus **noise**:

We may assume that $n(\mathbf{r})$ is **zero-mean** and **uncorrelated** with the signal,

and consider the **noise covariance**:

In other words: we've got **noisy** and **incomplete** data, on a sphere, Ω.

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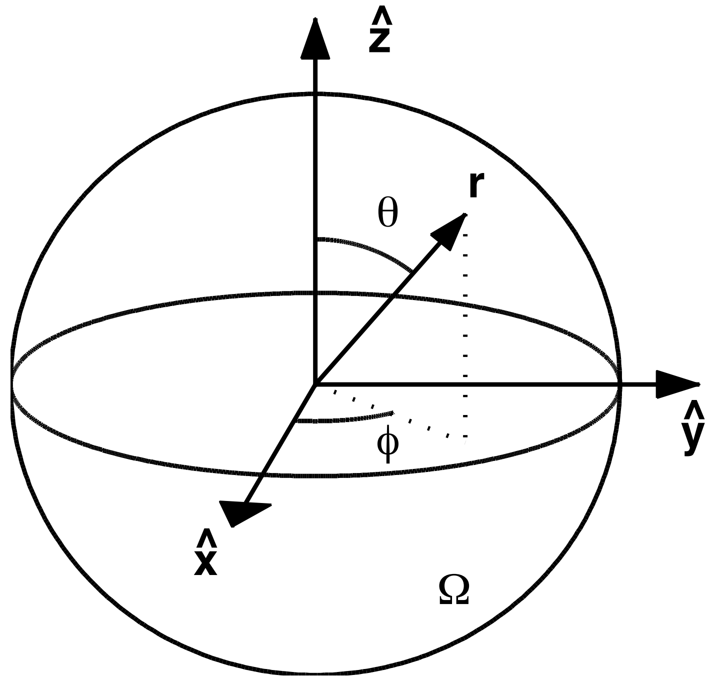
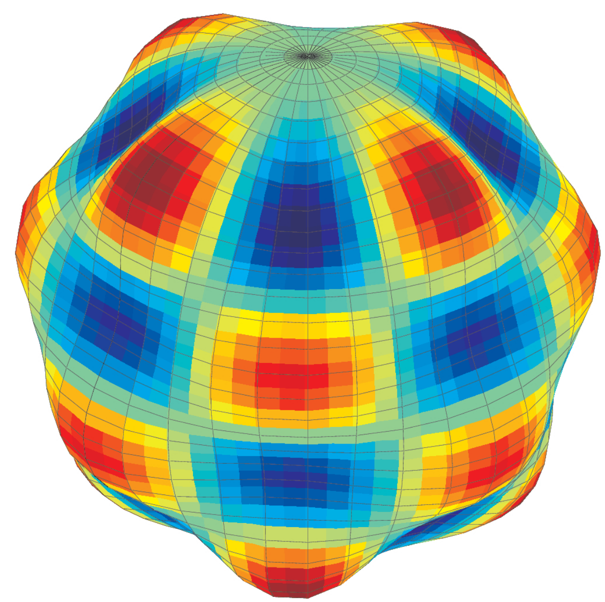
To honor the spherical shape of the Earth,

we work in the **spherical-harmonic** basis.

Spherical harmonics

Scalar signals $s(\mathbf{r})$ modeled on a unit sphere Ω:





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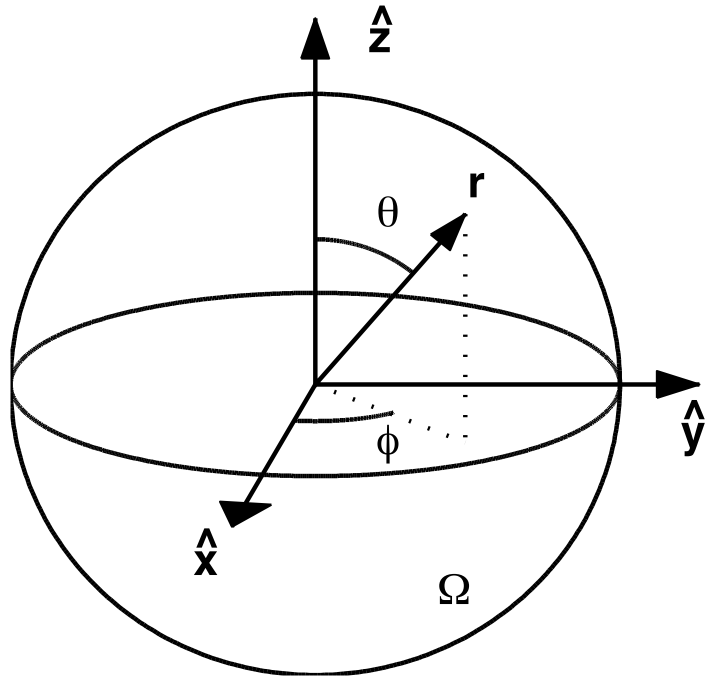

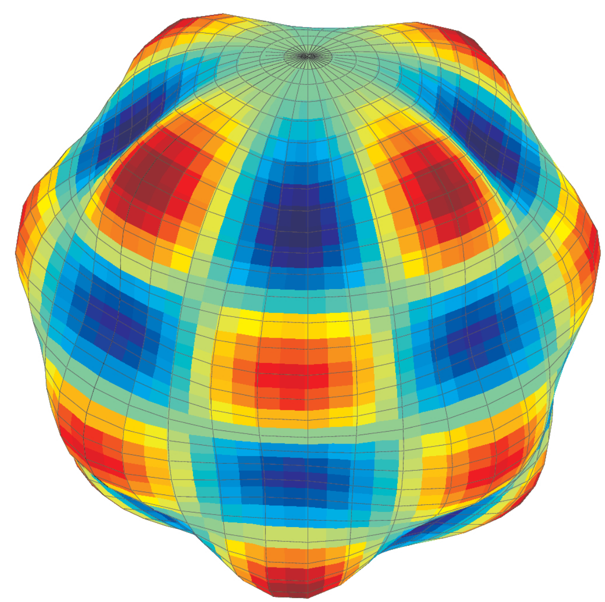

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$$\int_\Omega Y_{lm} Y_{l'm'}\, d\Omega = \delta_{ll'}\delta_{mm'} \quad \text{and} \quad s(\mathbf{r}) = \sum_{lm}^{\infty} s_{lm} Y_{lm}(\mathbf{r}).$$

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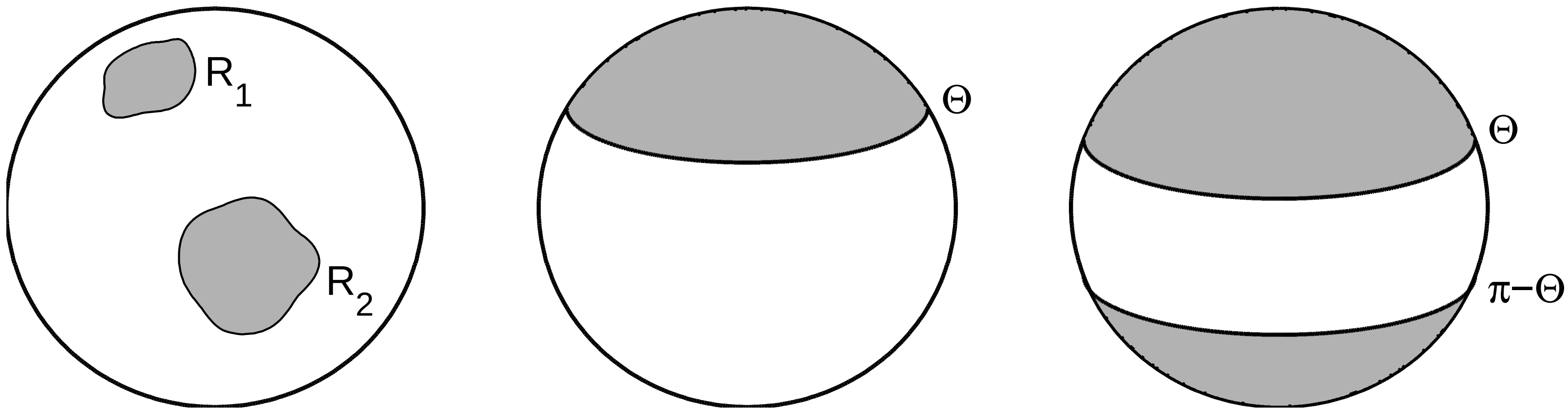

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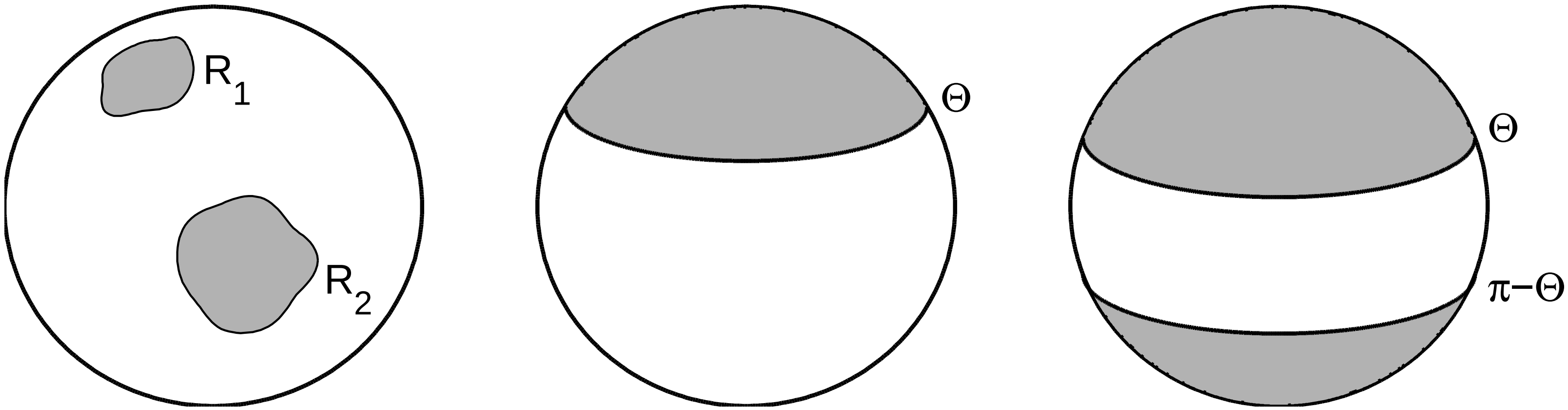

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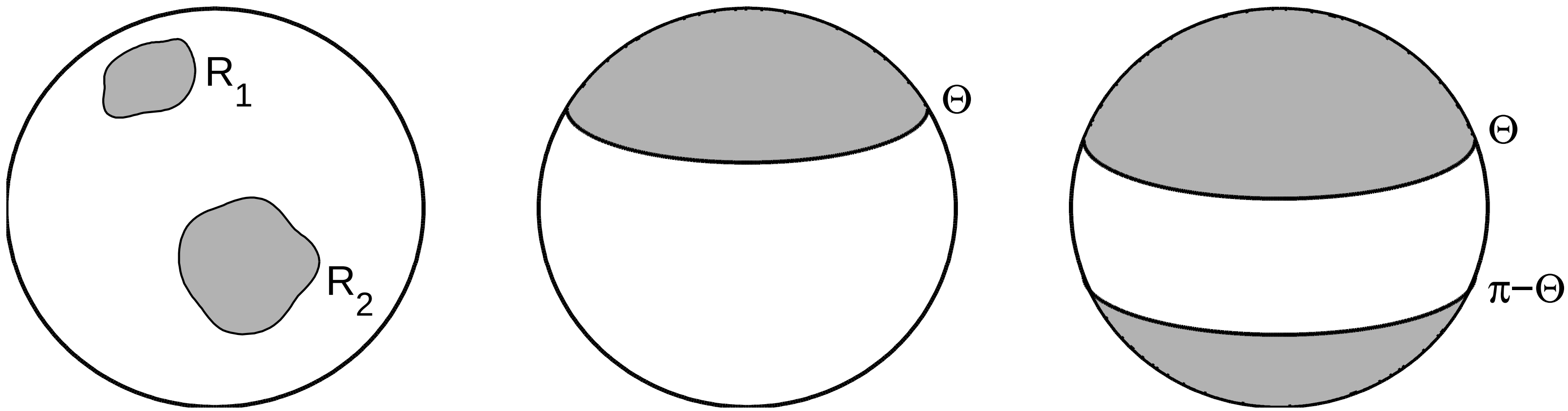

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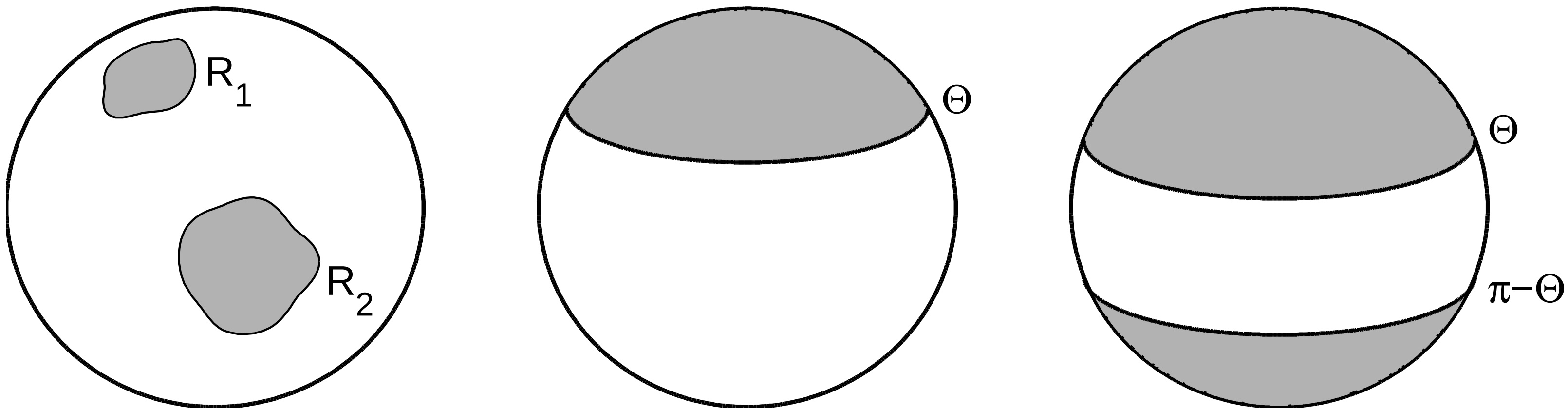

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These new, **doubly orthogonal**, functions are called **Slepian functions**, $g(\mathbf{r})$.

