Gravity, Topography, Magnetics

Geoscience Data Analysis

in Spherical and Planar Geometry

Frederik J Simons

Princeton University

Christopher Harig
Princeton University

Kevin W. Lewis
Johns Hopkins University

Alain M. Plattner
Cal State Fresno

Theme

*“If we are to make progress in data analysis, as it is important that we should, we need to pay attention to our **tools** and our **attitudes**. If these are adequate, our **goals** will take care of themselves.”*

Tukey, 1962

Theme

"The greatest value of a ***picture*** *is when it forces us to notice what we never expected to see."*

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An article about [...] science in a scientific publication is not the scholarship itself, it is merely advertising of the scholarship. The actual scholarship is the complete software development environment and the complete set of instructions which generated the figures.

Sussman & Wisdom, 2001

Tools

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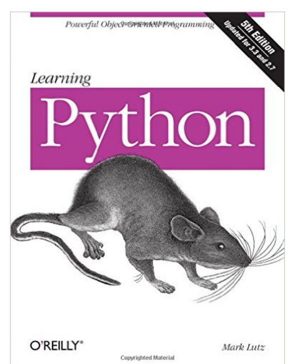

$50 book, free code

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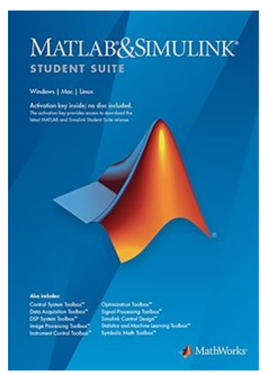

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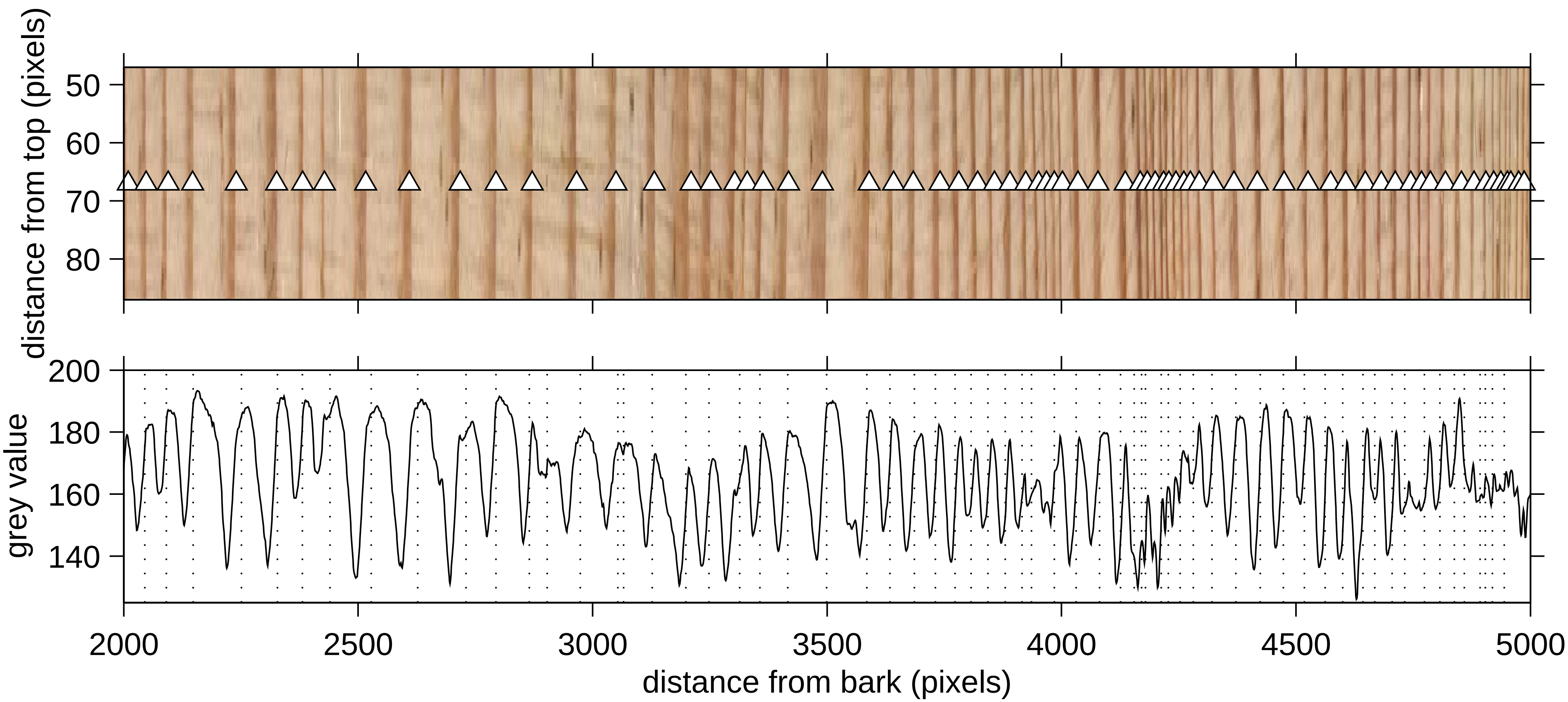
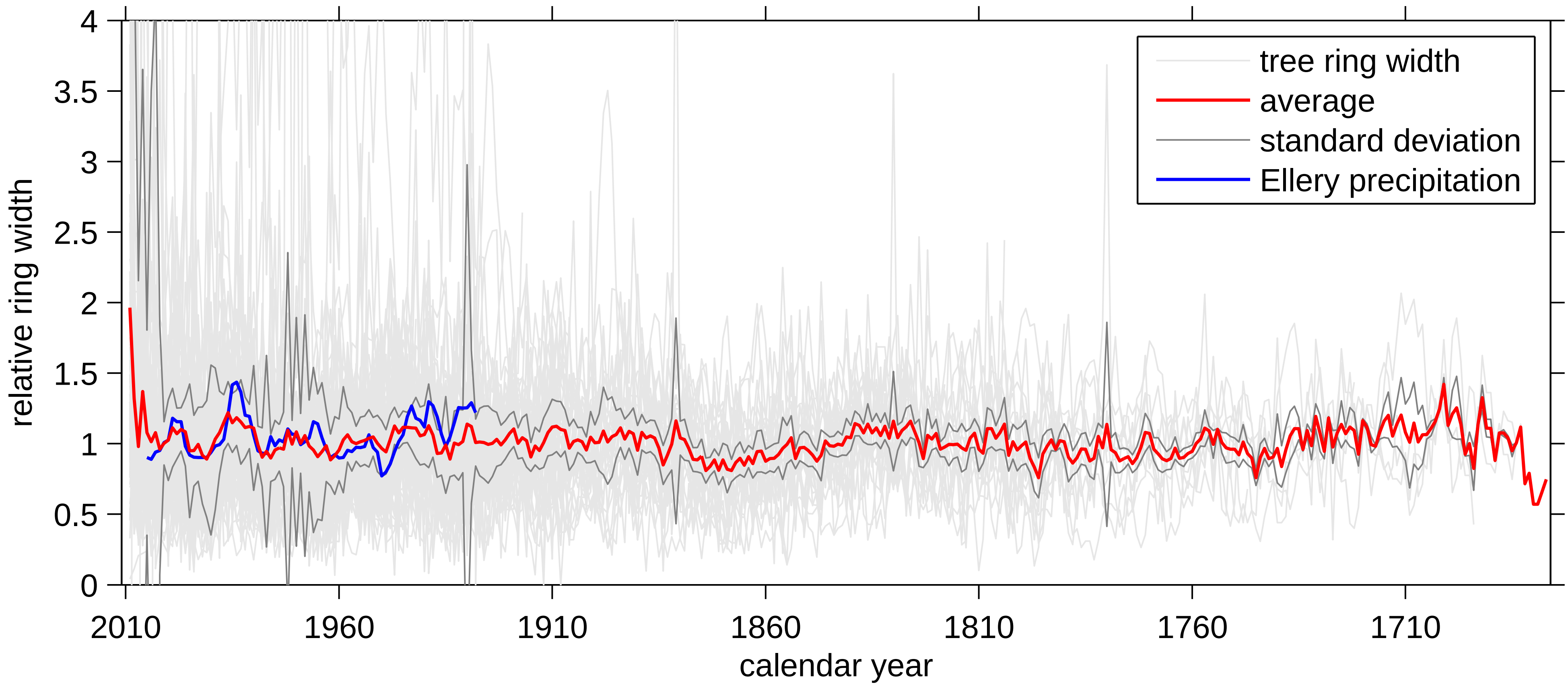
Fomel & Claerbout, 2009

