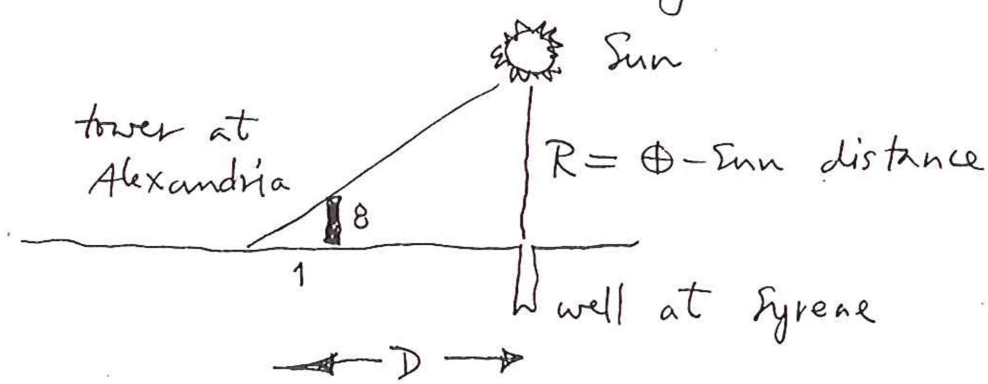
Size and shape of the Earth — "Third Rock from the Sun"

<u>Radius</u> of $\oplus$ first measured by Eratosthenes, librarian of Alexandra ~ 225 BC

Observed that at noon on summer solstice Sun was directly overhead at Syrene (near Aswan — on Tropic of Cancer), whereas at noon it cast an 8:1 shadow

Flat-earth interpretation of this observation:



Complementary triangles : $\frac{R}{D} = \frac{8}{1}$

$R = 8D$

D = 5000 stadia
one stadium = 160 m
D = 800 km

R = 6400 km

Eratosthenes rejected this view in favor of spherical $\oplus$ interpretation

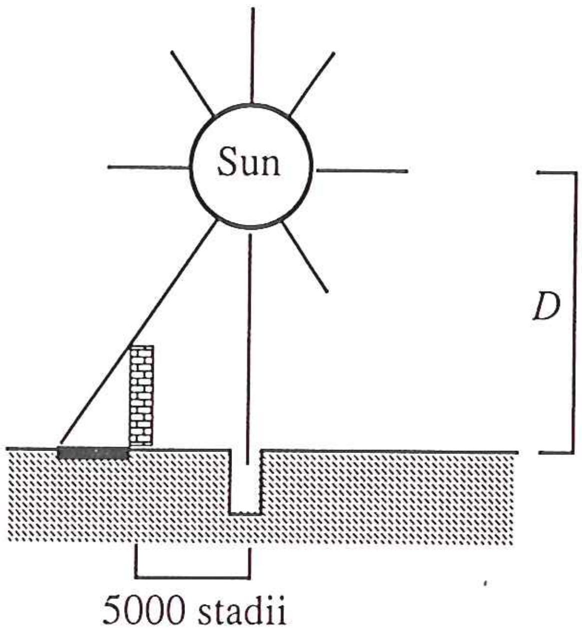


FIGURE 1.6 A flat Earth view of Eratosthenes' measurement. The differences in the shadows might be used to conclude that the Sun was a relatively short distance, *D*, away from Earth.