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Eigenvectors of D expand to **bandlimited Slepian functions**:

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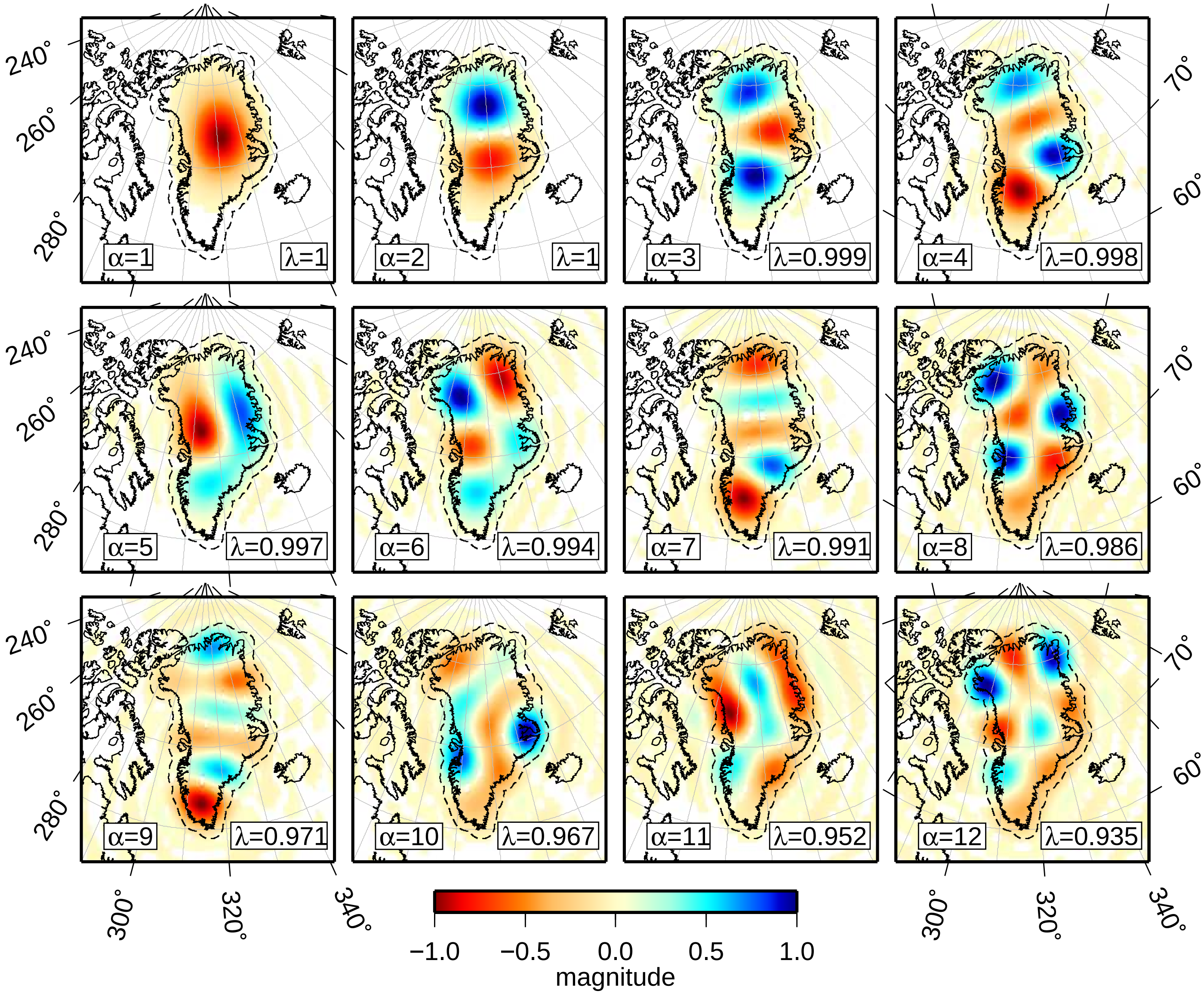
Voilà! We have *concentrated* a poorly localized basis of $(L+1)^2$ functions, Y_{lm}, both *spatially* and *spectrally*, to a new basis with only about K functions, g.

Slepian functions for Greenland, $L = 60$



Harig & Simons, *PNAS* 2012

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3. Experiment with the **bandwidth** L of the signal as considered, allow for small **buffers** outside the region of interest. Monitor the statistics.

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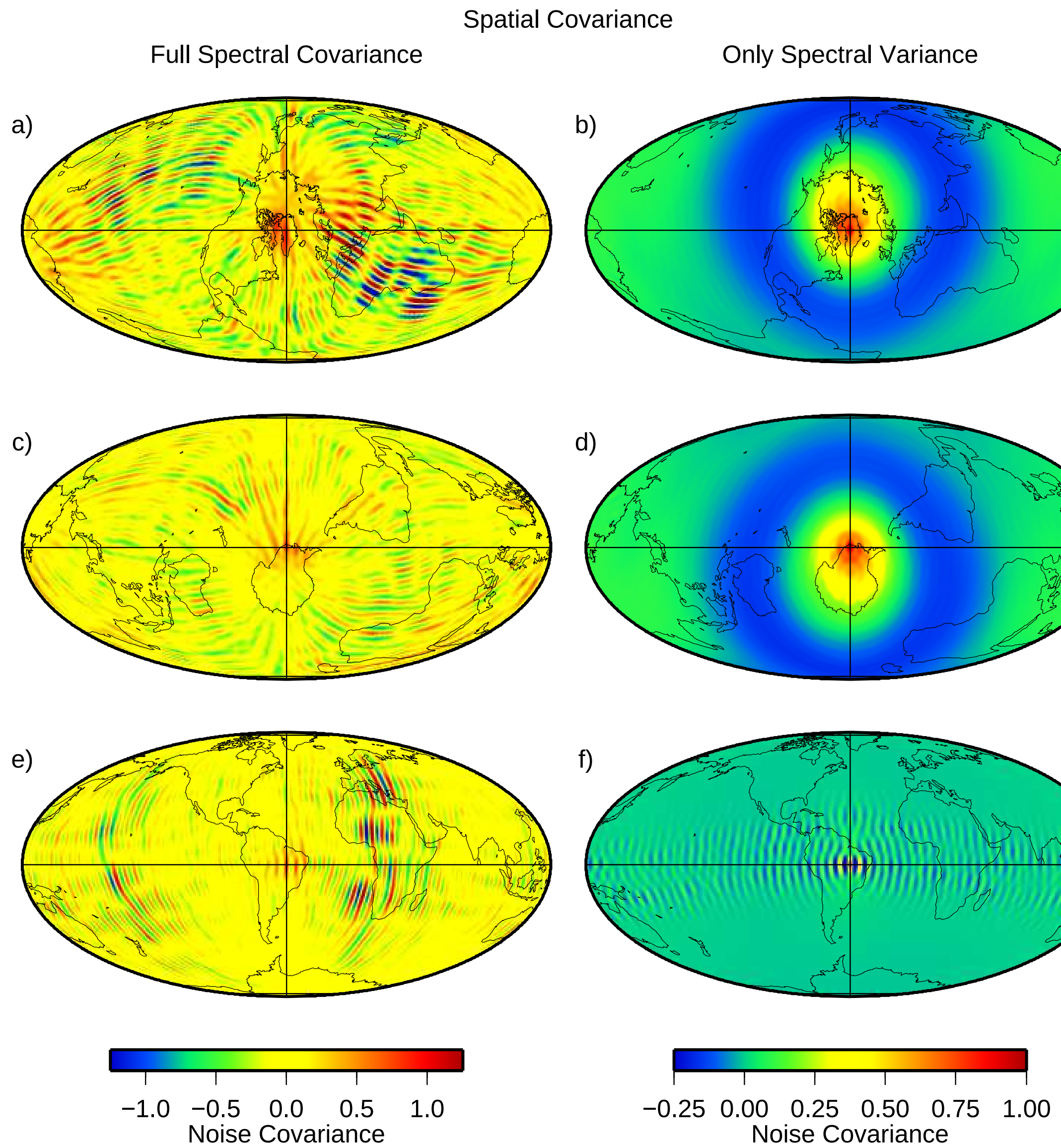
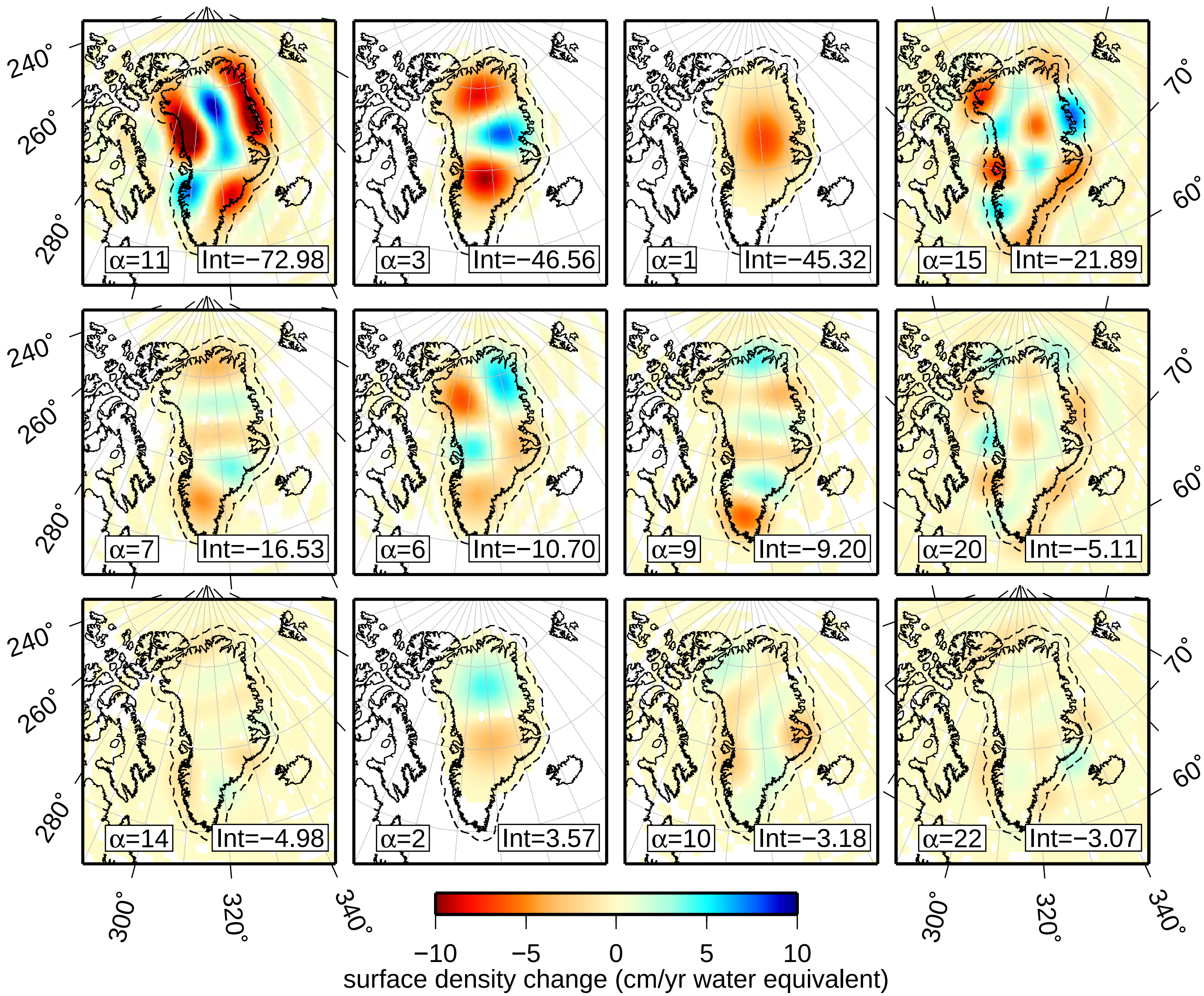
5. This is *very* different from most other approaches, though in spirit, it is *identical* to the stuff Slepian, Shannon and Wiener figured out in the 1950s.