Pictures — 1a

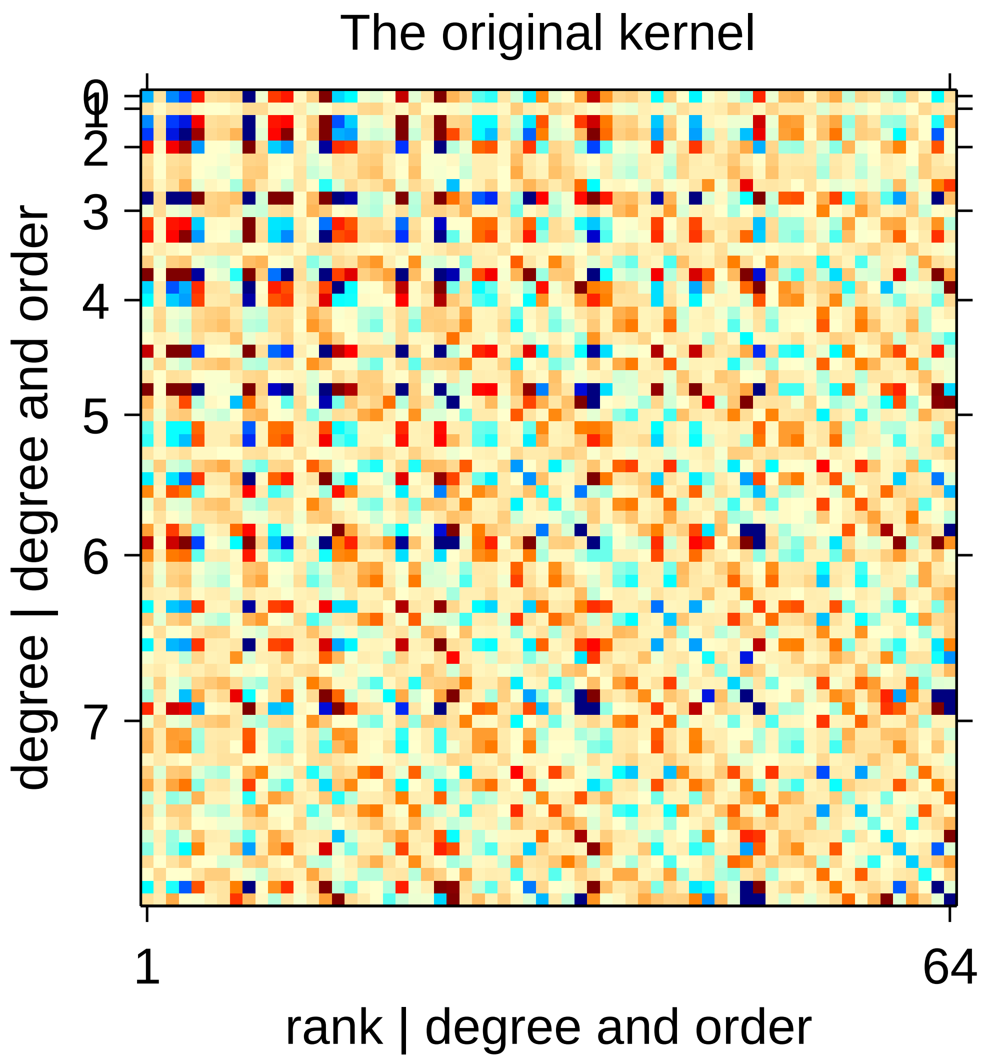


distance from top (pixels)
50
60
70
80
grey value
200
180
160
140
2000
2500
3000
3500
4000
4500
5000
distance from bark (pixels)

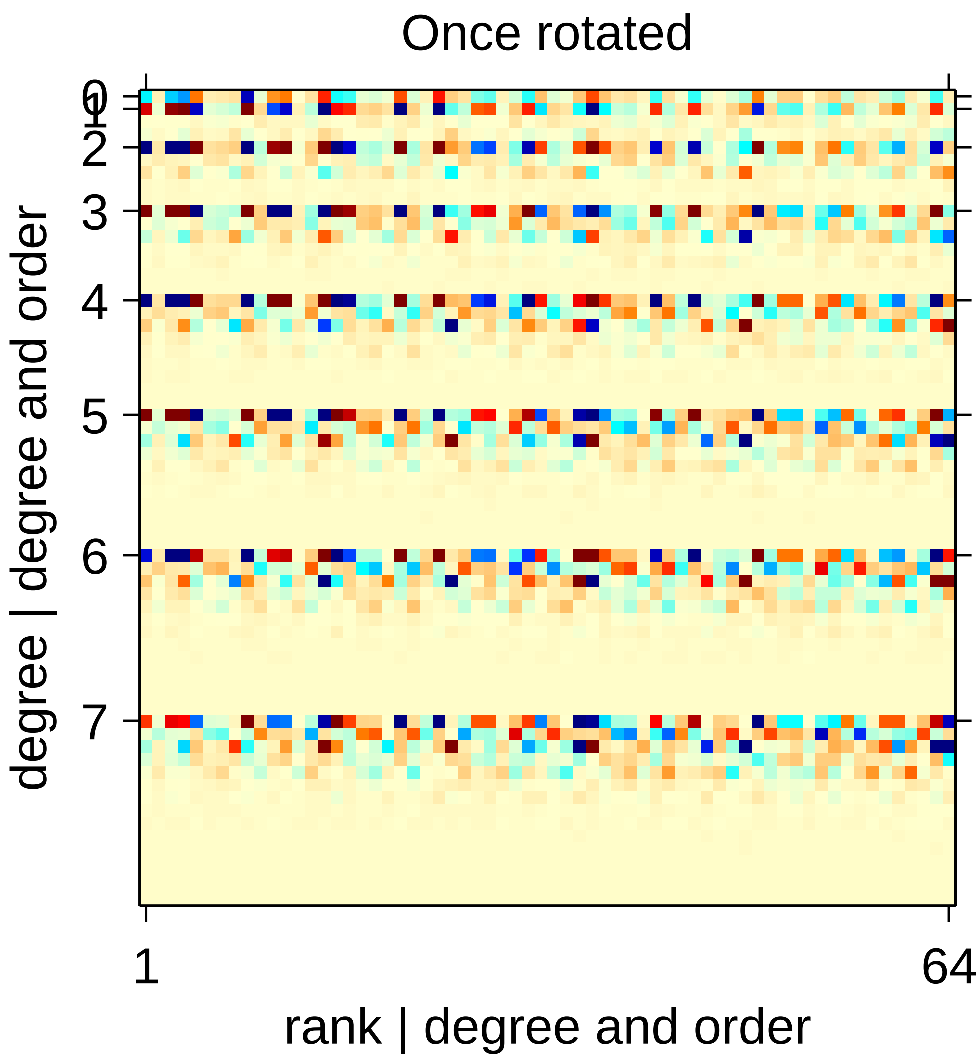
Pictures — 1b



tree ring width
average
standard deviation
Ellery precipitation
relative ring width
0
0.5
1
1.5
2
2.5
3
3.5
4
2010
1960
1910
1860
1810
1760
1710
calendar year


The original kernel
0
1
2
3
4
5
6
7
degree | degree and order
1
64
rank | degree and order

