# Automatic **detection** & rapid **determination** of **earthquake magnitude** by **wavelet** multiscale **analysis** of the **primary arrival**

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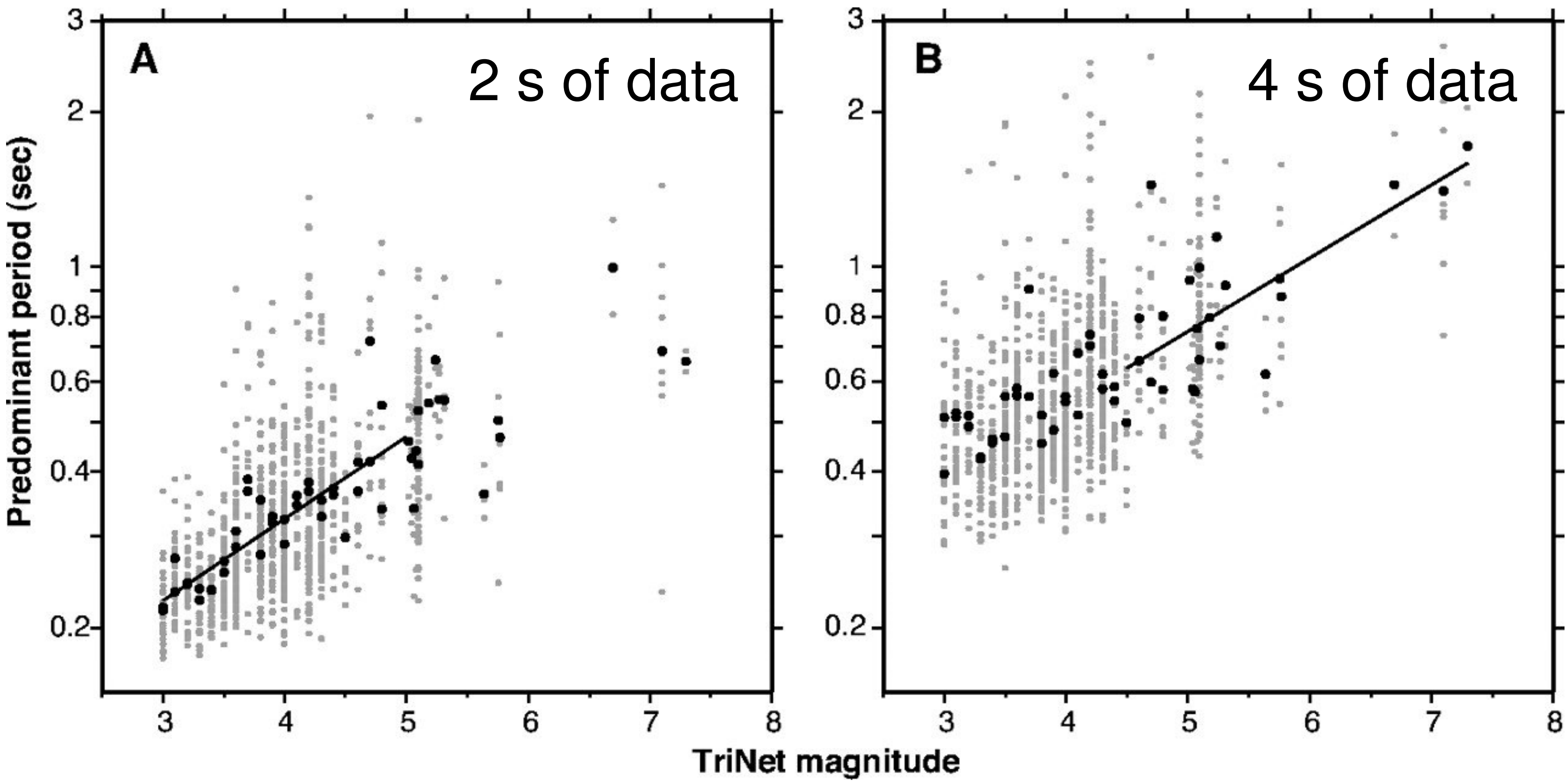
# Observation