I. Look at the noise (in the pixel basis)

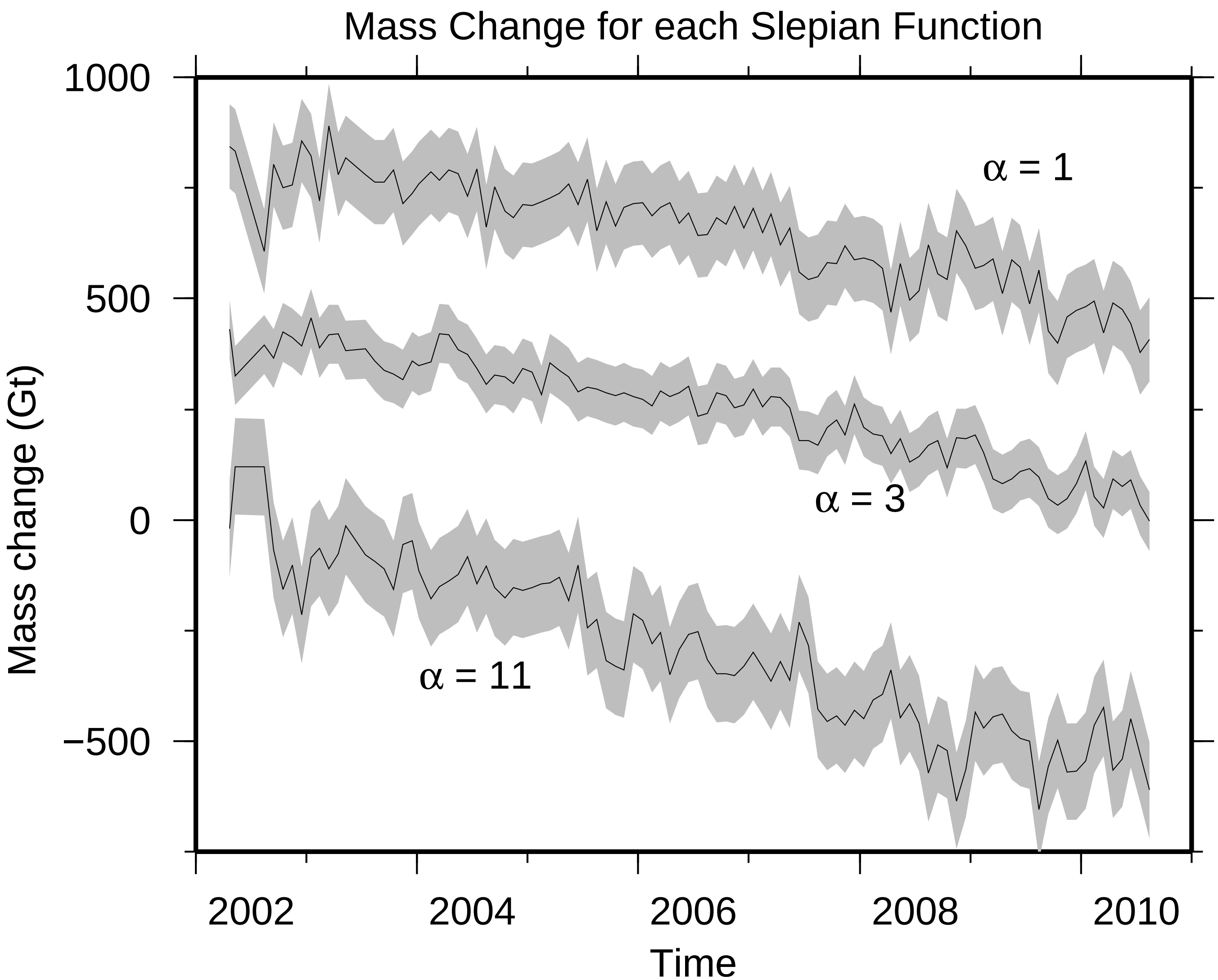


Harig & Simons, *PNAS* 2012

II. Project the signal onto the Slepian basis

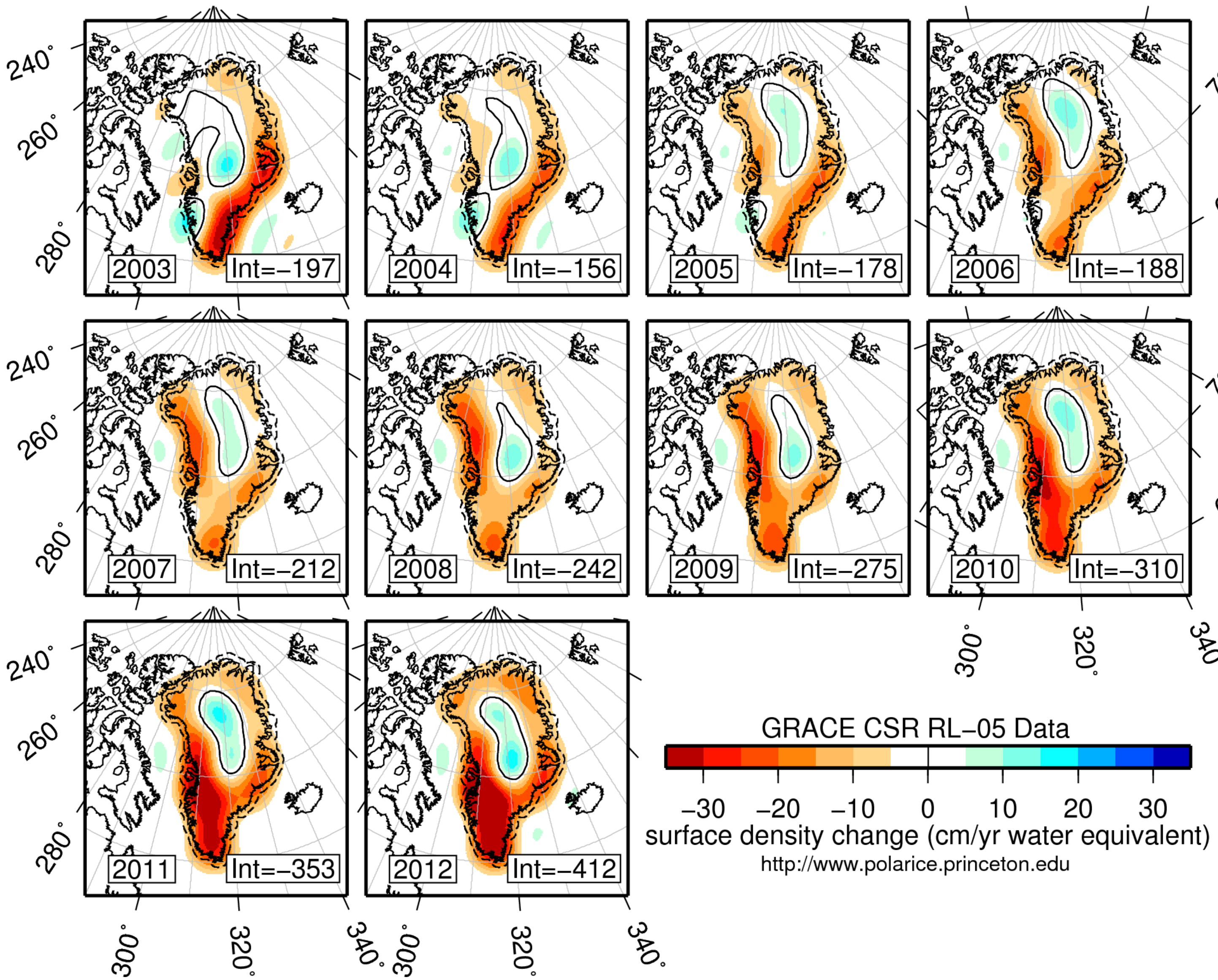


III. Solve for the time-dependence



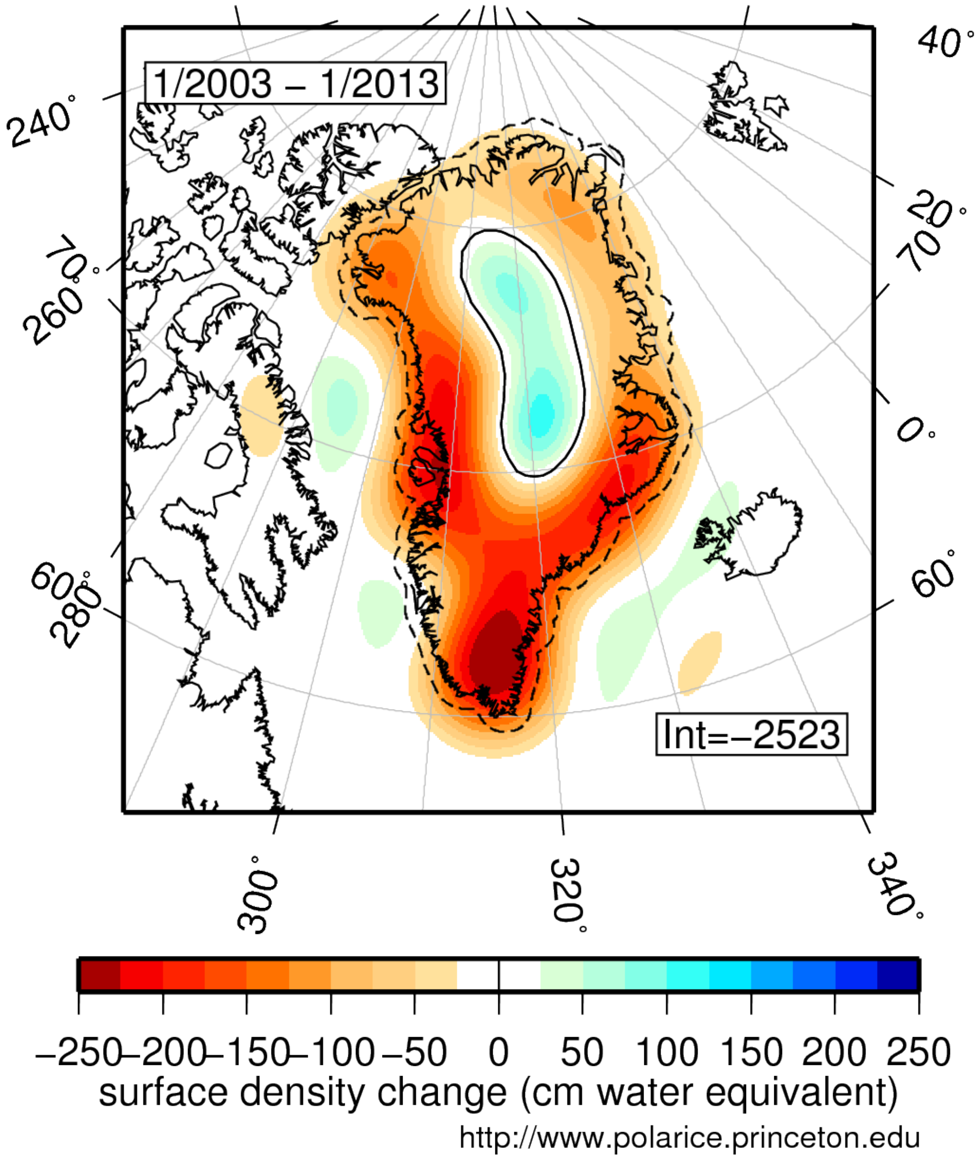
Harig & Simons, *PNAS* 2012

IV. Temporal variations of the spatial pattern



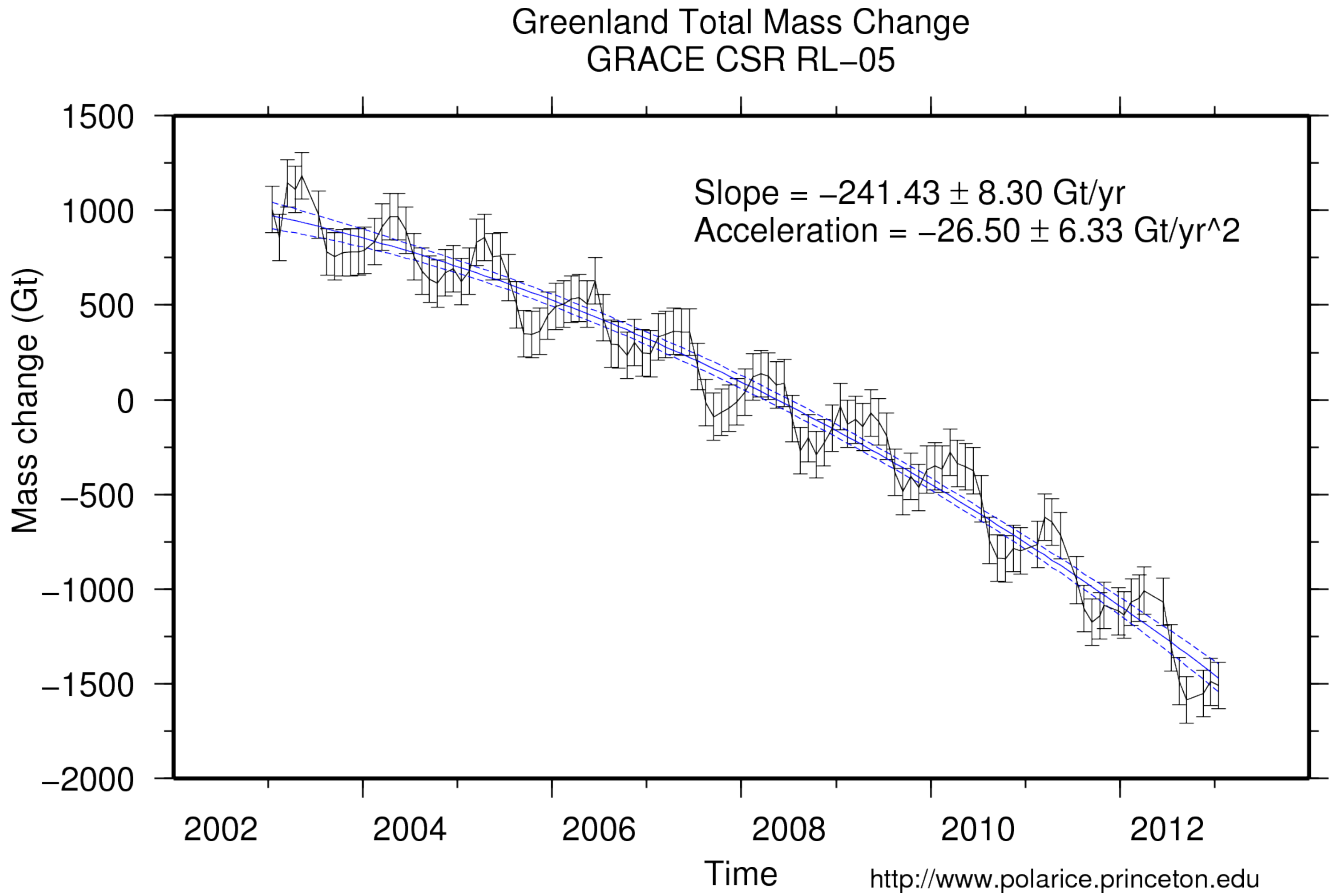
Harig & Simons, *PNAS* 2012

V. Spatial pattern 2003–2013



Harig & Simons, *PNAS* 2012

V. Invert for the total budget (if you must)



Harig & Simons, *PNAS* 2012

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- Modeling by Slepian functions requires very few *ad hoc* assumptions. Moreover, in addition to regional mass-average estimates, we get **maps**.
- Maps of the time-averaged mass loss show a marked concentration at the **outlet glaciers**. Observed rates compare well with GPS surveys.