Pictures — 2b


Once rotated
0
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degree | degree and order
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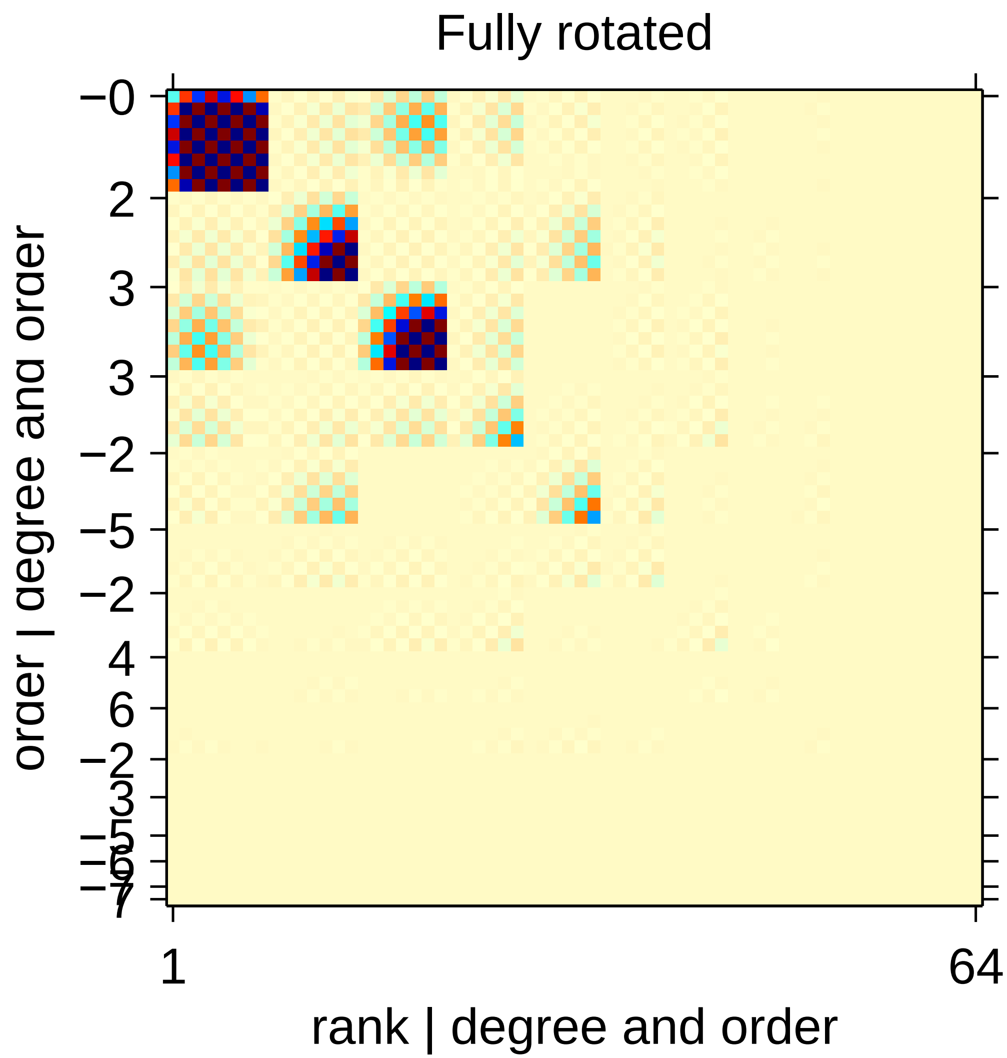
rank | degree and order


Fully rotated
−0
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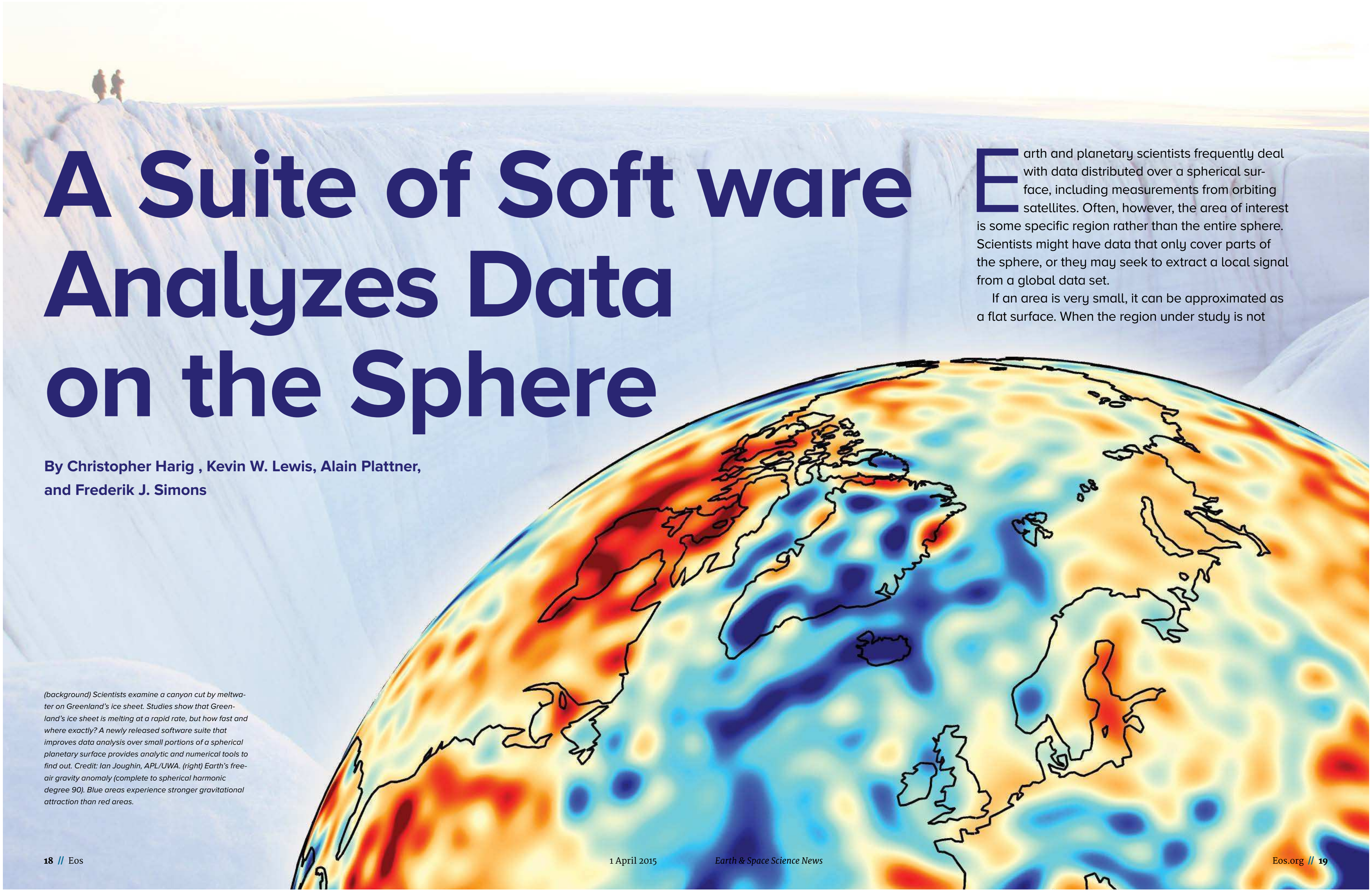
3
−2
−5
−2
4
6
−2
1
64
rank | degree and order
order | degree and order

A suite of software: on `GitHub` and CSDMS



A Suite of Soft ware Analyzes Data on the Sphere

By Christopher Harig , Kevin W. Lewis, Alain Plattner, and Frederik J. Simons

Earth and planetary scientists frequently deal with data distributed over a spherical surface, including measurements from orbiting satellites. Often, however, the area of interest is some specific region rather than the entire sphere. Scientists might have data that only cover parts of the sphere, or they may seek to extract a local signal from a global data set.

If an area is very small, it can be approximated as a flat surface. When the region under study is not

(background) Scientists examine a canyon cut by meltwater on Greenland's ice sheet. Studies show that Greenland's ice sheet is melting at a rapid rate, but how fast and where exactly? A newly released software suite that improves data analysis over small portions of a spherical planetary surface provides analytic and numerical tools to find out. Credit: Ian Joughin, APL/UWA. (right) Earth's free-air gravity anomaly (complete to spherical harmonic degree 90). Blue areas experience stronger gravitational attraction than red areas.