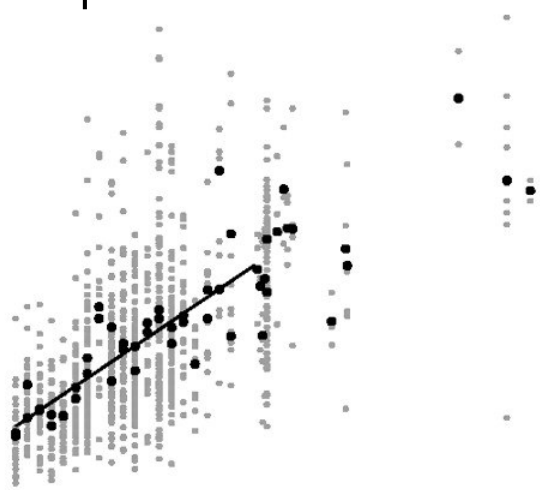



*Allen & Kanamori, 2003*: Predominant period of *P* wave scales with magnitude

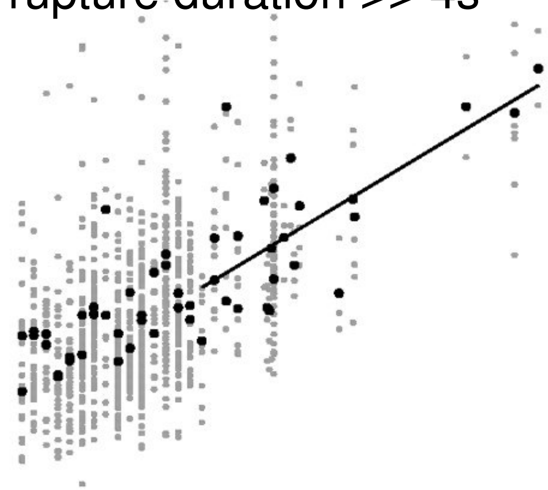
# Interpretation

The first few seconds (*think:* **~4 s**)
of the seismogram (*think:* ***P*** **wave**)
are diagnostic of the **magnitude**
of both large and small **earthquakes**