Conclusions — I

- The battle to detect the **slight secular mass changes** from melting ice caps using GRACE has been about *estimating* the signal with realistic **uncertainties**, both in terms of **overall mass loss** and as a function of **position and time**.

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- On balance, the Greenland ice loss accounts for only a **minor fraction** of the Earth's sea level rise rate.
- Let us turn to the geological record to study **sea level change** on a global and regional scale.

Data Example I

San Salvador, Bahamas

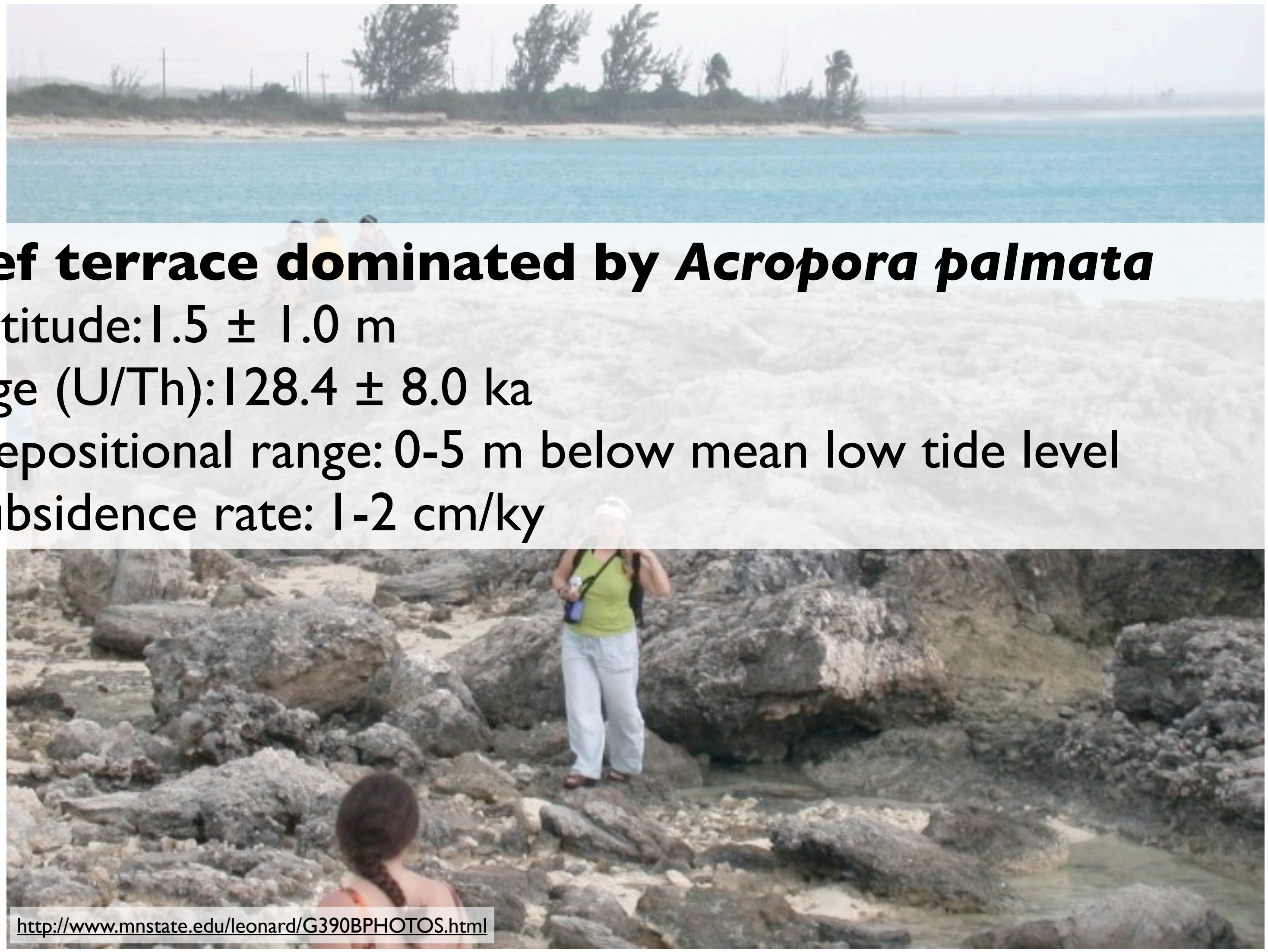
Reef terrace dominated by *Acropora palmata*

Altitude: 1.5 ± 1.0 m

Age (U/Th): 128.4 ± 8.0 ka

Depositional range: 0-5 m below mean low tide level

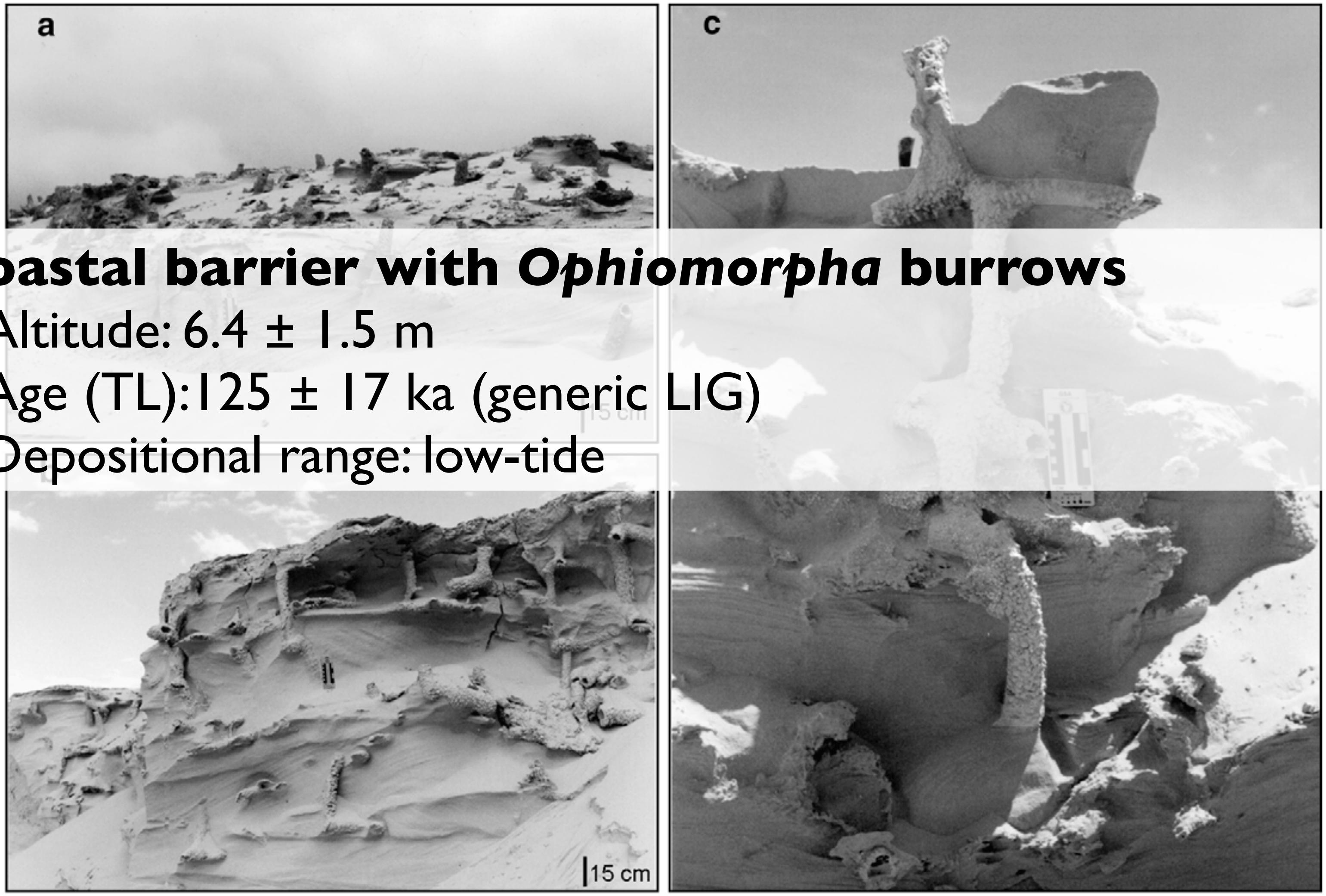
Subsidence rate: 1-2 cm/ky



Chen et al. (1991)

Data Example II

Rio Grande do Sol, Brazil



Coastal barrier with *Ophiomorpha* burrows

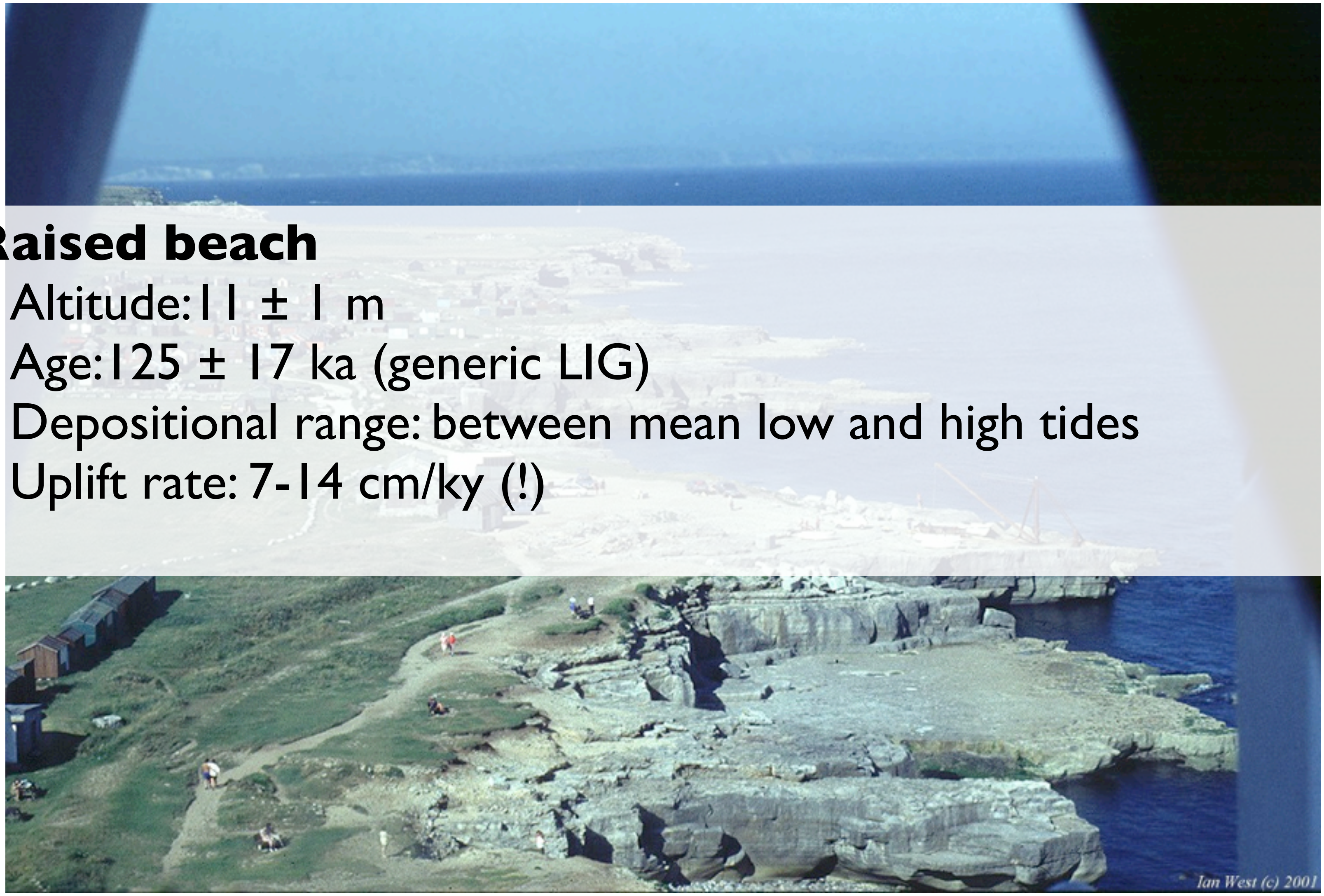
Altitude: 6.4 ± 1.5 m

Age (TL):125 ± 17 ka (generic LIG)