Harig et al., EOS 2015

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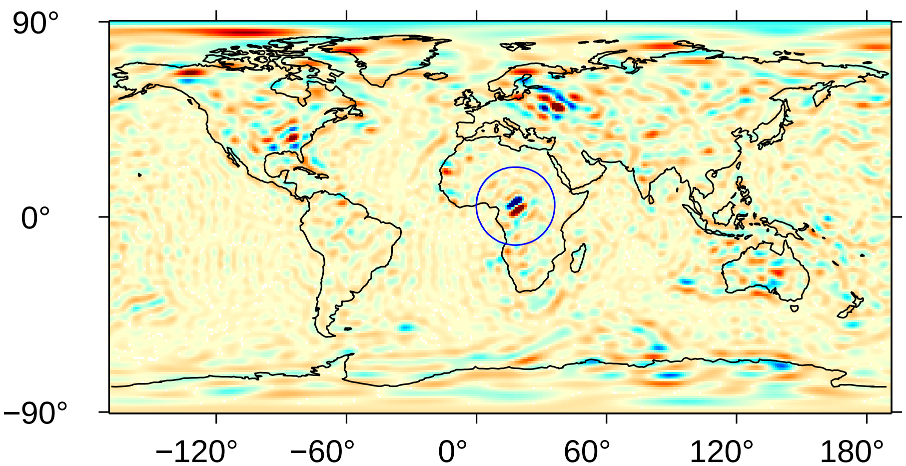
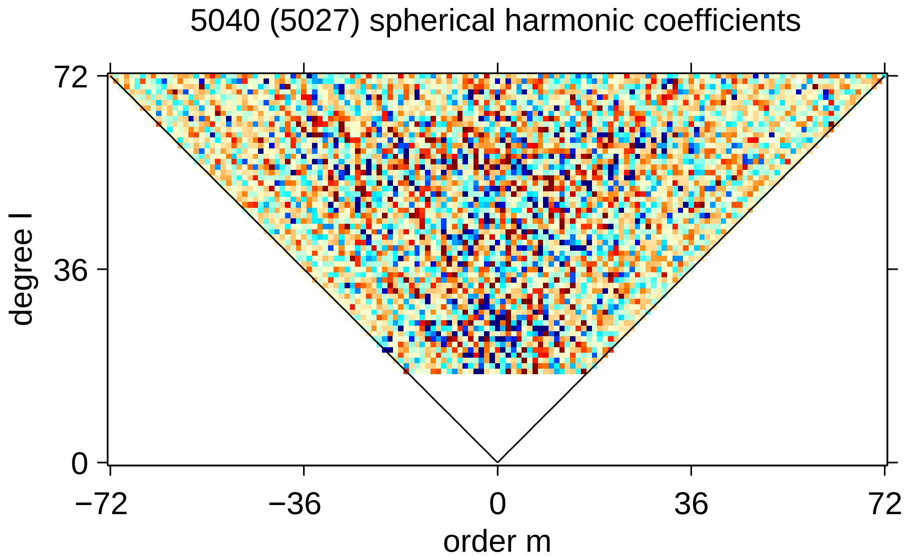
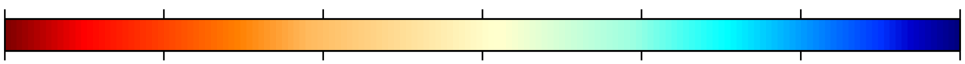
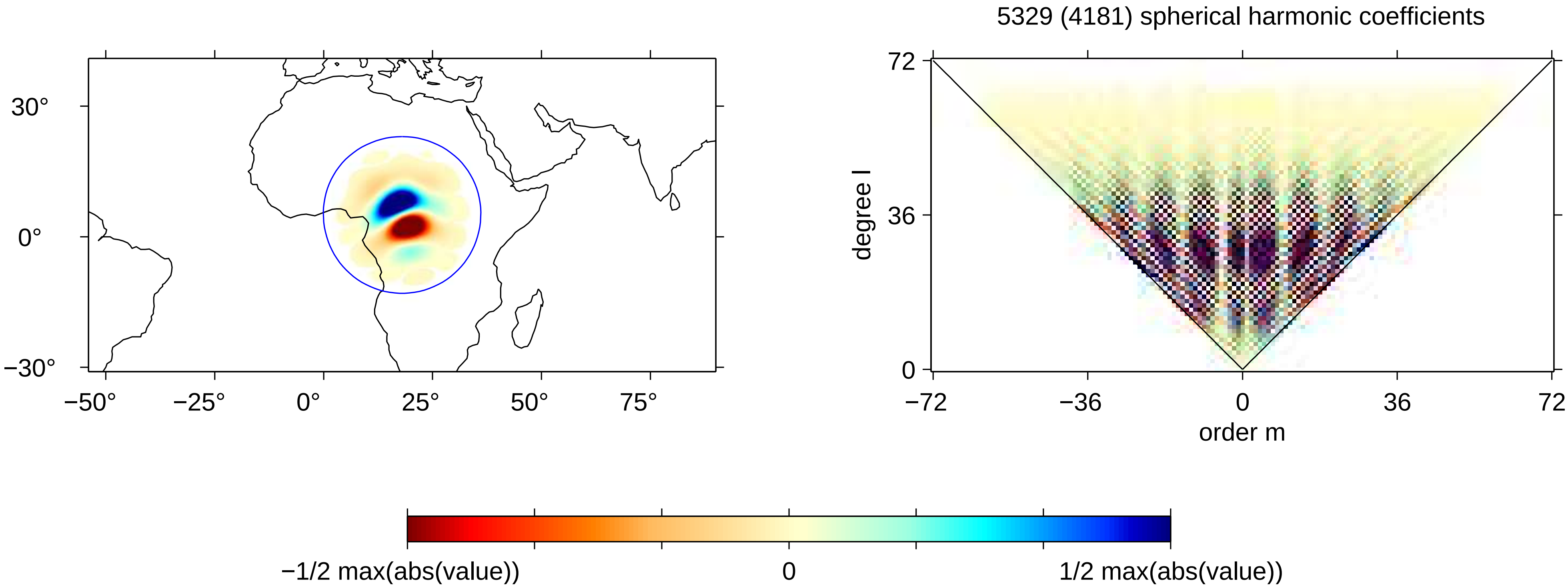
Here, R can be a region of arbitrary geographical description.



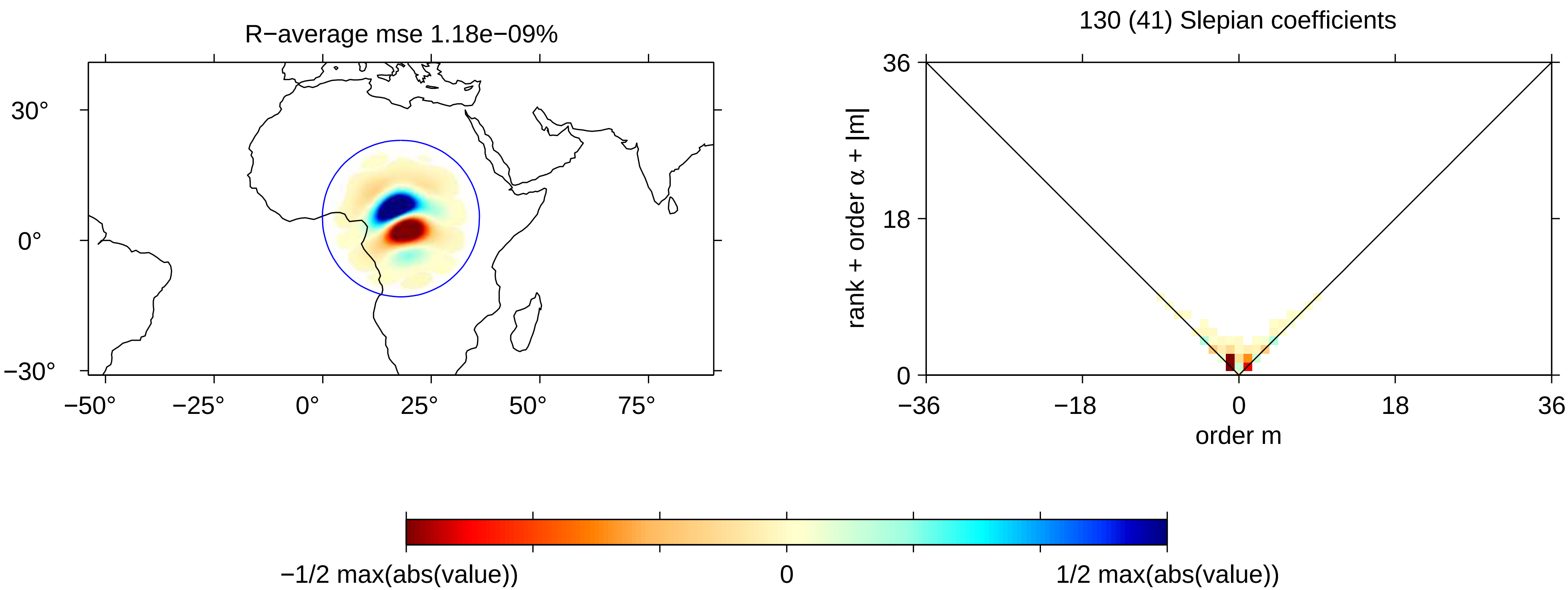




A *global* basis, **good** for *global* problems.