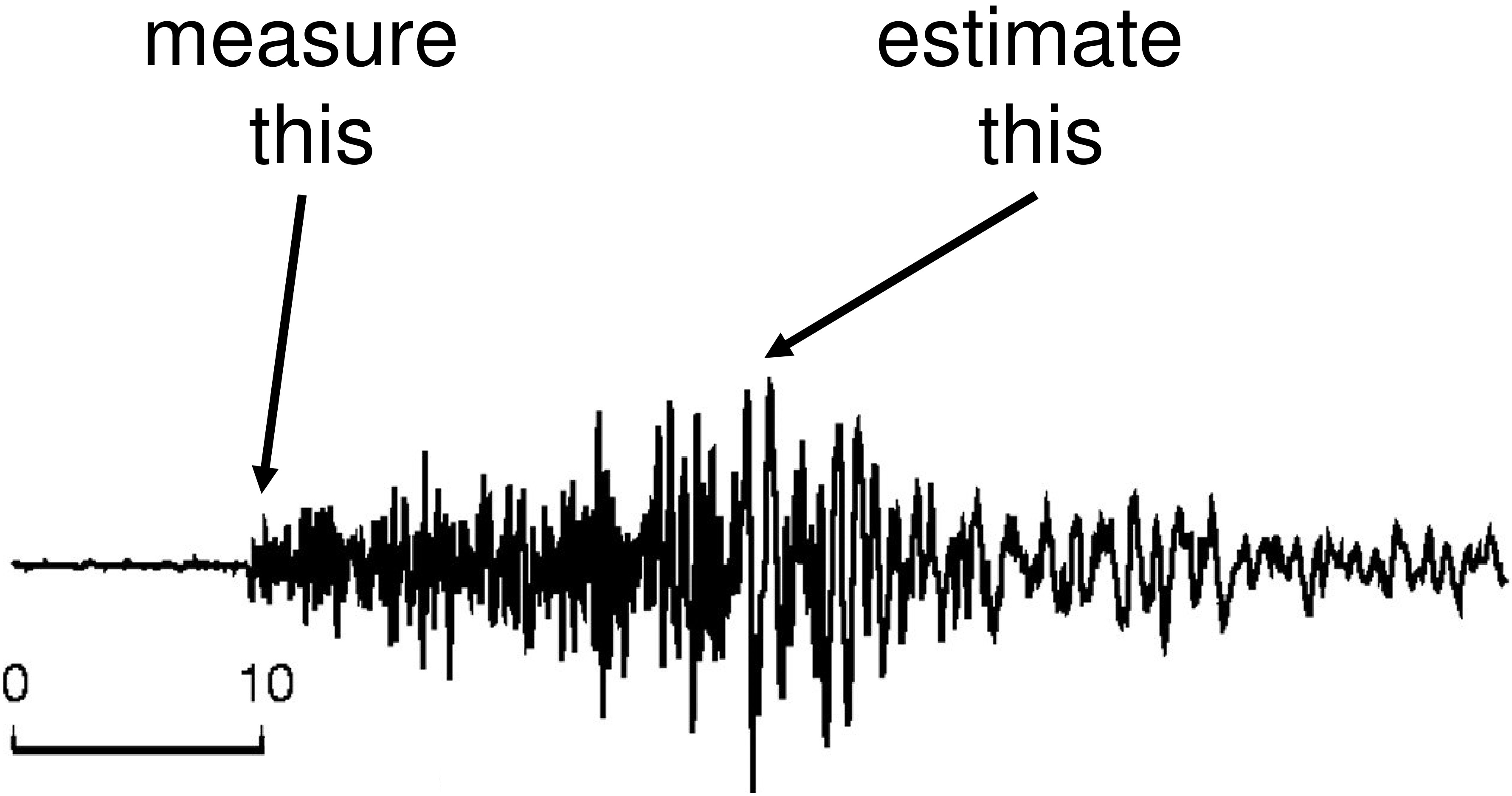
rupture duration < 4s



rupture duration >> 4s



# Interpretation



# Implication

*P* waves carry **information**

Early **warning** systems will work

Magnitude **before rupture is complete**

# Speculation

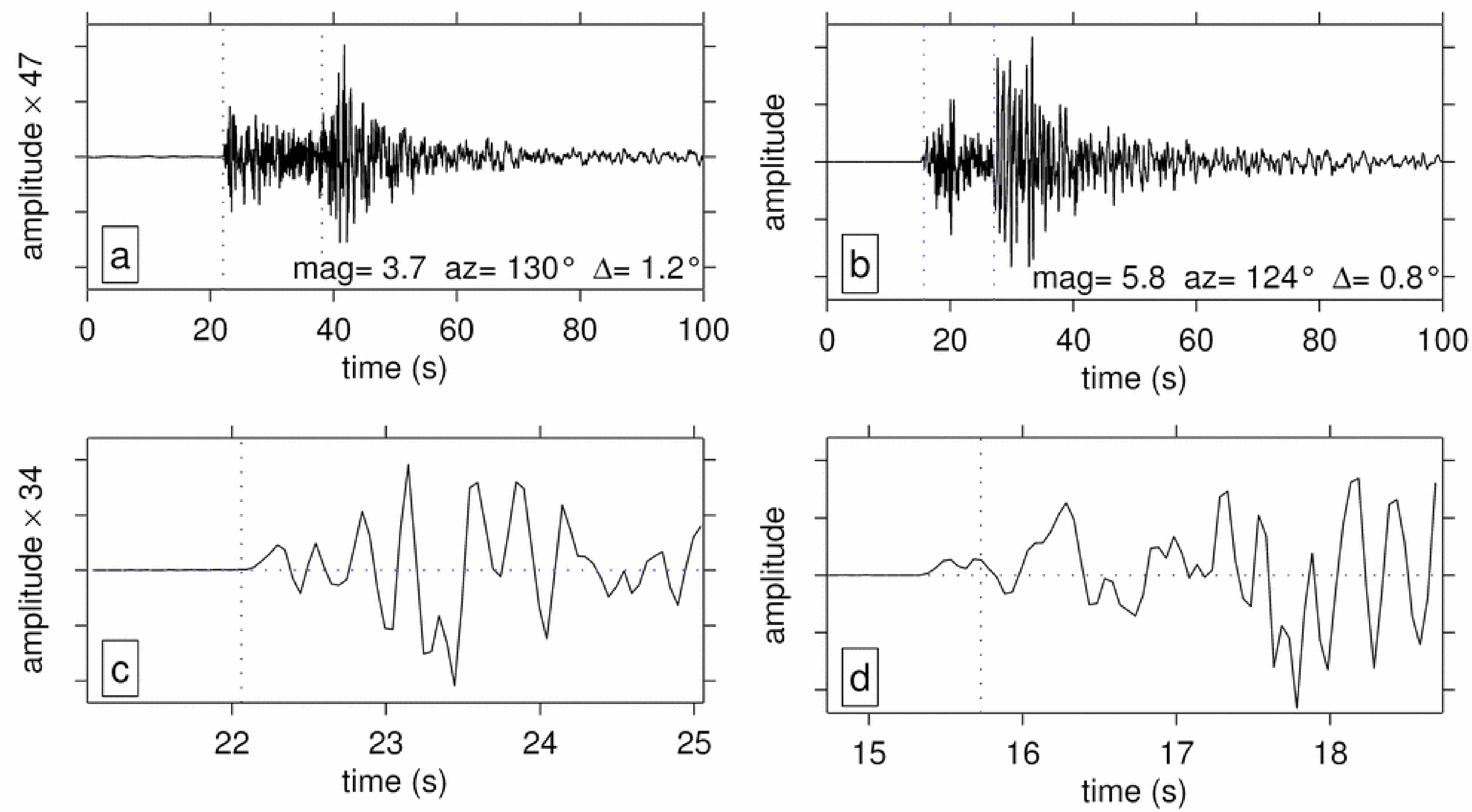
Earthquake rupture is **deterministic**

Small/large earthquakes start **differently**

**Source physics** needs rethinking

# Data

$u(t)$ [small] $u(t)$ [large]