Depositional range: low-tide

Tomazelli et al. (2007)

Data Example III
Portland East, England

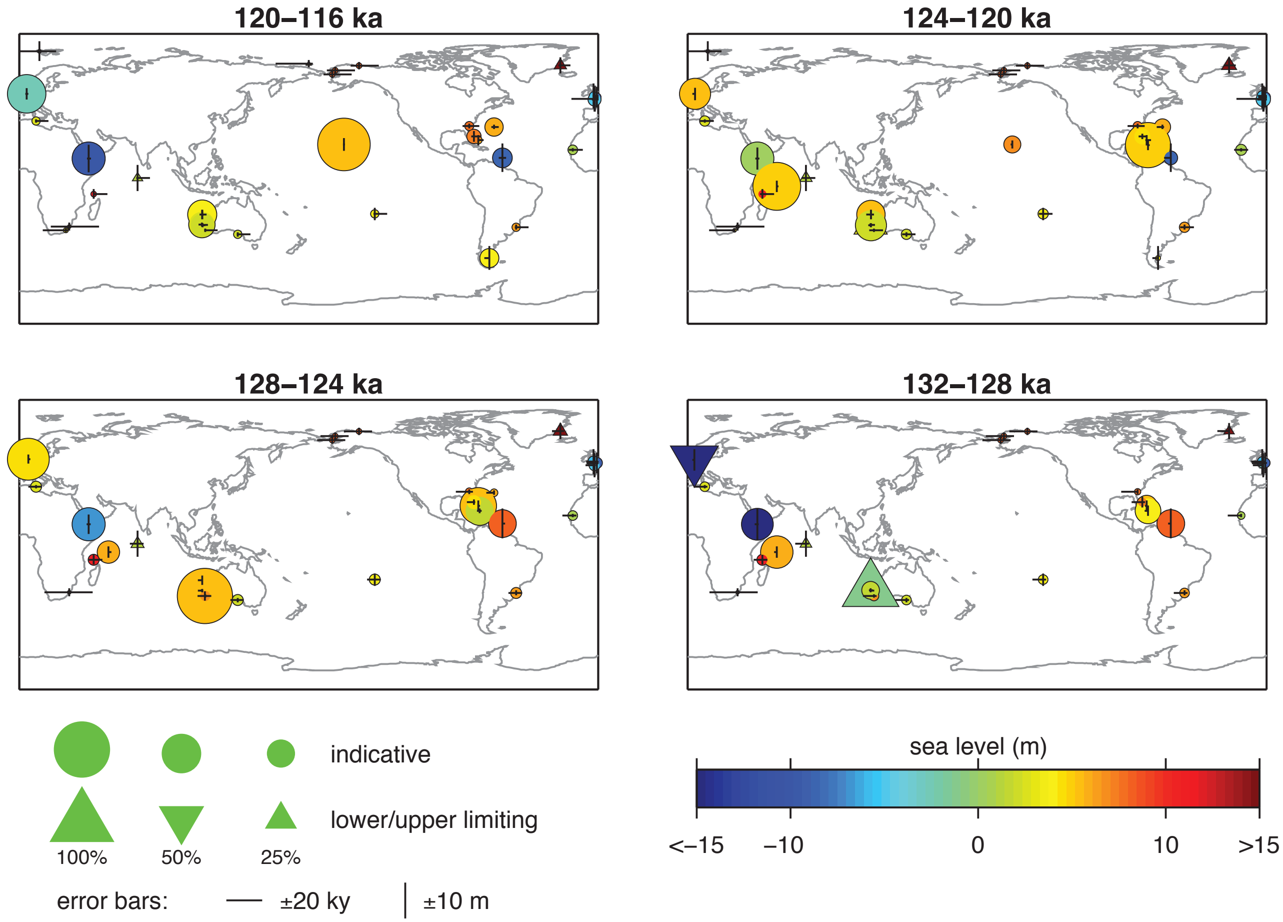


Raised beach

Altitude: 11 ± 1 m
Age: 125 ± 17 ka (generic LIG)
Depositional range: between mean low and high tides
Uplift rate: 7-14 cm/ky (!)

Westaway et al. (2006)

Geological Sea Level Indicators



A *very* sparse and noisy sample of local sea level indicators

Kopp, Simons et al., *Nature*, 2009

What makes it hard

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- We need to build acceptable prior solutions, which we do from a variety of data sources (e.g. global $^{18}\delta O$ curves, perturbed ice melting histories, ...)

And then we sample thousands and thousands of models to come up with
a **global sea level curve** for the Last InterGlacial

Kopp, Simons et al., *Nature*, 2009

Using the *physics* to derive the *statistics*:

Any **dynamic sea level modelling** must include gravitational, elastic, rotational, isostatic, shoreline migrations, isostasy and tectonics! From our prior solutions and constraints, Jerry Mitrovica built a series of sea level curves for us, which we turned it our posterior:

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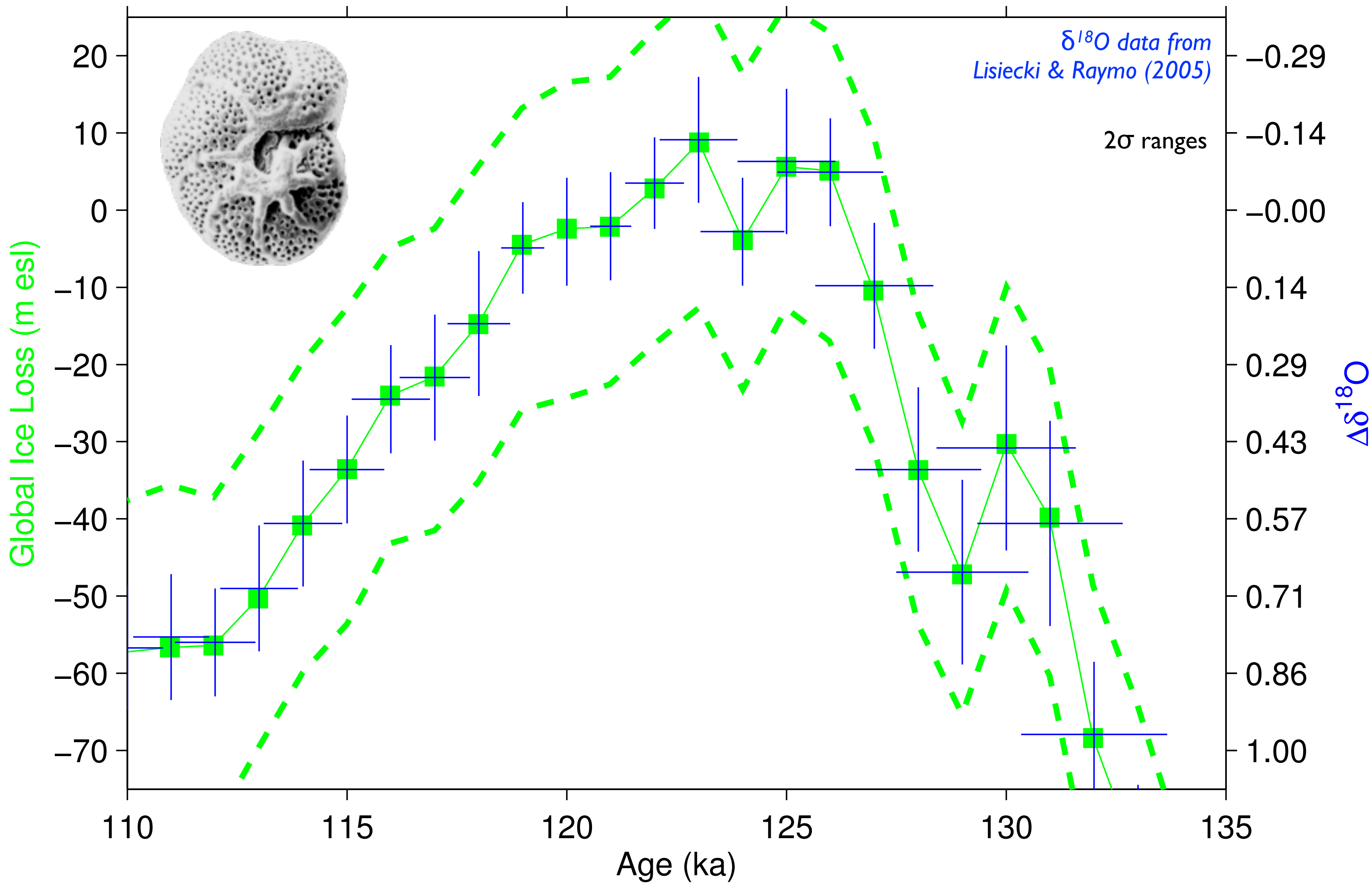
$$\underbrace{p(\mathbf{s})}_{\text{prior}} \quad \rightarrow \quad \underbrace{p(\mathbf{d}|\mathbf{s})}_{\text{“model”}} \quad \rightarrow \quad \underbrace{p(\mathbf{s}|\mathbf{d})}_{\text{posterior}}$$

Kopp, Simons et al., *Nature*, 2009

Oxygen isotopic record of global ice volume



Cartoon from: http://earthguide.ucsd.edu/virtualmuseum/climatechange2/01_1.shtml

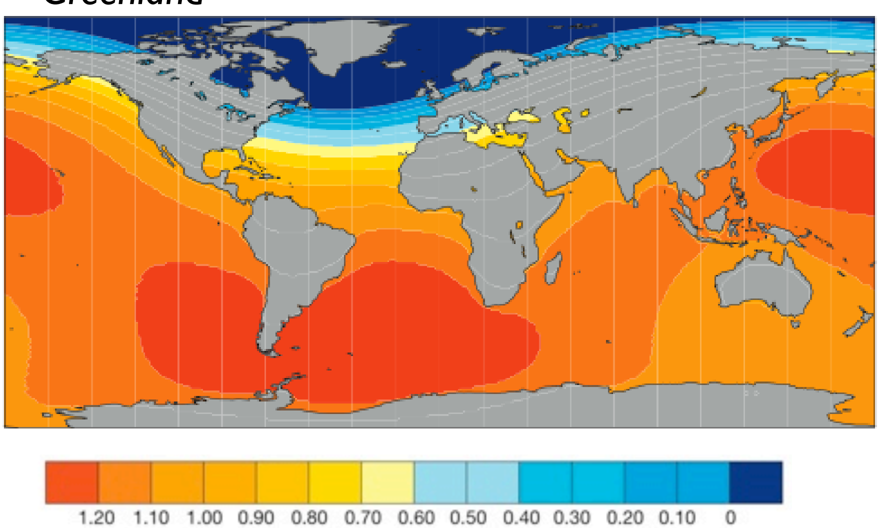
Our Sea Level Model

Effects included:

Gravitational, elastic, rotational, isostatic, shoreline migrations

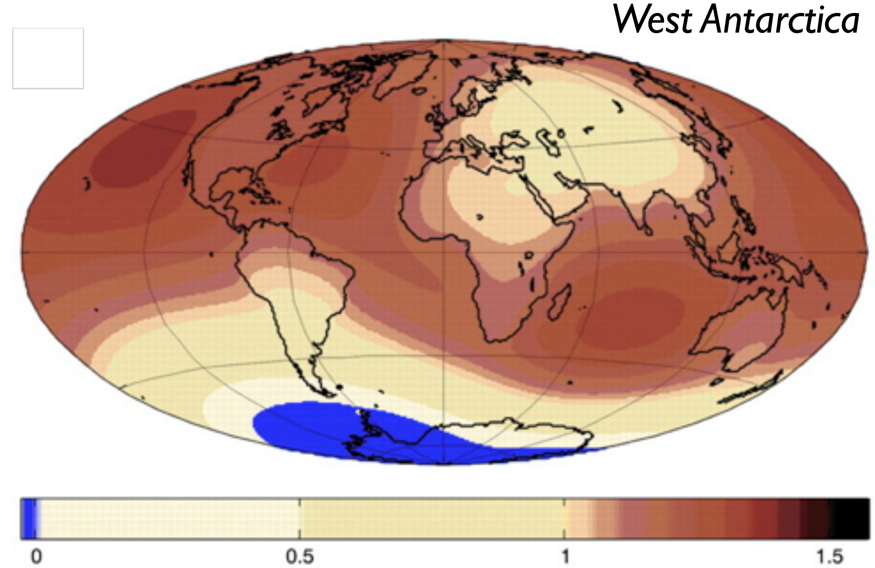
Example: "*Fingerprints*" of Greenland and West Antarctic Ice Sheet melting, per meter global sea level rise