A *global* basis, **bad** for *local* problems.



A *local* basis, **good** for *local* problems. Sparsity!

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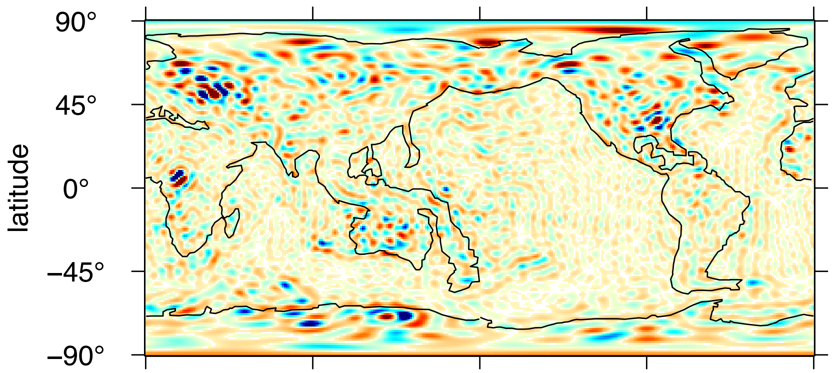
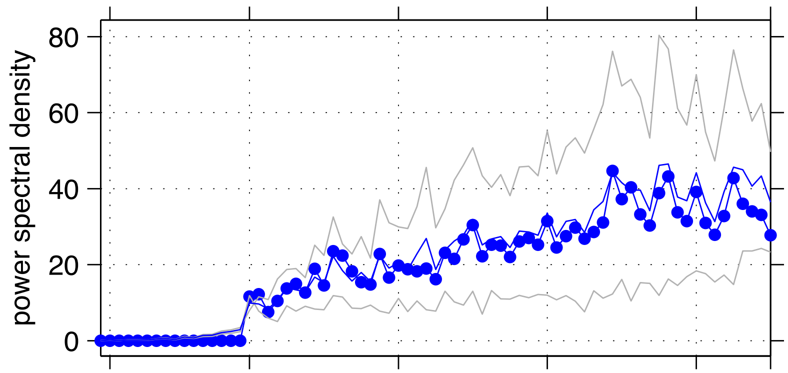
$$\hat{S}_l^{\mathrm{MT}} = \sum_{\alpha} \lambda_\alpha \left(\frac{1}{2l+1} \sum_m \left| \int_\Omega g_\alpha(\mathbf{r})\, d(\mathbf{r})\, Y_{lm}(\mathbf{r})\, d\Omega \right|^2 \right).$$

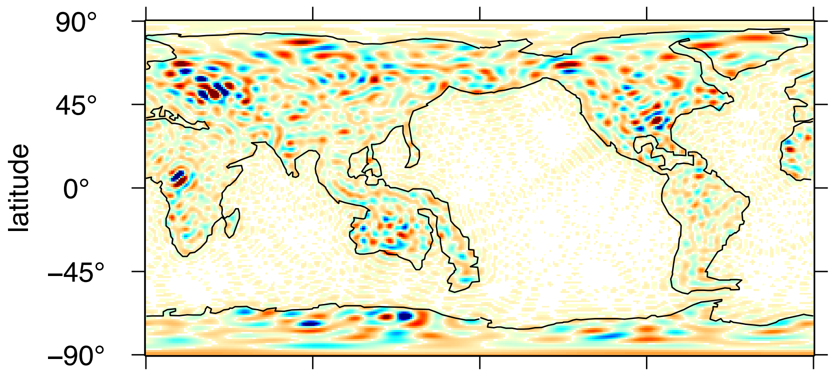
SLEPIAN_Charlie



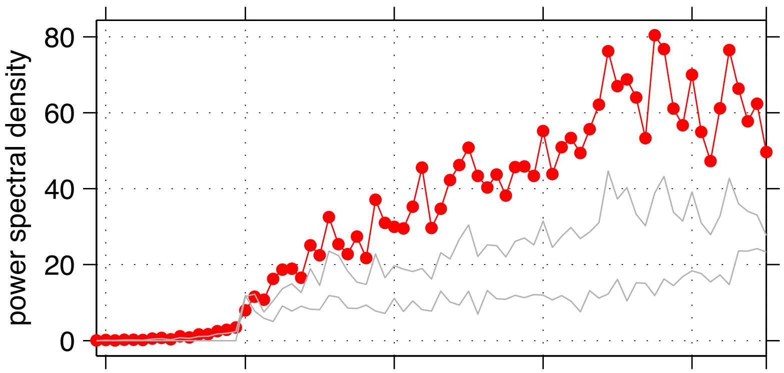
“Whole-sphere”



power spectral density

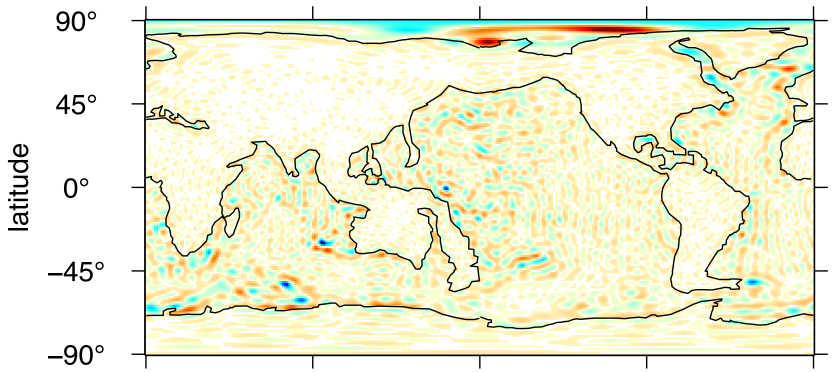


“Continents-only”

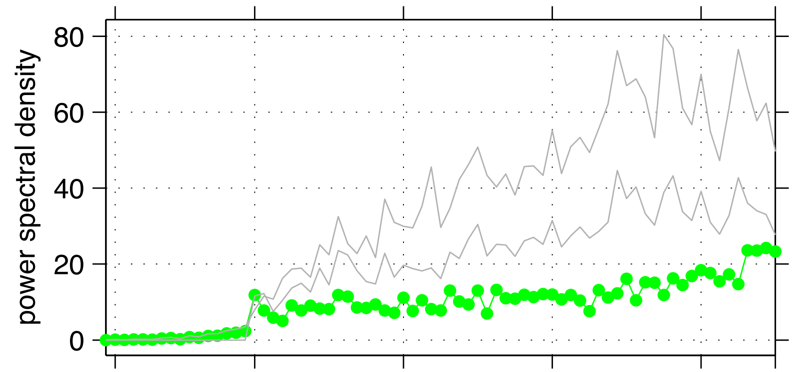


power spectral density

Beggan et al., GJI 2013



“Oceans-only”