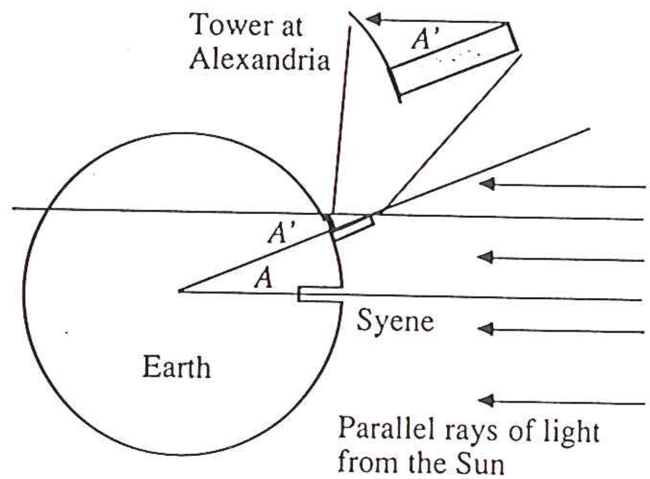
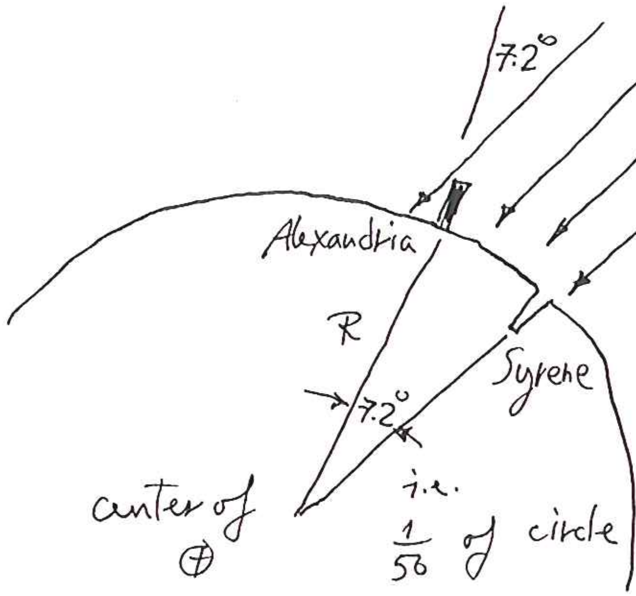


FIGURE 1.7 Eratosthenes' actual way of determining the circumference of Earth, assuming a round Earth and a very distant Sun. The size of the tower and well are greatly exaggerated.

Table 3-1. Characteristics of the orbits of the nine planets and of the largest object in the asteroid belt:

Planet	Radius of orbit (10^{13} centimeters)	Time for one revolution about Sun (years)	Inclination of orbit* (degrees)	Eccentricity of orbit†	Length of day (days)
Mercury	0.58	0.24	7.0	0.21	59
Venus	1.08	0.62	3.4	0.01	243
Earth	1.50	1.00	0.0	0.02	1.0
Mars	2.29	1.88	1.9	0.09	1.0
Asteroid Ceres	4.15	4.60	10.6	0.08	0.4
Jupiter	7.70	11.9	1.3	0.05	0.4
Saturn	14.3	29.5	2.5	0.06	0.4
Uranus	28.3	84.0	0.8	0.05	1.0
Neptune	45.1	164.8	1.8	0.01	0.9
Pluto	59.2	247.7	17.2	0.25	6.2

*The plane of the Earth's orbit is used as the reference.
†A measure of the deviation from circularity.



Sun so distant that rays of light appear parallel

same numerical answer as before

radius $R = \frac{50}{2\pi} D = 8D = 6400$ km

Modern accurate measurement — <u>mean</u> radius

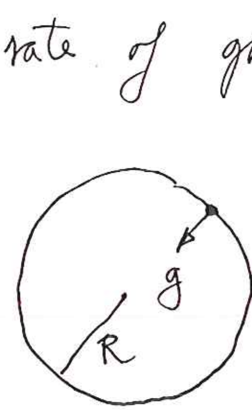
$R = 6371$ km

Volume of Earth : $V = \frac{4}{3}\pi R^3 = 1.08 \cdot 10^{21}\ m^3$

Two ways to measure <u>mass</u> of Earth :

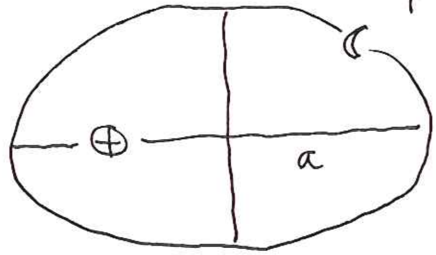
(1) rate of gravitational acceleration (free fall)