Greenland



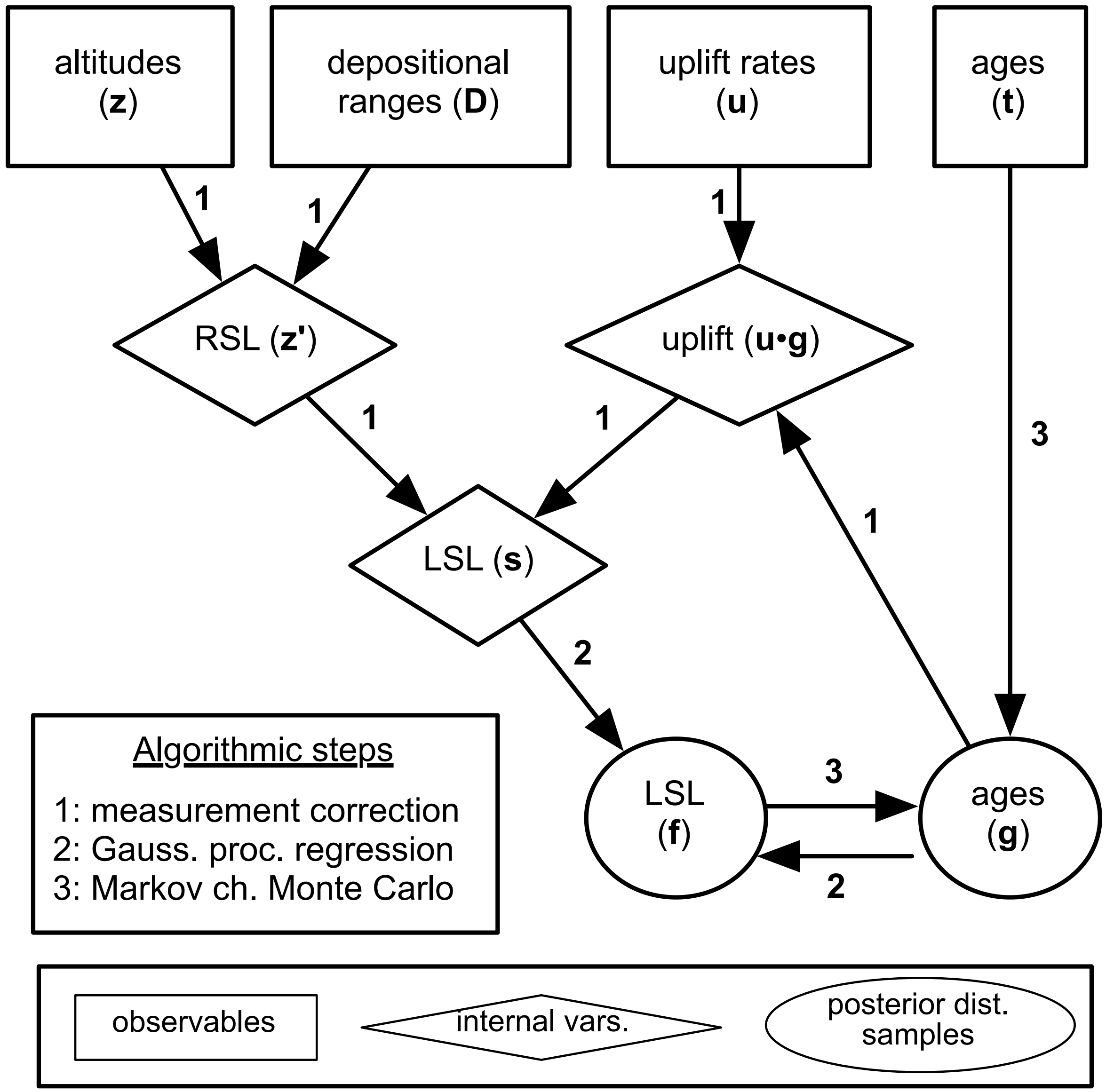
Mitrovica et al. (2001)

West Antarctica

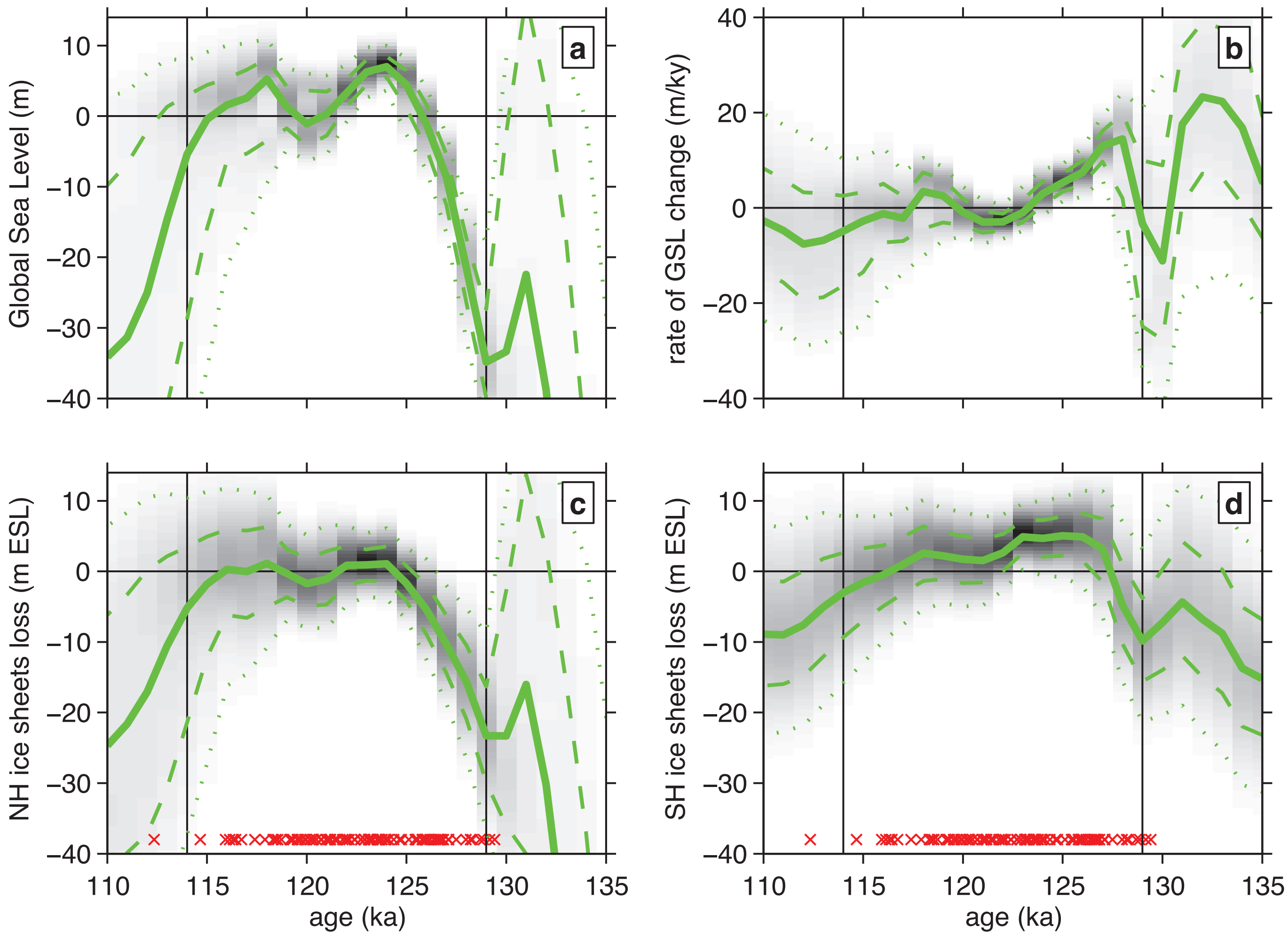


Mitrovica et al. (2009)

Markov Chain Monte Carlo analysis

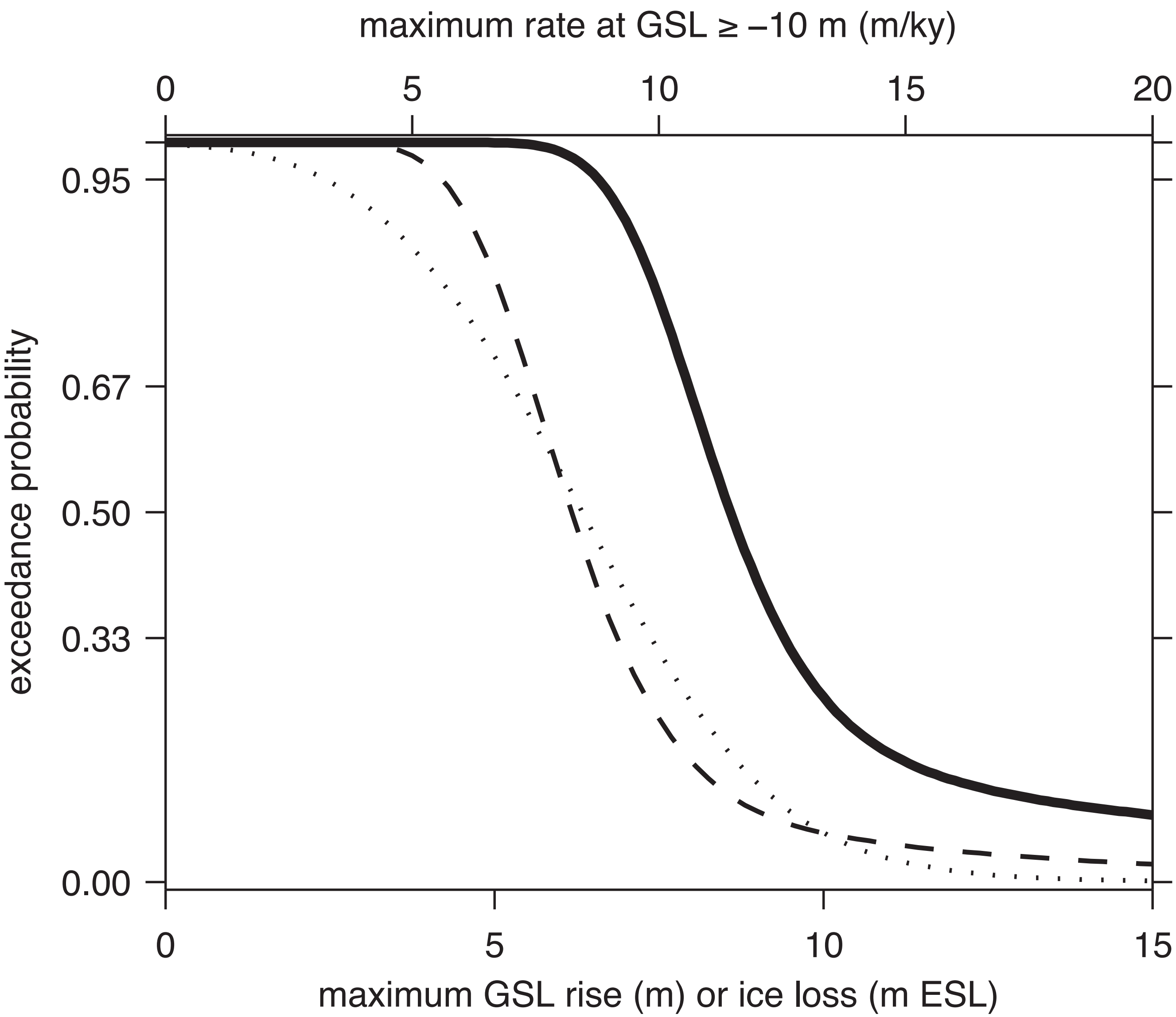


Kopp, Simons et al., *Nature*, 2009



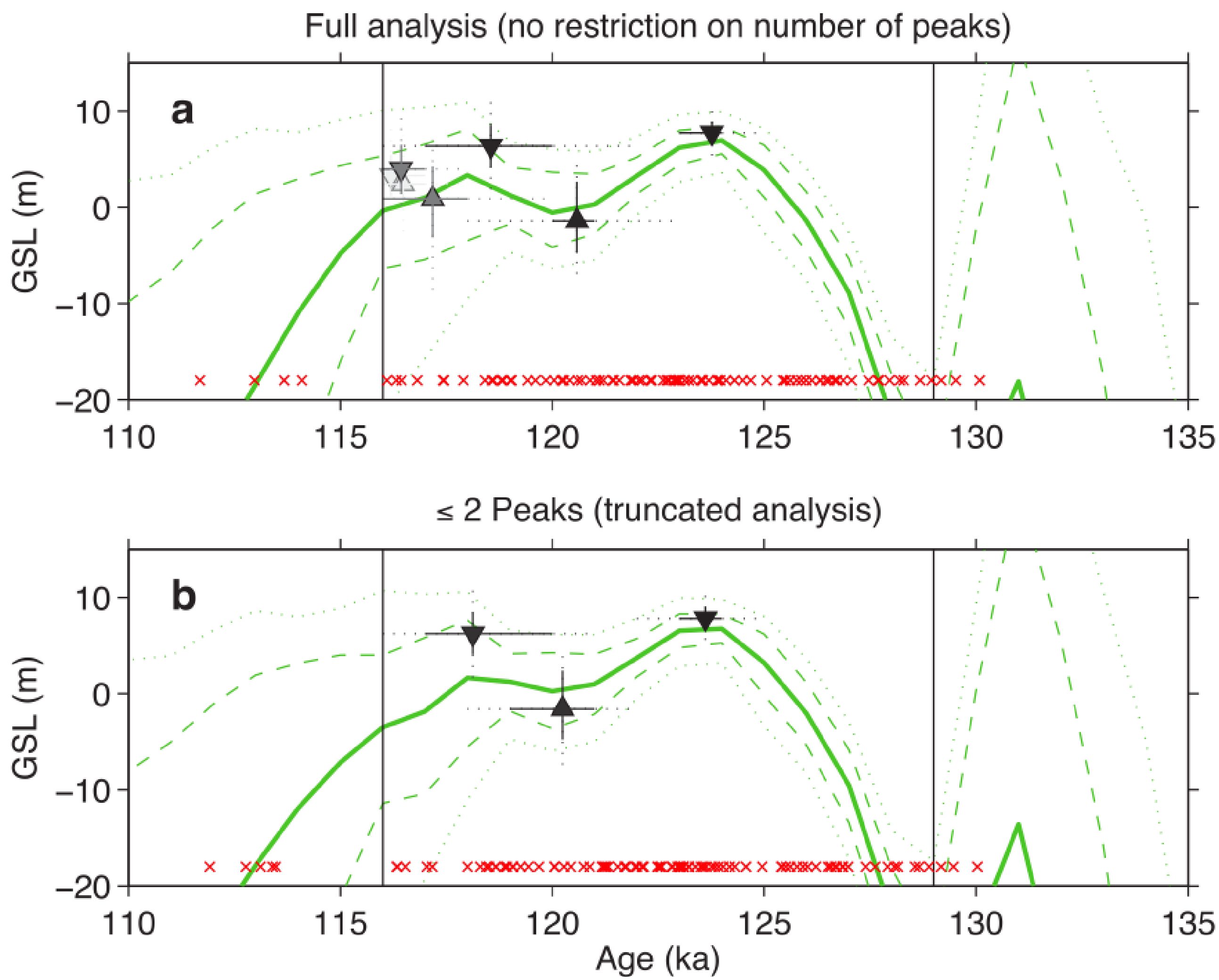
Kopp, Simons et al., *Nature*, 2009

...a clue to future sea level?