power spectral density

Beggan et al., GJI 2013

SLEPIAN_Delta

Time-dependent potential-field estimation
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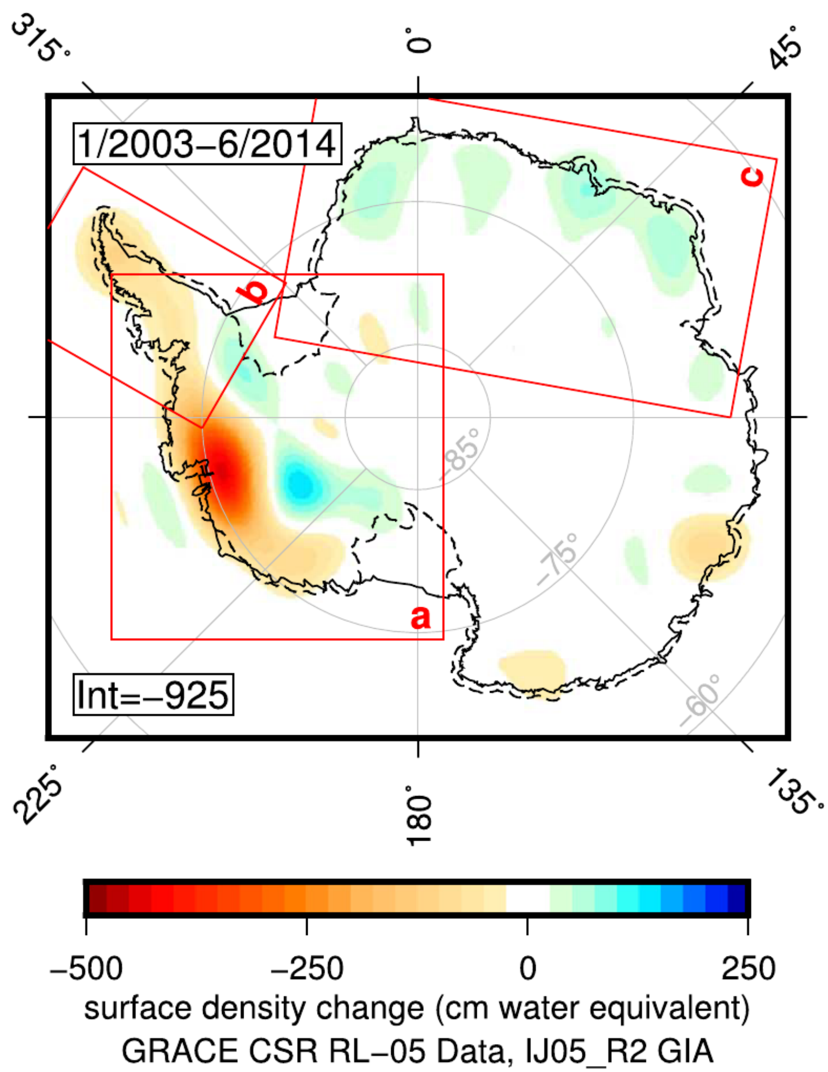
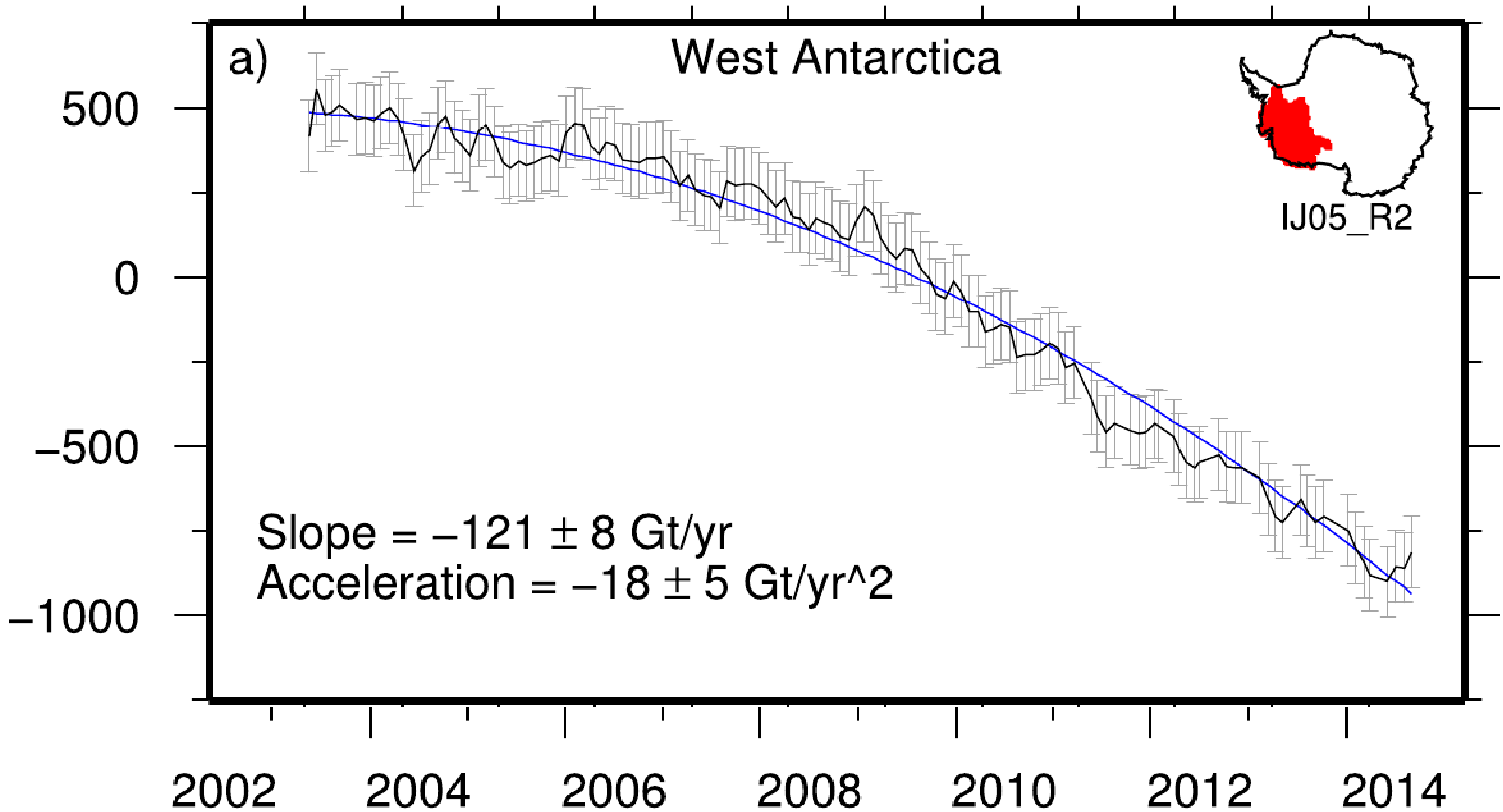
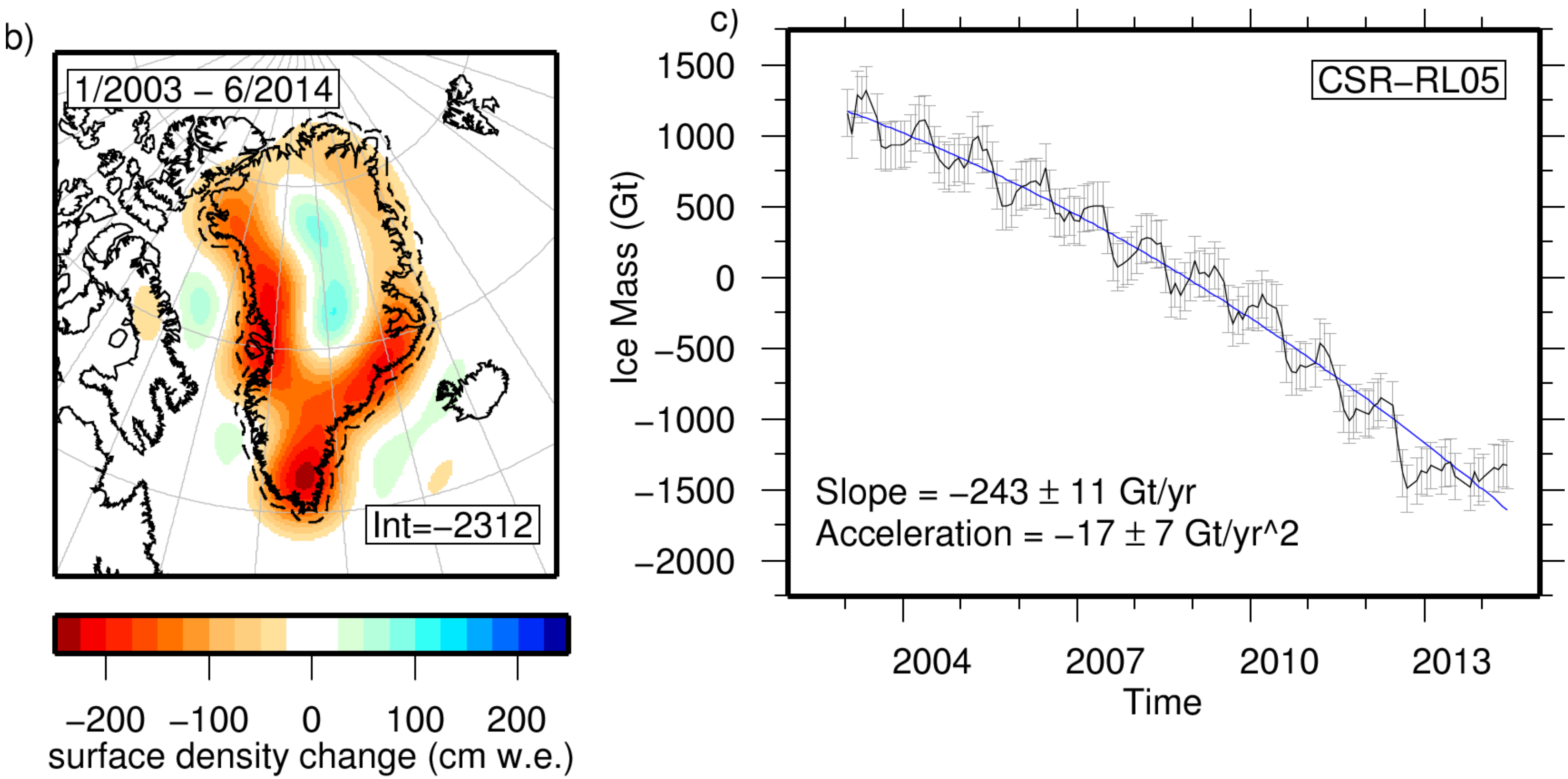
The "data" from the GRACE satellites (at *altitude*), are a non-local mixture $[s_{lm} + n_{lm}](t)$ from which, using the **Slepian basis** we extract the **spatio-temporal signature** of the source —
the time-dependent **ice mass loss** function (at the *surface*).