$$g = \frac{GM}{R^2} = 9.8\ m/sec^2$$



$GM_{\oplus} = 4 \cdot 10^{14}\ m^3/sec^2$

(2) Kepler's third law



period of lunar orbit
$T \approx 28$ days

$$\frac{4\pi^2 a^3}{T^2} = G\left(M_\oplus + M_{\leftmoon}\right)$$

semi-major axis:
$a \approx 60\, R_\oplus$

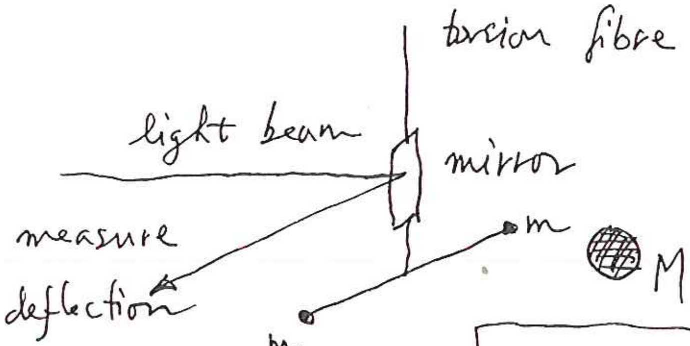
$M_{\leftmoon} \approx \frac{1}{81.3} M_\oplus$

again gives
$GM_\oplus = 4 \cdot 10^{14}\ m^3/sec^2$

Modern determinations – artificial satellite orbits – very accurately known

$$GM_\oplus = 3.986013 \cdot 10^{14}\ m^3/sec^2$$

Newton's constant G – measured by Lord Cavendish (1798) – "weighing the Earth"



$$G = 6.67 \cdot 10^{-11}\ m^3\, kg^{-1}\, sec^{-2}$$

$M_{\oplus} \simeq 5.98 \cdot 10^{24}$ kg

Mean density $\bar{\rho} = \frac{M}{V} = 5500$ kg/m^3

Decompression correction:

$\bar{\rho}_{decompressed} = 4300$ kg/m^3

Heavier than common surface rocks (e.g., basalt)

$\rho_{rocks} = 2500 - 3400$ kg/m^3

First clue regarding <u>bulk composition of Earth</u>: what is it made of?

Mass $M = 4\pi \int_0^R \rho(r)\, r^2\, dr$

↑ density distribution as function of radius

<u>Moment of inertia</u>: provides another constraint

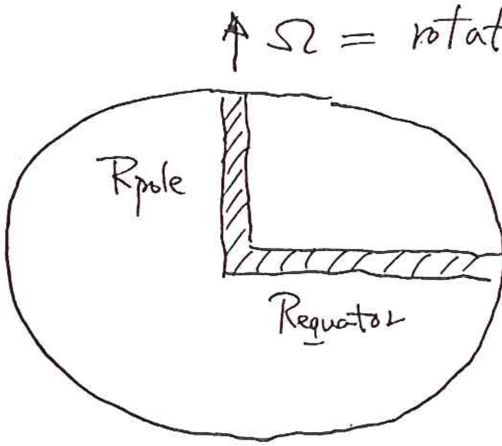
$$C = \frac{8\pi}{3} \int_0^R \rho(r)\, r^4\, dr$$

For a uniform sphere : $\rho(r) = \text{constant}$

$$C = \frac{2}{5} MR^2 = 0.4 MR^2$$ ← uniform sphere

Flattening of Earth (Newton, Principia, 1687)

$$\Omega = \text{rotation rate} = \frac{2\pi}{\text{one sidereal day}}$$



$$\Omega = 7.292 \cdot 10^{-5} \text{ rad/sec}$$

$$m = \frac{\text{centrifugal force}}{\text{gravity}} = \frac{\Omega^2 R}{GM/R^2} = \frac{\Omega^2 R^3}{GM}$$

$$\varepsilon = \frac{R_{eq} - R_{pole}}{R_{eq}}$$ flattening or ellipticity

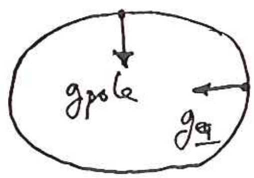
weight of $1 \times 1 \text{ (meter)}^2$ polar column
$= \frac{1}{2} R_{pole}\, g_{pole}$

weight of equatorial column $= \frac{1}{2} R_{eq}\, g_{eq} (1-m)$

centrifugal dilution factor → $(1-m)$

Balance columns:

$$1 - m = \frac{R_{pole}}{R_{eq}} \frac{g_{pole}}{g_{eq}} = (1-\varepsilon) \frac{g_{pole}}{g_{eq}}$$



$$\frac{g_{pole}}{g_{eq}} = 1 + \frac{1}{5}\varepsilon$$

$$1 - m = 1 - \frac{4}{5}\varepsilon \qquad \boxed{\varepsilon = \frac{5}{4} m}$$

$$m = \frac{1}{290} \Rightarrow \boxed{\varepsilon = \frac{1}{230}}$$

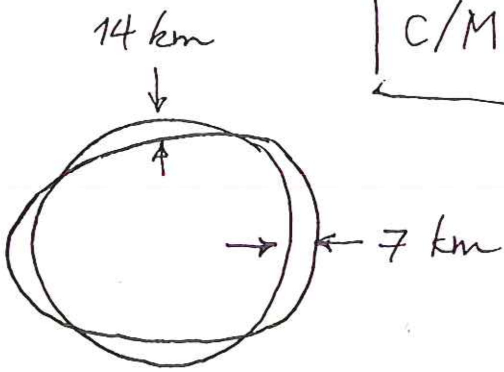
More general result due to Clairaut and Radau

$$\boxed{\varepsilon = \frac{10m}{4 + 25\left(1 - \frac{3}{2}\frac{C}{MR^2}\right)^2}}$$