# Predominant period ?

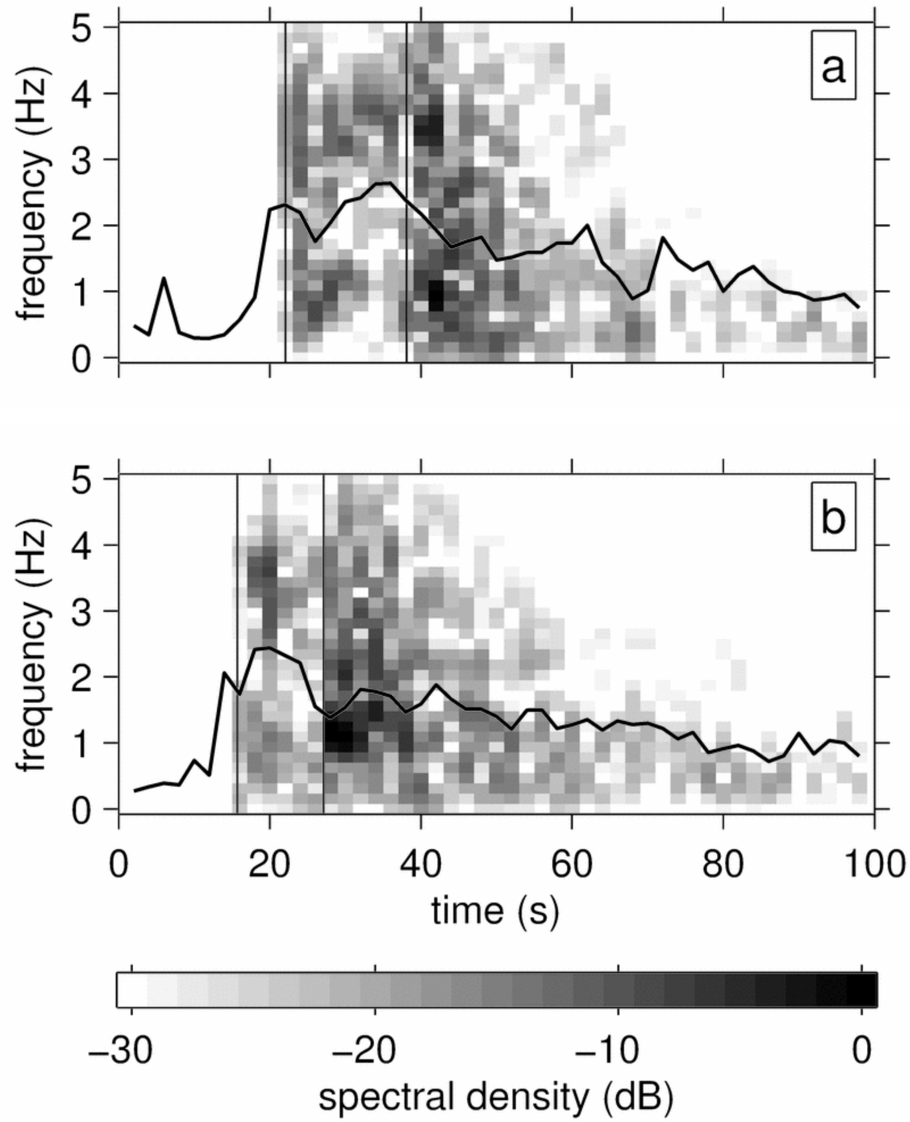
measure this → 

$$\tau_c^2 = \frac{\int_0^\infty |\hat{u}(f)|^2 \, df}{\int_0^\infty f^2 \, |\hat{u}(f)|^2 \, df}$$

← get this

A funnily weighted measure of
**average spectral density**
in the waveform interval of interest

# Stability & Significance ?




$t, f, |\widehat{u}(f)|^2, \tau_c$
[small]

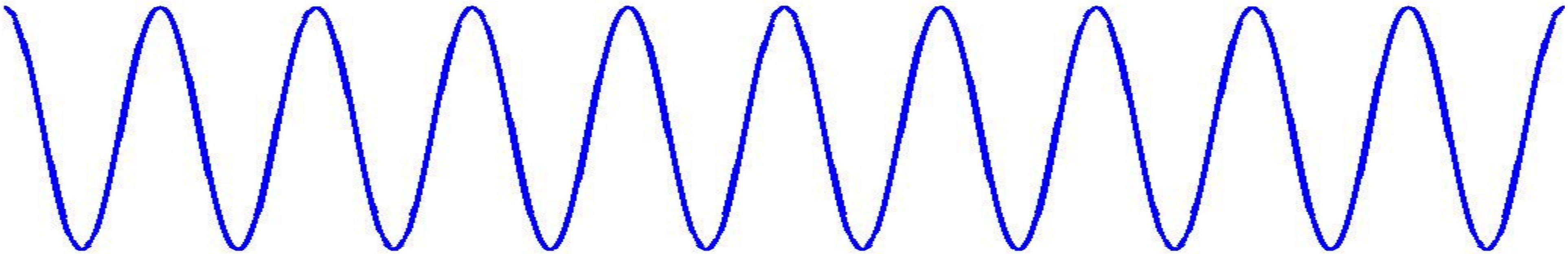
$t, f, |\widehat{u}(f)|^2, \tau_c$
[large]

# Why

Use a redundant transform ?
*[The windowed Fourier transform]*

Reduce the time-frequency information to a single number ?
*[The weighted average of spectral density]*

Suffer from instability problems in estimating that number ?
*[Calculating predominant frequency is notoriously hard]*