Harig et al., EOS 2015



Harig & Simons, EPSL 2015


a)
West Antarctica
IJ05_R2
500
0
–500
–1000
Slope = –121 ± 8 Gt/yr
Acceleration = –18 ± 5 Gt/yr^2
2002
2004
2006
2008
2010
2012
2014


b)
1/2003 – 6/2014
Int=–2312
–200 –100 0 100 200
surface density change (cm w.e.)
c)
CSR–RL05
1500
1000
500
0
–500
–1000
–1500
–2000
Ice Mass (Gt)
Slope = –243 ± 11 Gt/yr
Acceleration = –17 ± 7 Gt/yr^2
2004 2007 2010 2013
Time

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- **Matlab** *is* a tool to grow with students from their freshmen days to their professional academic environment — not the *only* language, but an *excellent* one
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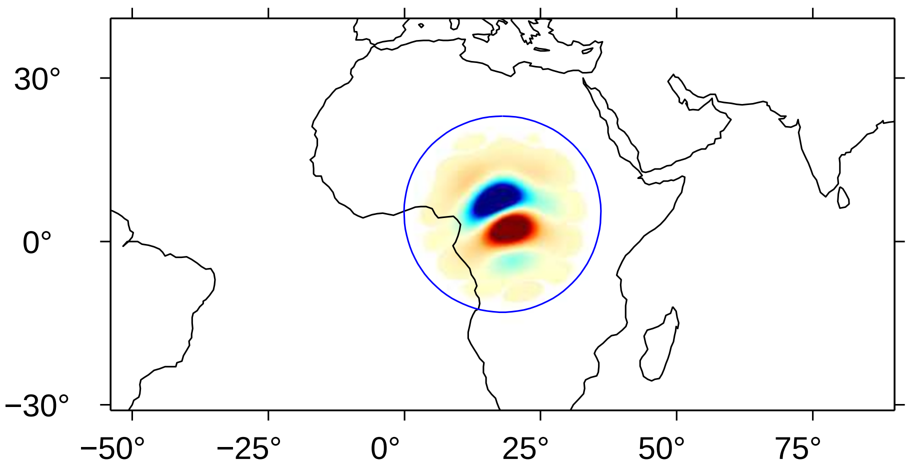
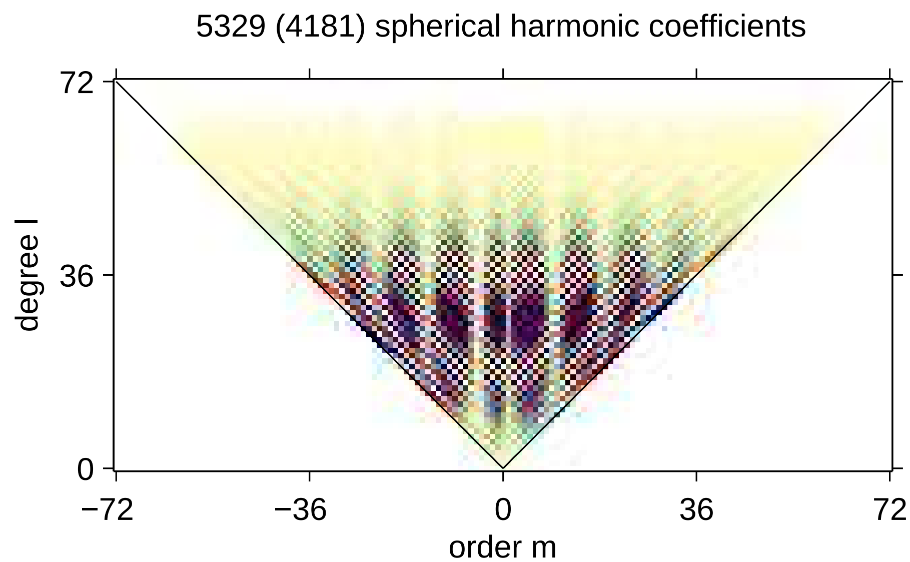
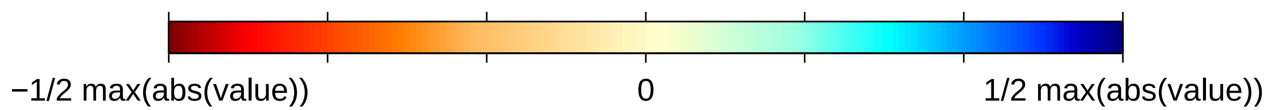
- `SLEPIAN_Delta`: Time-dependent estimation for GRACE satellite data