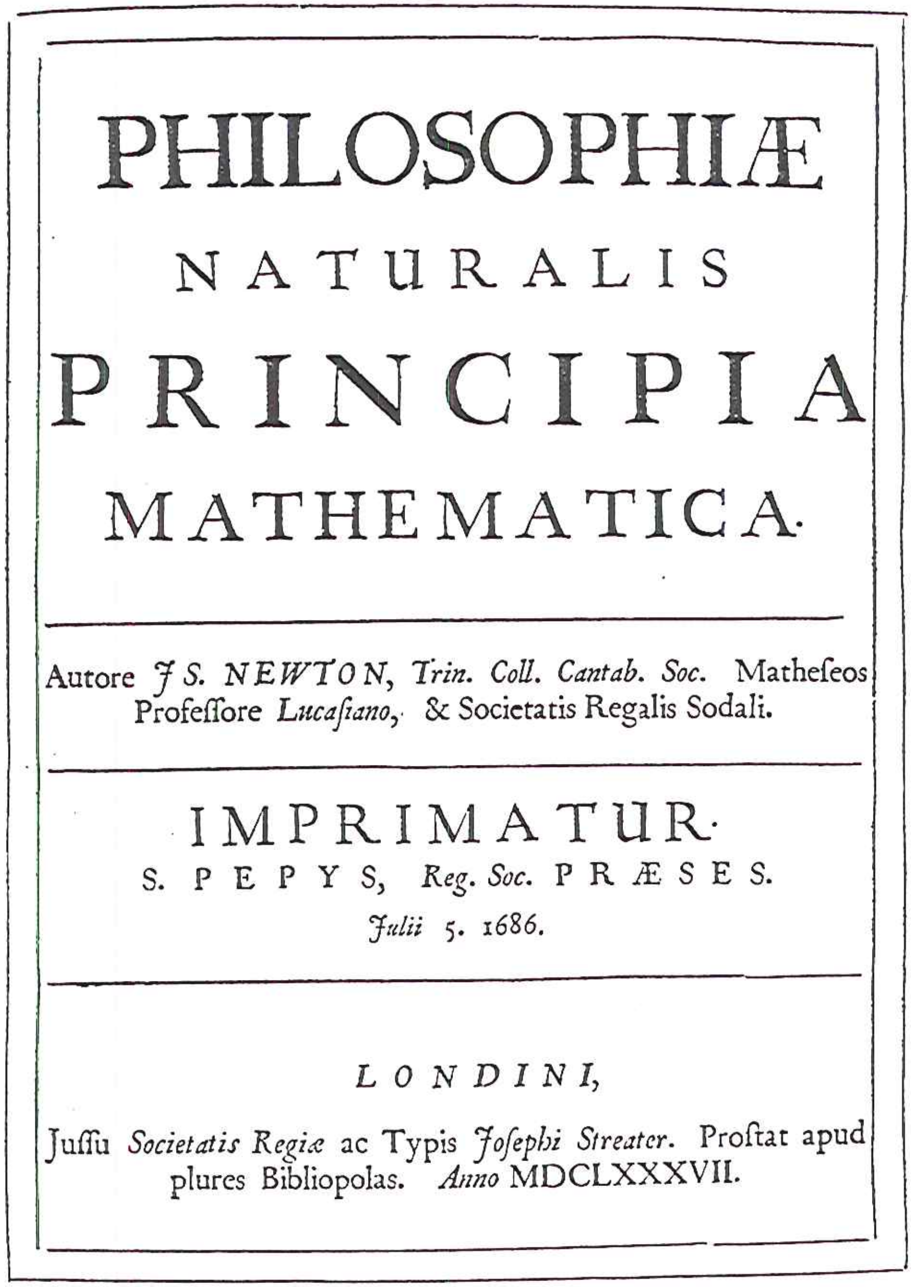
Maupertuis & Cassini history

Modern data

$$\varepsilon = 1/298.25$$
$$C/MR^2 = 0.33089$$



$$R_{pole} = 6357 \text{ km}$$
$$R_{eq} = 6378 \text{ km}$$
$$R_{eq} - R_{pole} = 21 \text{ km}$$



PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

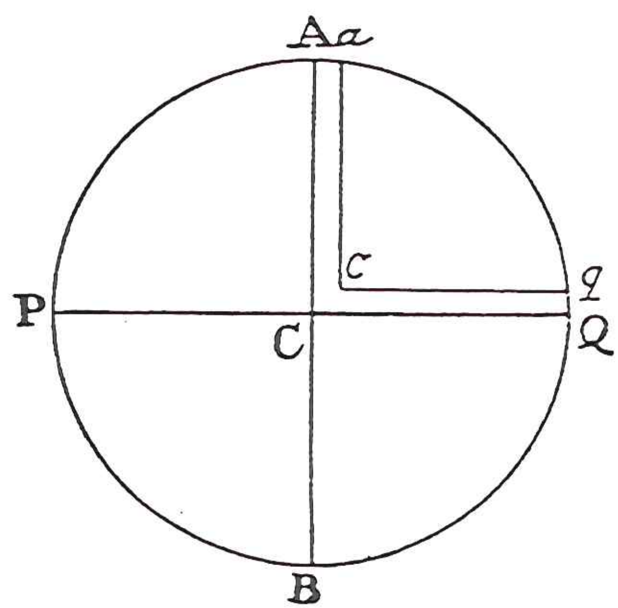
IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