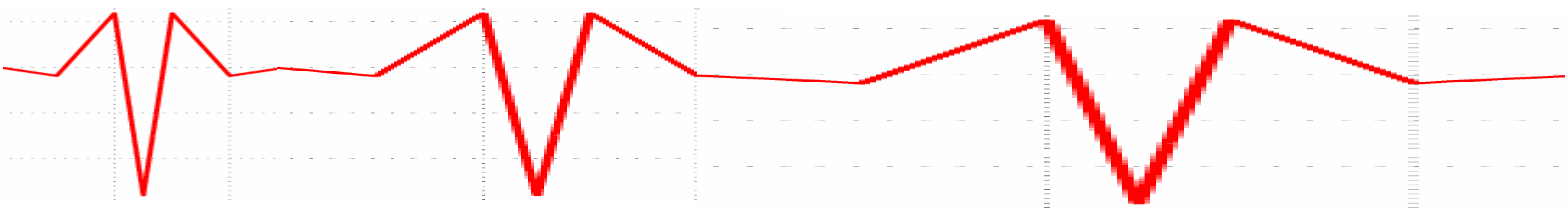


# If, instead

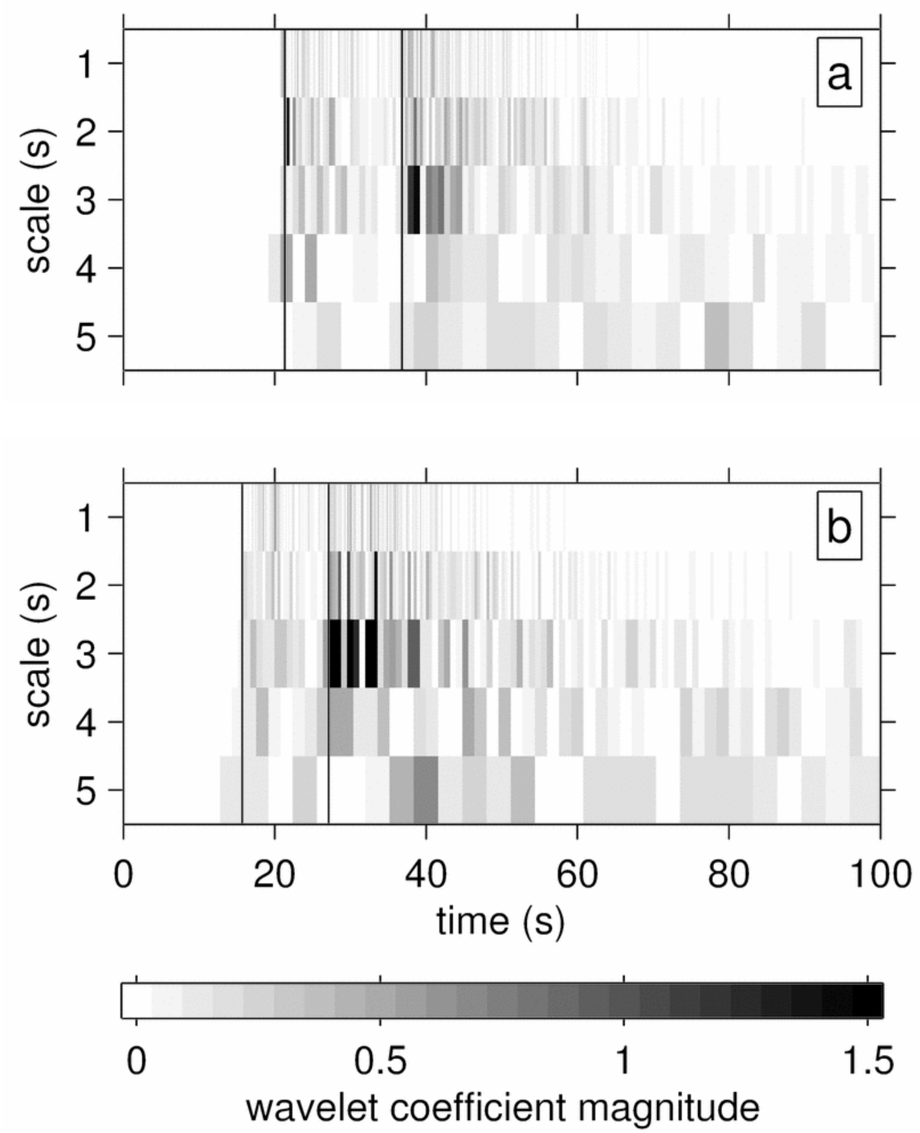
We can use a non-redundant transform !
*[The discrete wavelet transform]*

We can use the full time and scale information !
*[All in one go with the fast lifting implementation]*

We can get stable detection and reliable discrimination !
*[By studying wavelet coefficients after thresholding]*