Basis I: spherical harmonics







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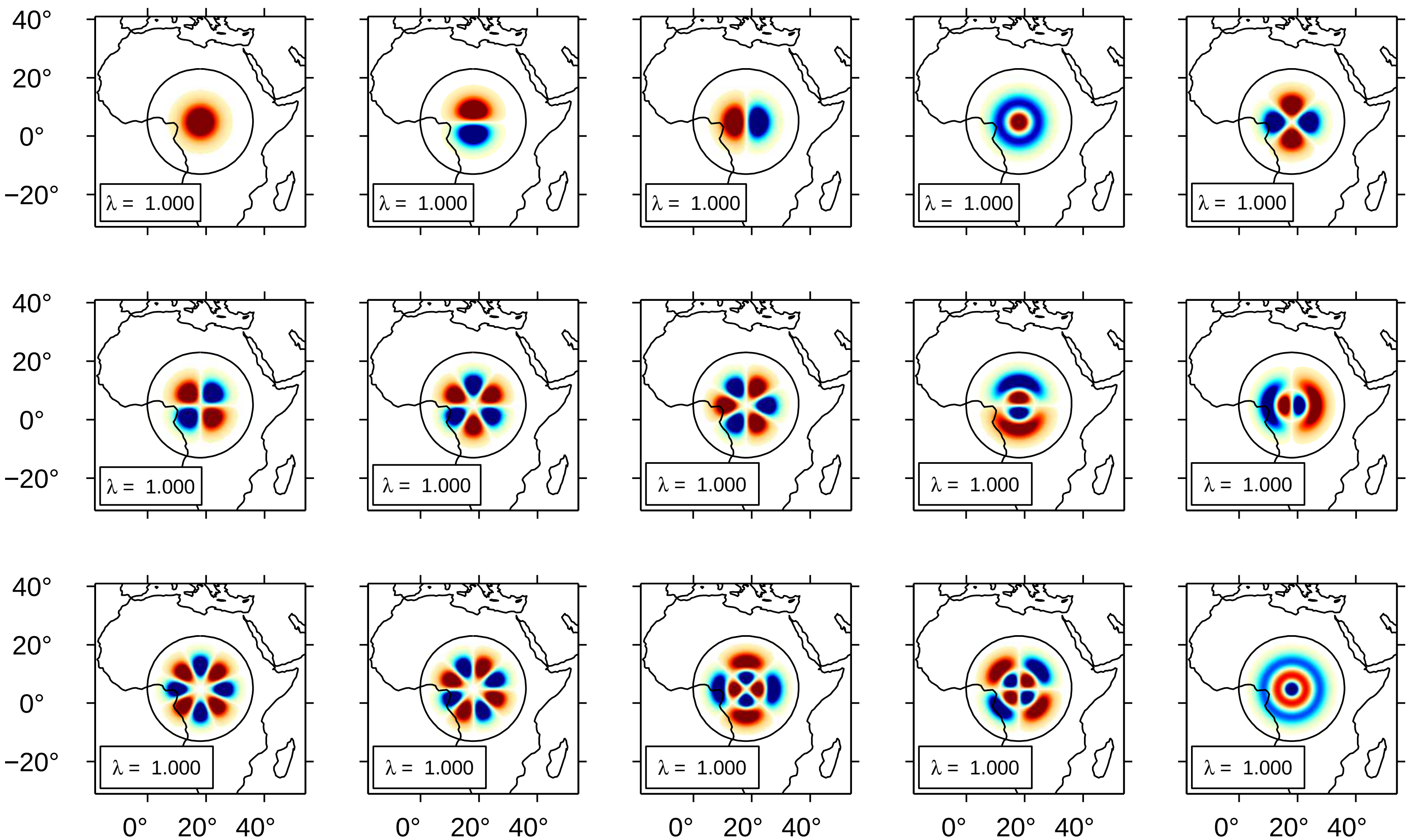
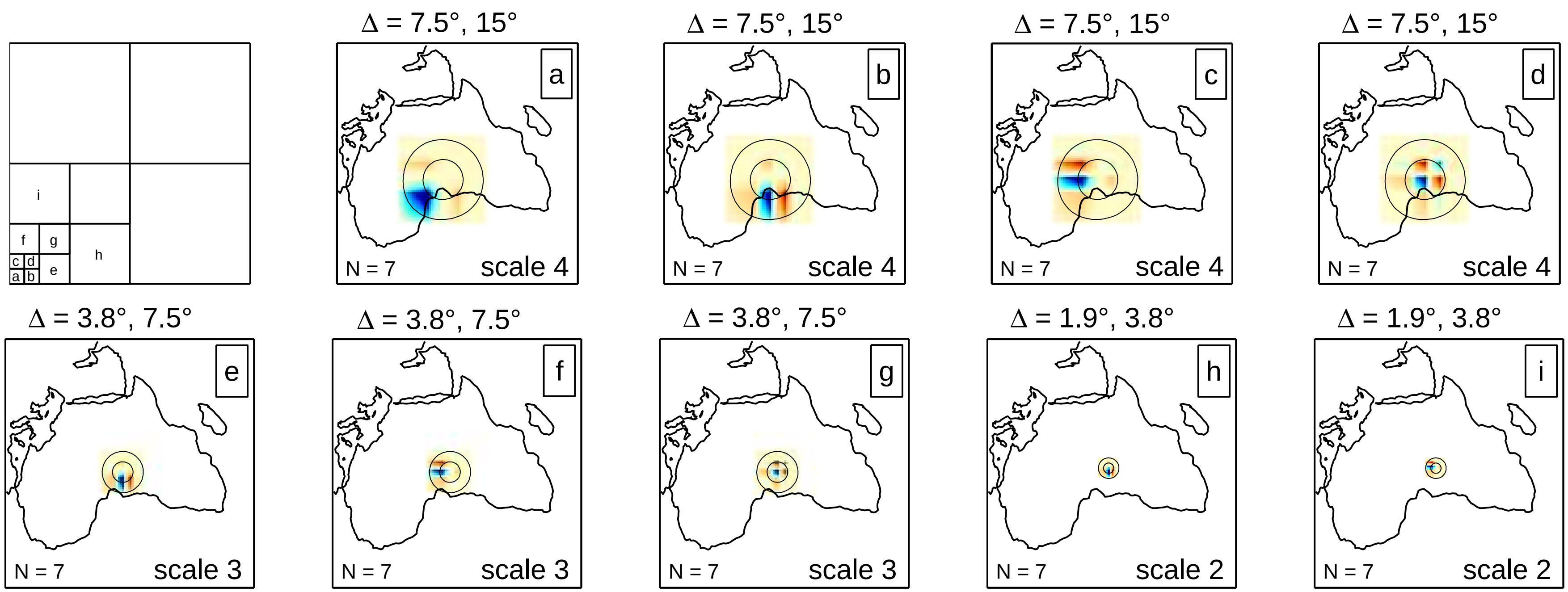
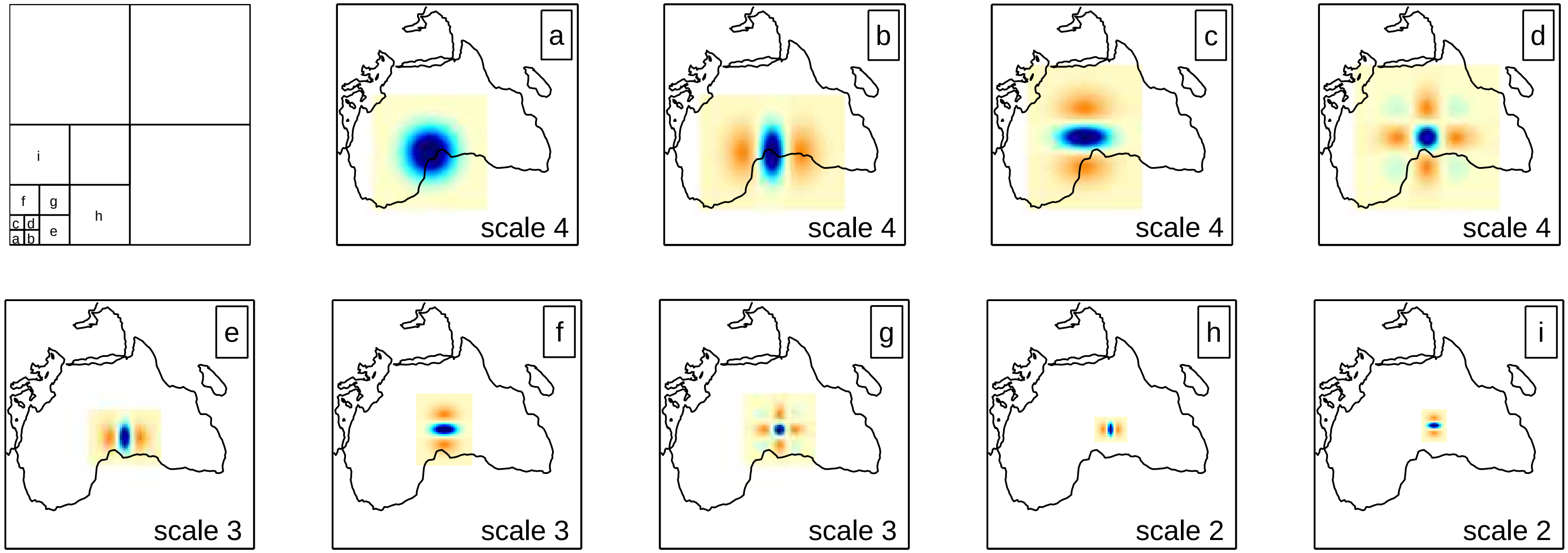
Basis II: Slepian functions



Basis III: cubed-spherical orthogonal wavelets



Loris, Simons et al., *GJI*, 2011



Simons, Loris *et al.*, *GJI*, 2011