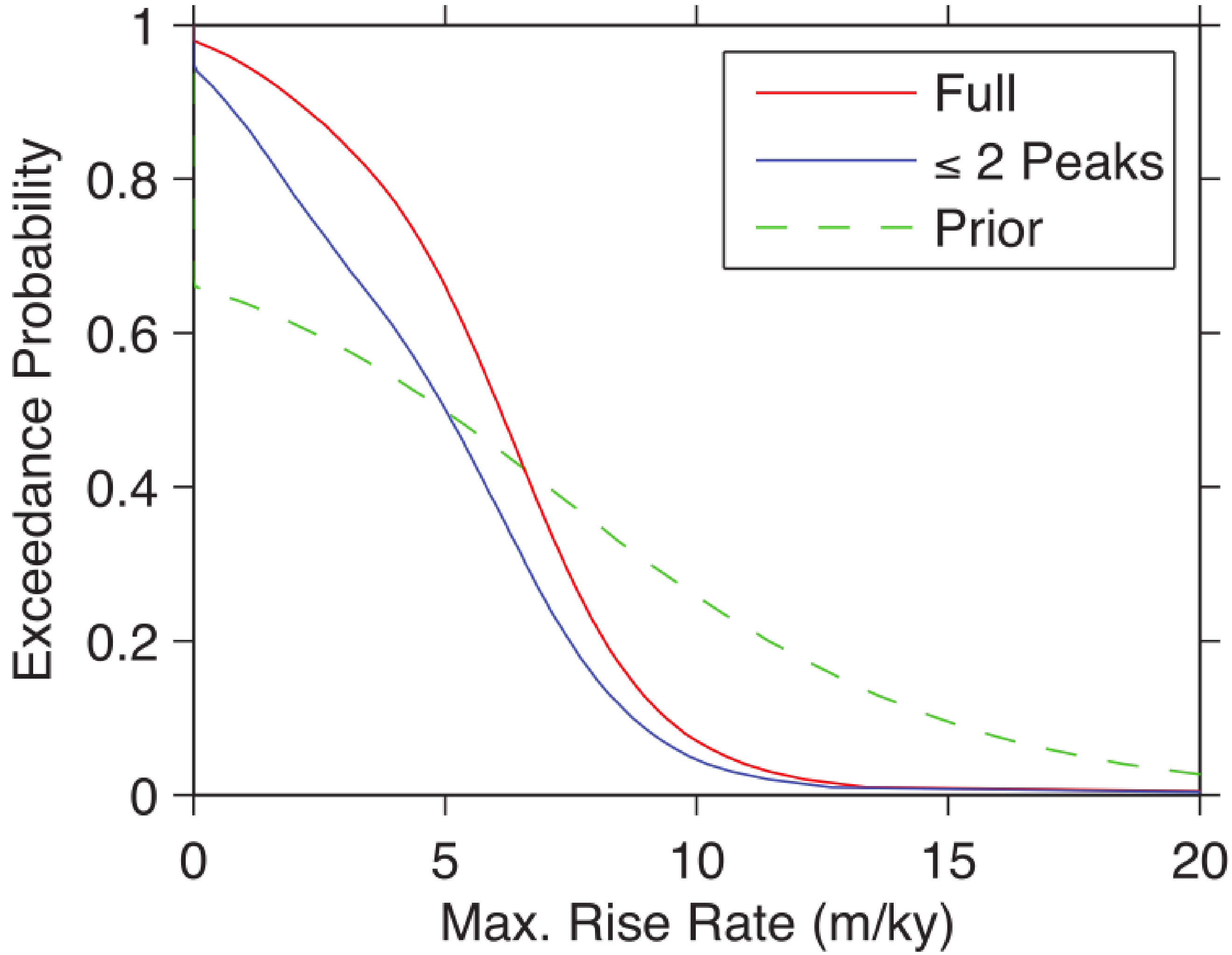
Kopp, Simons et al., *Nature*, 2009

Sea level *within* the Last InterGlacial...



Kopp, Simons et al., *GJI*, 2013

...a clue to future sea level rise *rates*?



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Conclusions — II

Using **spatiospectral localization techniques** and basis projection we recover subtle changes in Earth's gravitational and magnetic fields from noisy and incomplete satellite data.

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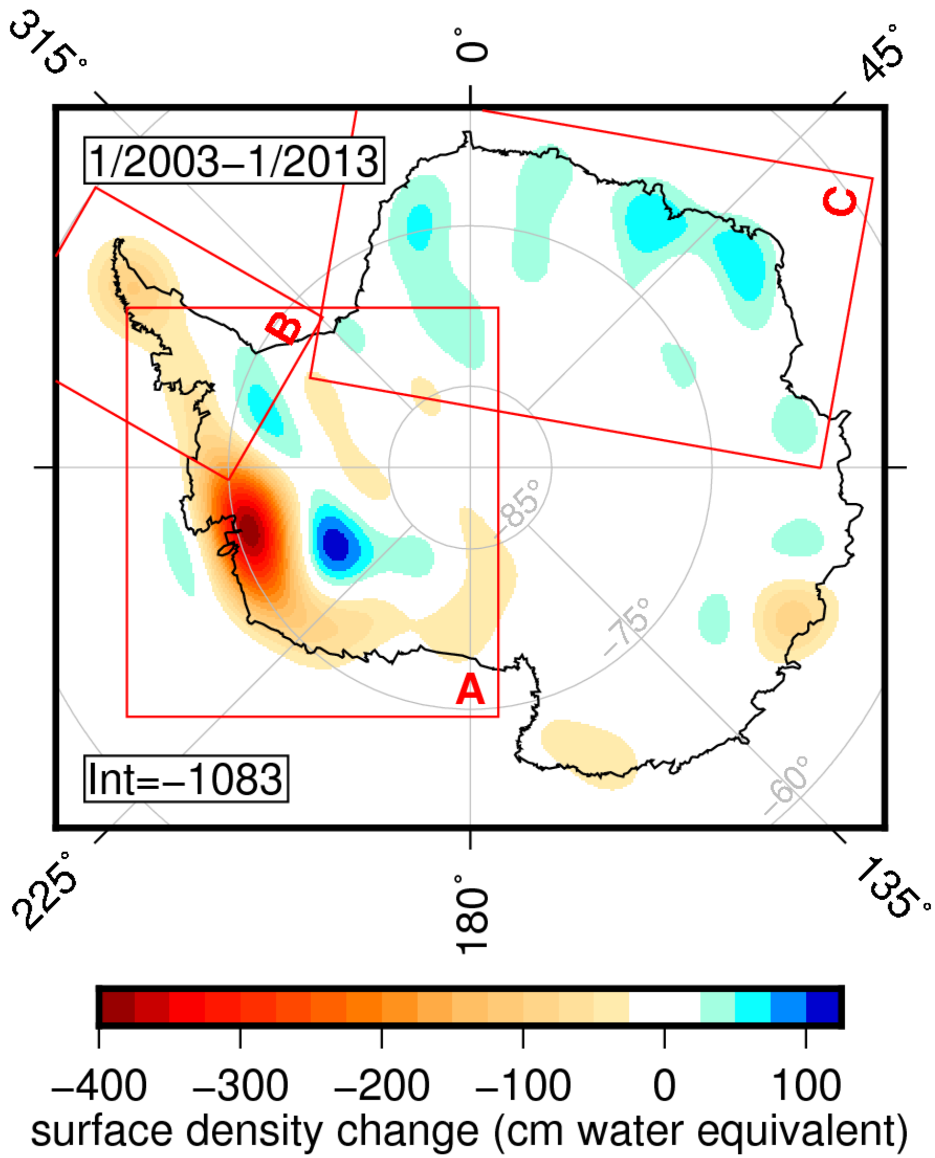
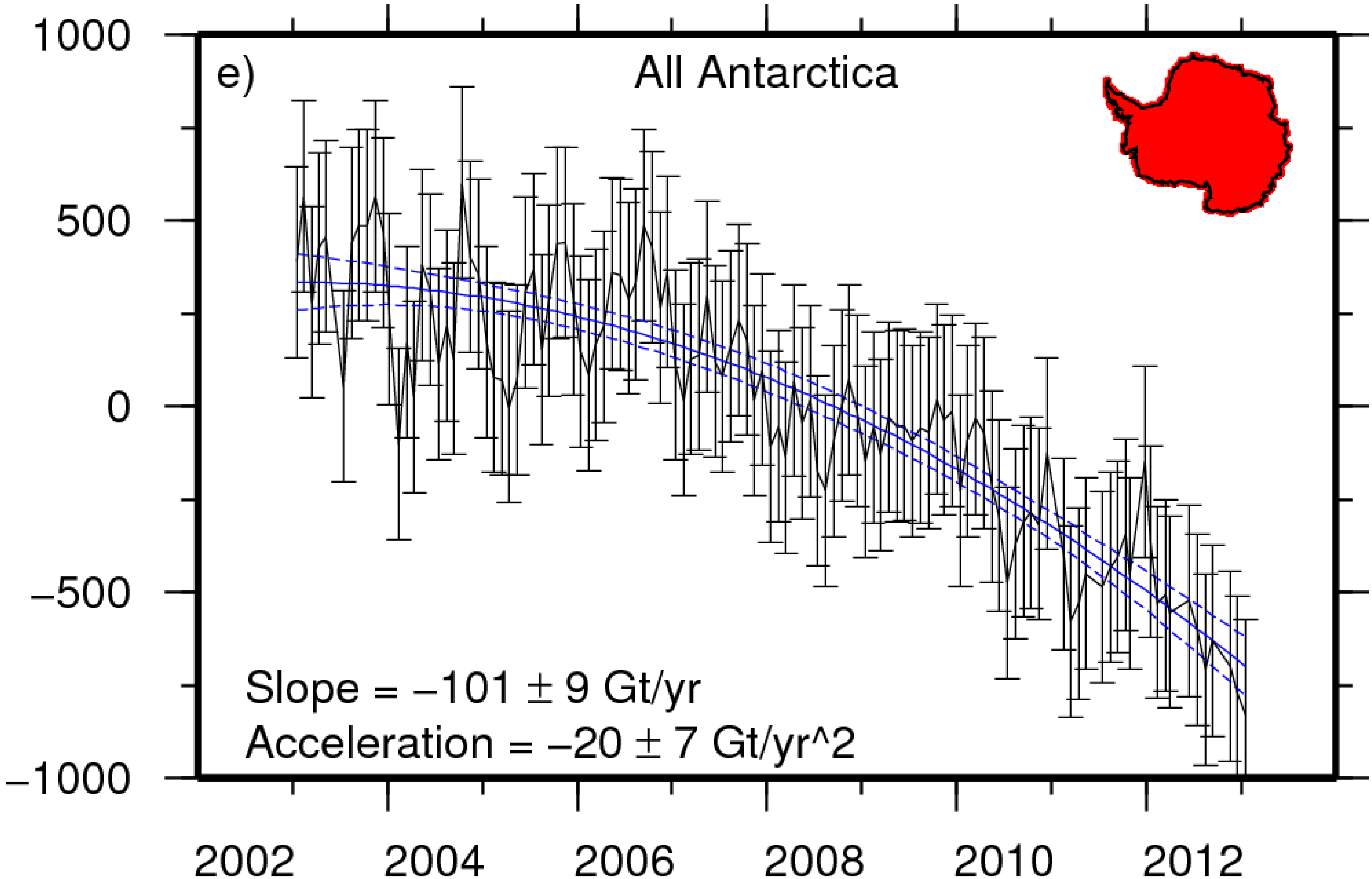
Using **adaptive sampling techniques** and **Gaussian process modelling** we can turn messy geological data into a coherent statistical model of the history of geophysical processes such as sea level change through time.