# Stable & Significant !



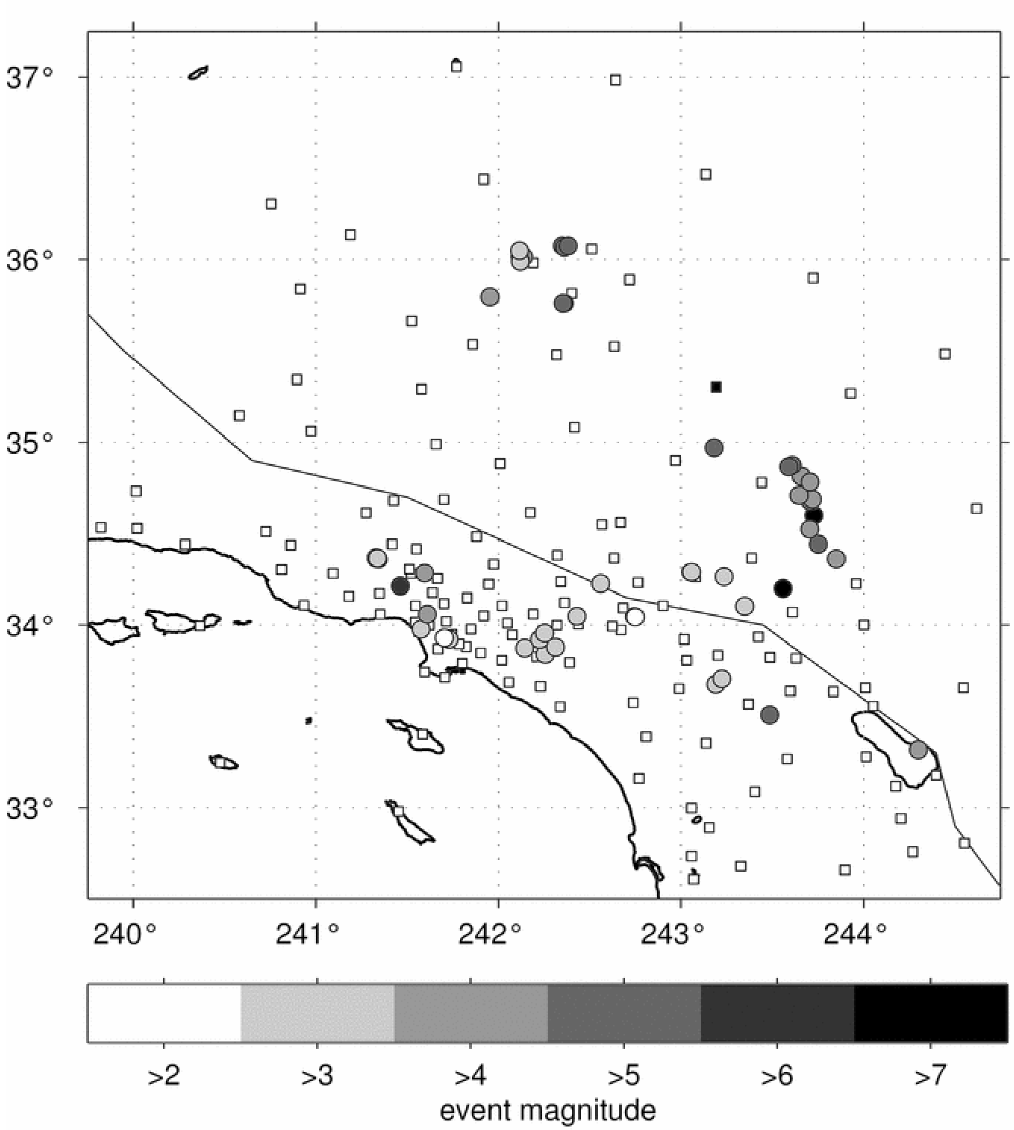

$t, s, |\tilde{u}(s)|$

[small]

$t, s, |\tilde{u}(s)|$

[large]

# Southern Californian data

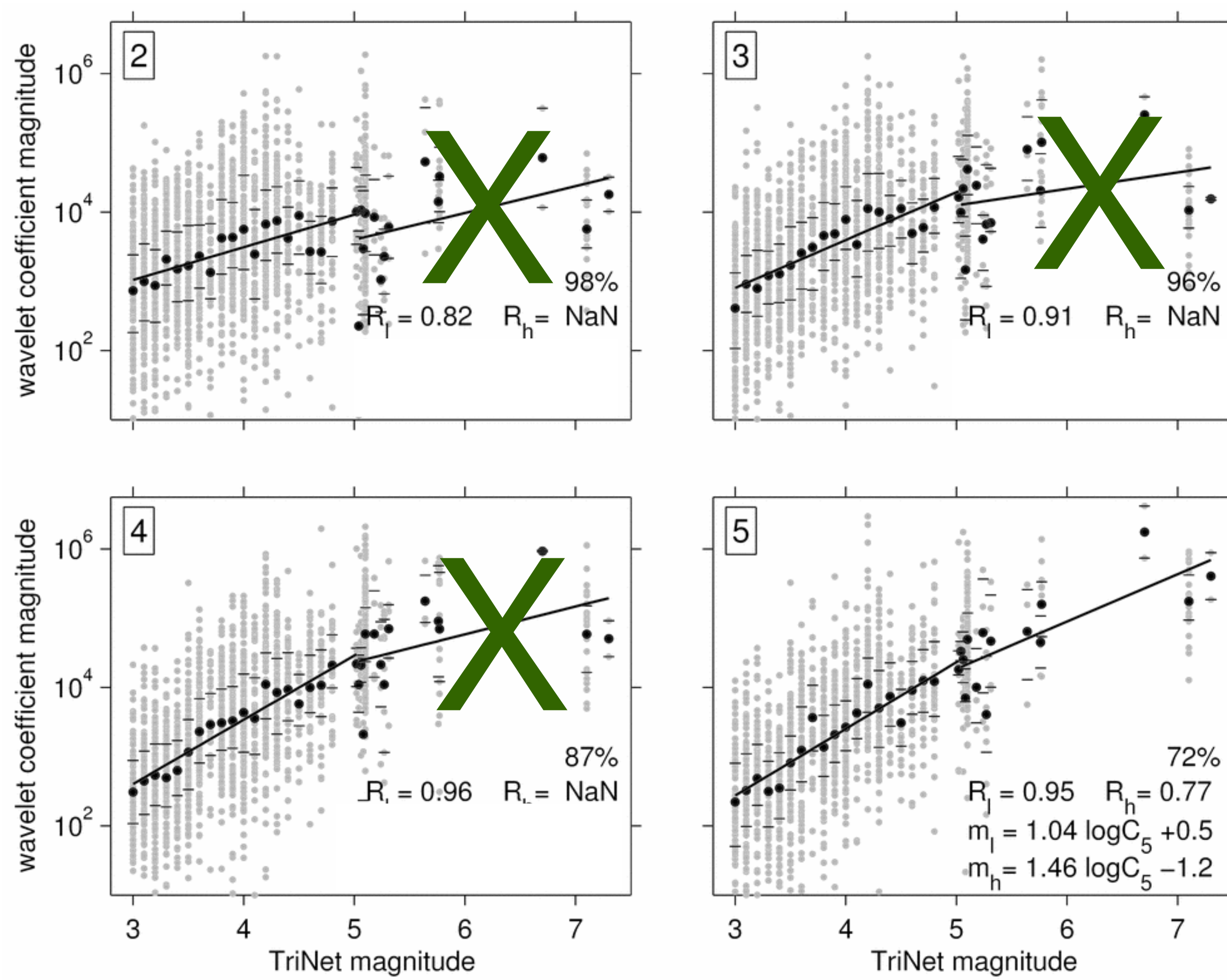



2272 records
142 stations
53 events

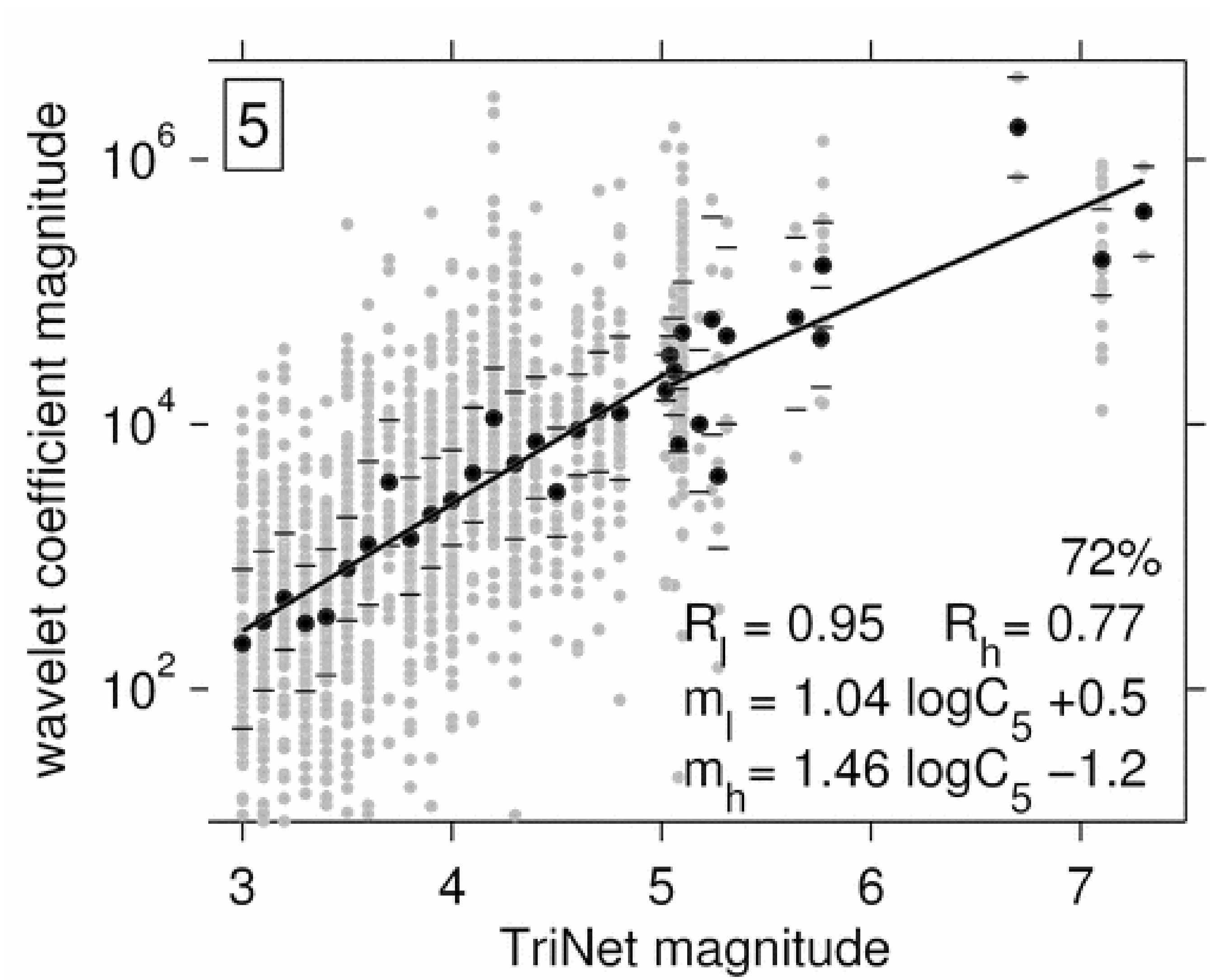
<150 km distance

fast lifted
thresholded
CDF 2,4 wavelet
detection &
discrimination

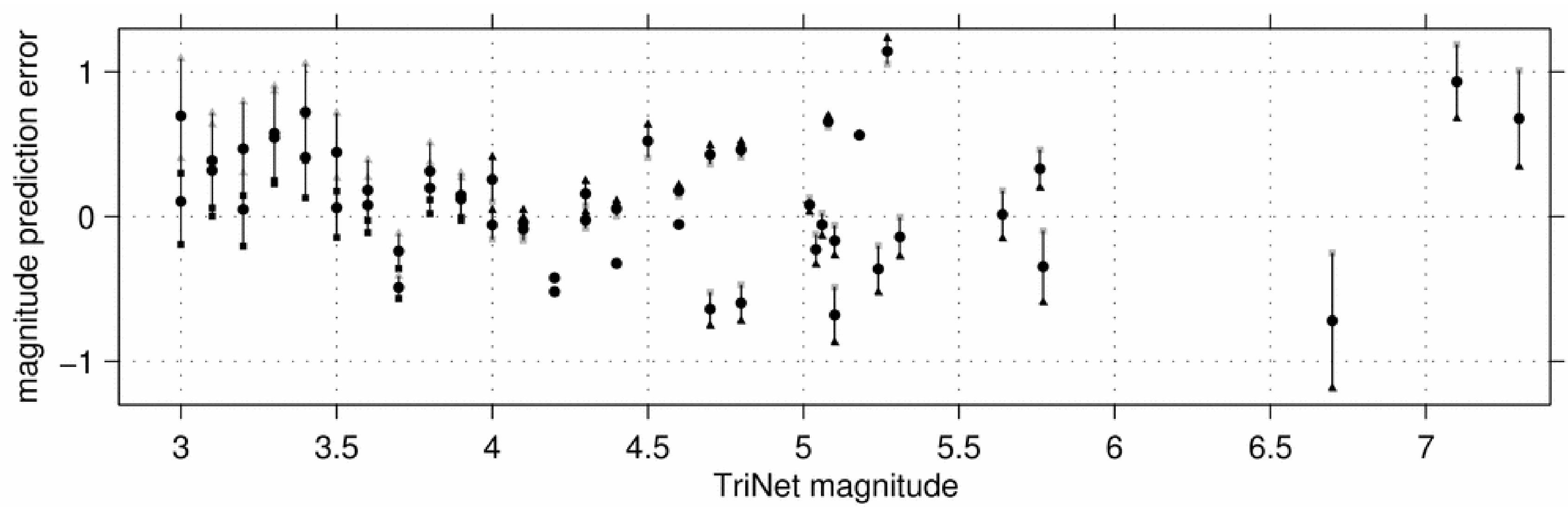
# Results (2 coarse → 5 fine)



# Significant correlations



# Magnitude prediction error




Regardless of the actual magnitude
or the number of reporting stations,
the prediction error is within one magnitude unit

# Conclusions

**No !** Predominant frequency is not the best diagnostic for what happens in a waveform, and it is hard to measure

**Yes !** Discrete wavelet analysis provides much more stable results, and is easily and rapidly implemented

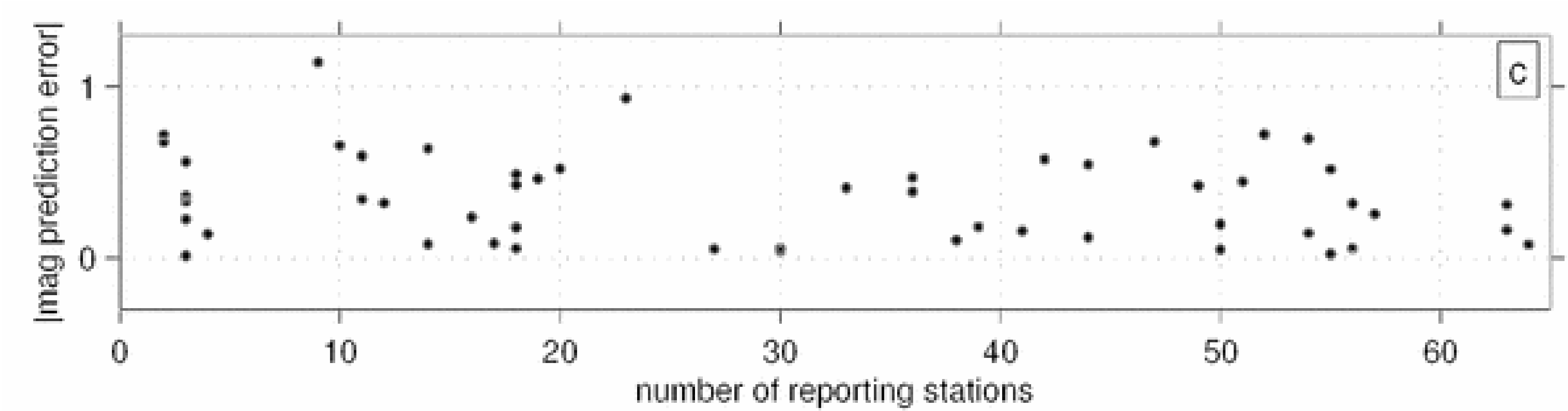
**Yes !** The first few seconds of the waveform still significantly correlate with the final observed earthquake magnitude

**No !** We haven't looked at the significant data gap between magnitudes 6 and 8… yet

# Algorithm, anyone?

```
% Loop over M LIFTING STEPS with stored coefficients Pa and Ua
for index=1:M
  P=Pa{index};    U=Ua{index};  Pl=length(P);   Ul=length(U);
  % PREDICT ------------------------------------------------
  mlx2=mod(length(x),2);
  for l=ceil(Pl/2):ceil(length(x)/2)-floor(Pl/2)-(Pl==1)*mlx2
    Lp=l+[(1-ceil(Pl/2)):1:floor(Pl/2)];
    d(l)=d(l)-floor(P(:)'*a(Lp)+1/2);
  end
  % UPDATE -------------------------------------------------
  for l=1+floor(Ul/2):floor(length(x)/2)-ceil(Ul/2)+1
    Lu=l-[floor(Ul/2):-1:(1-ceil(Ul/2))];
    a(l)=a(l)+floor(U(:)'*d(Lu)+1/2);
  end
end
% SCALE ----------------------------------------------------
d=d*Ku; a=a*Kp;
```

# Magnitude prediction error

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