Harig & Simons, EPSL, 2015

Ice mass loss 2002–2013



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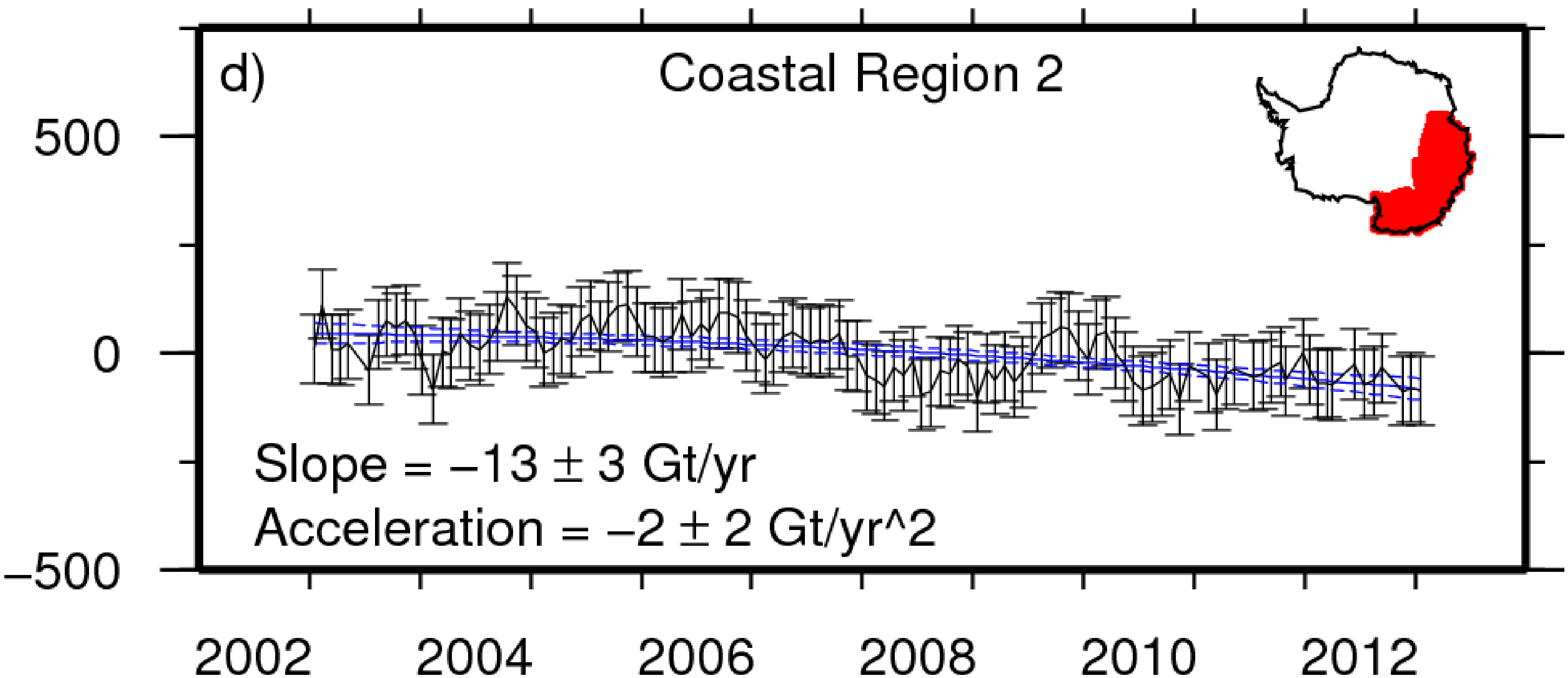

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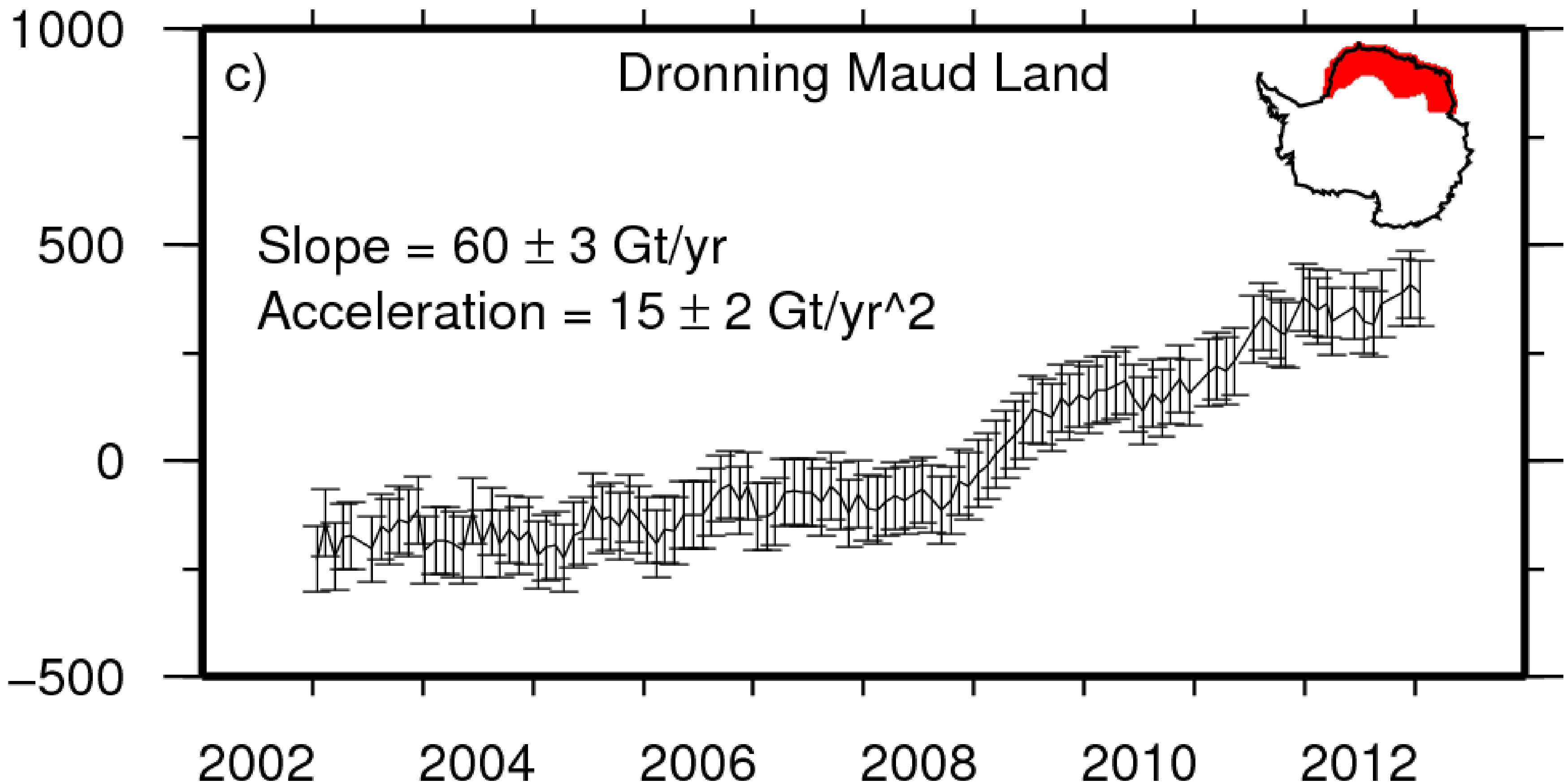

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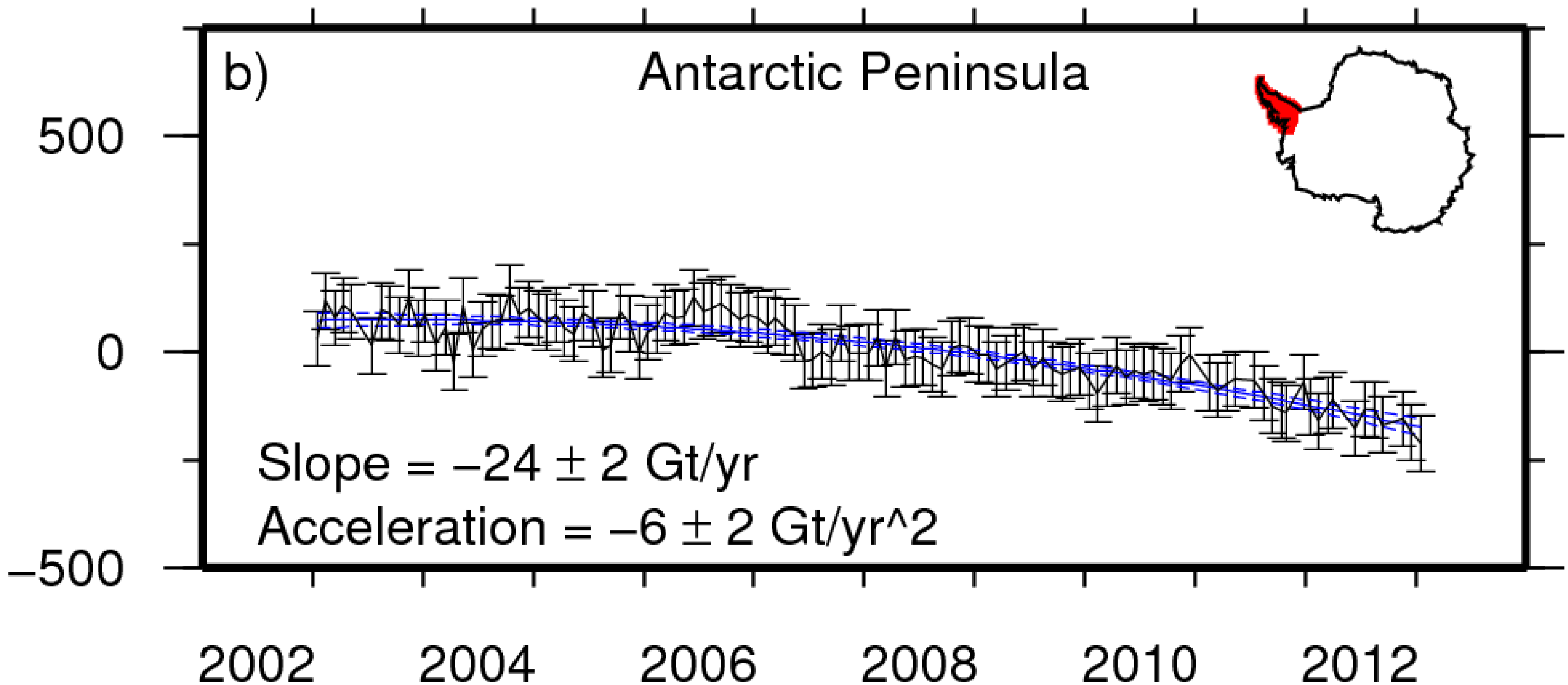

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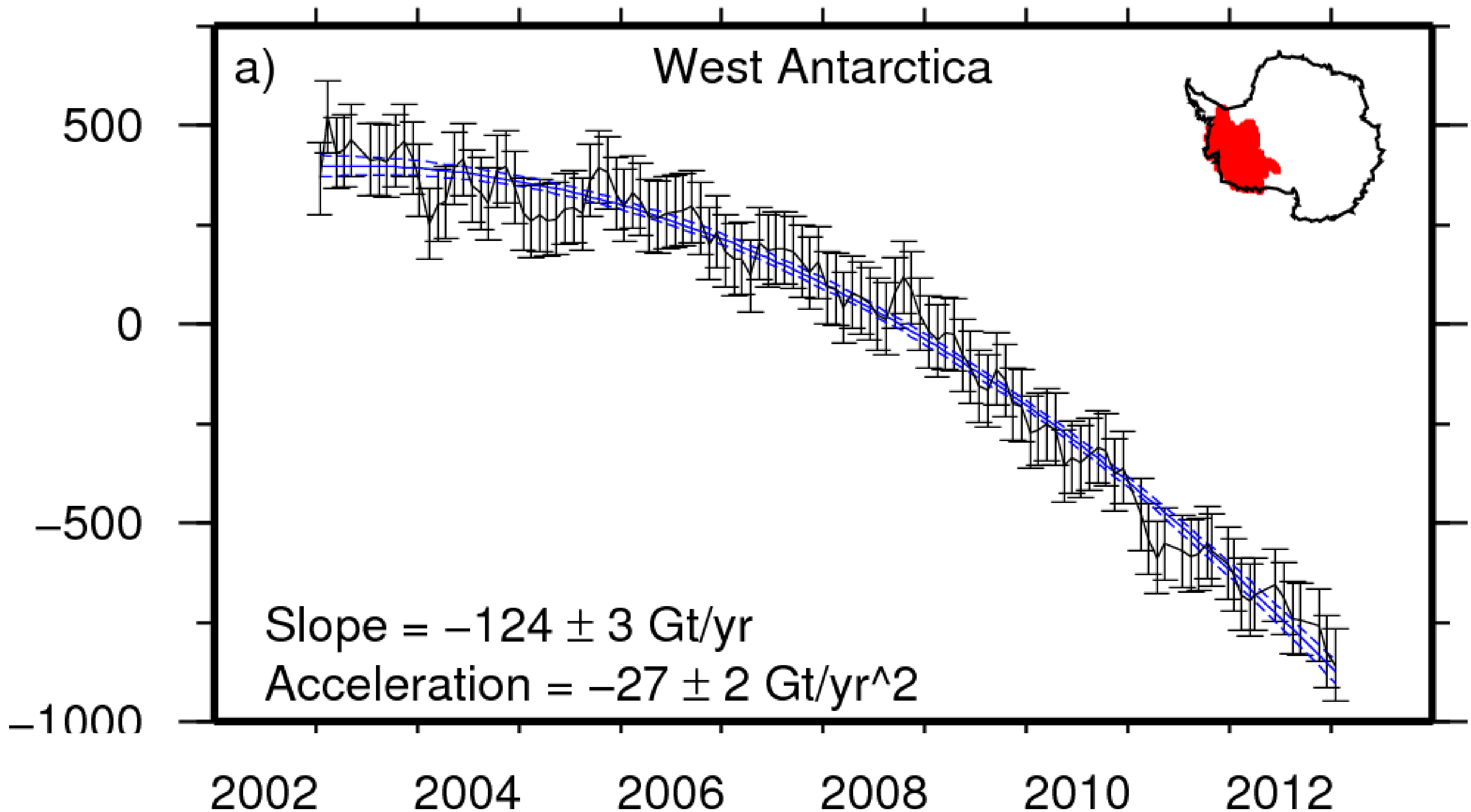

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