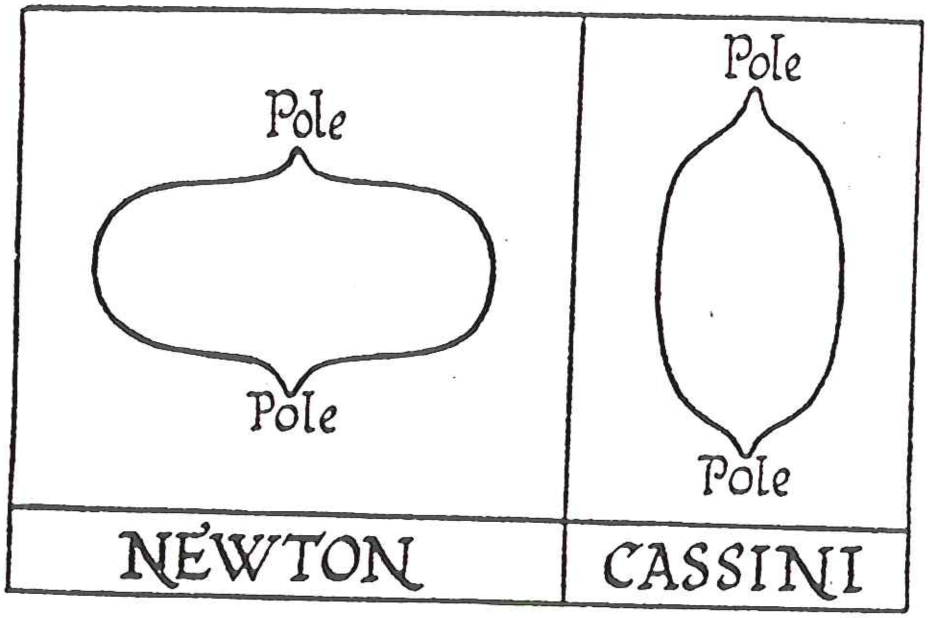
LONDINI,

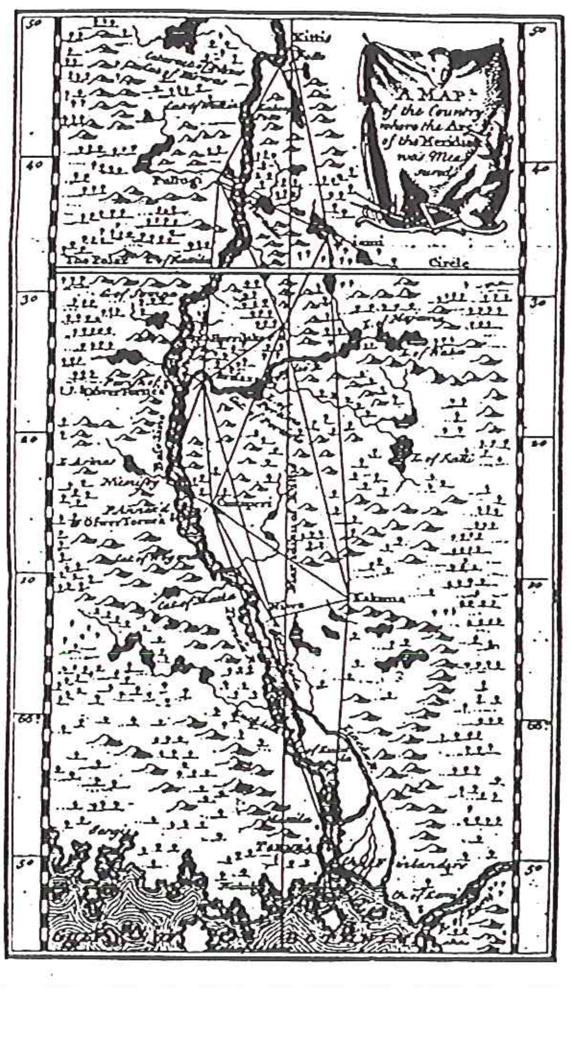
Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtat apud plures Bibliopolas. *Anno* MDCLXXXVII.

TITLE PAGE OF THE FIRST EDITION OF THE PRINCIPIA

Verum vis centrifuga partis cujusque est ad pondus ejusdem ut 1 et 289, hoc est, vis centrifuga, quæ deberet esse ponderis pars $\frac{4}{505}$, est tantum pars $\frac{1}{289}$. Et propterea dico, secundum Regulam auream, quod si vis centrifuga $\frac{4}{505}$ faciat ut altitudo aquæ in crure $ACca$ superet altitudinem aquæ in crure $QCcq$ parte centesimâ totius altitudinis: vis centrifuga $\frac{1}{289}$ faciet ut excessus altitudinis in crure $ACca$ sit altitudinis in crure altero $QCcq$ pars tantum $\frac{1}{229}$.







A map of Lapland compiled by the French scientist Maupertuis in 1736–37, to show his triangulation of a degree of a meridian. He began at Tornio (Torneå) at 65°50′N and stopped at Kittis Mountain just short of 66°50′N and just north of the Arctic Circle. His calculation, when compared with the length of a degree in France and on the equator in Peru, showed that the Earth was slightly flattened at the poles, as Newton's theory of gravitation had predicted.

Courriers de la physique, argonautes nouveaux,
Qui franchissez les monts, qui traversez les eaux,
Ramenez des climats soumis aux trois couronnes
Vos perches, vos secteurs, et sur-tout deux Lapones.
Vous avez confirmé dans ces lieux pleins d'ennui
Ce que Newton connut sans sortir de chez lui.

Voltaire

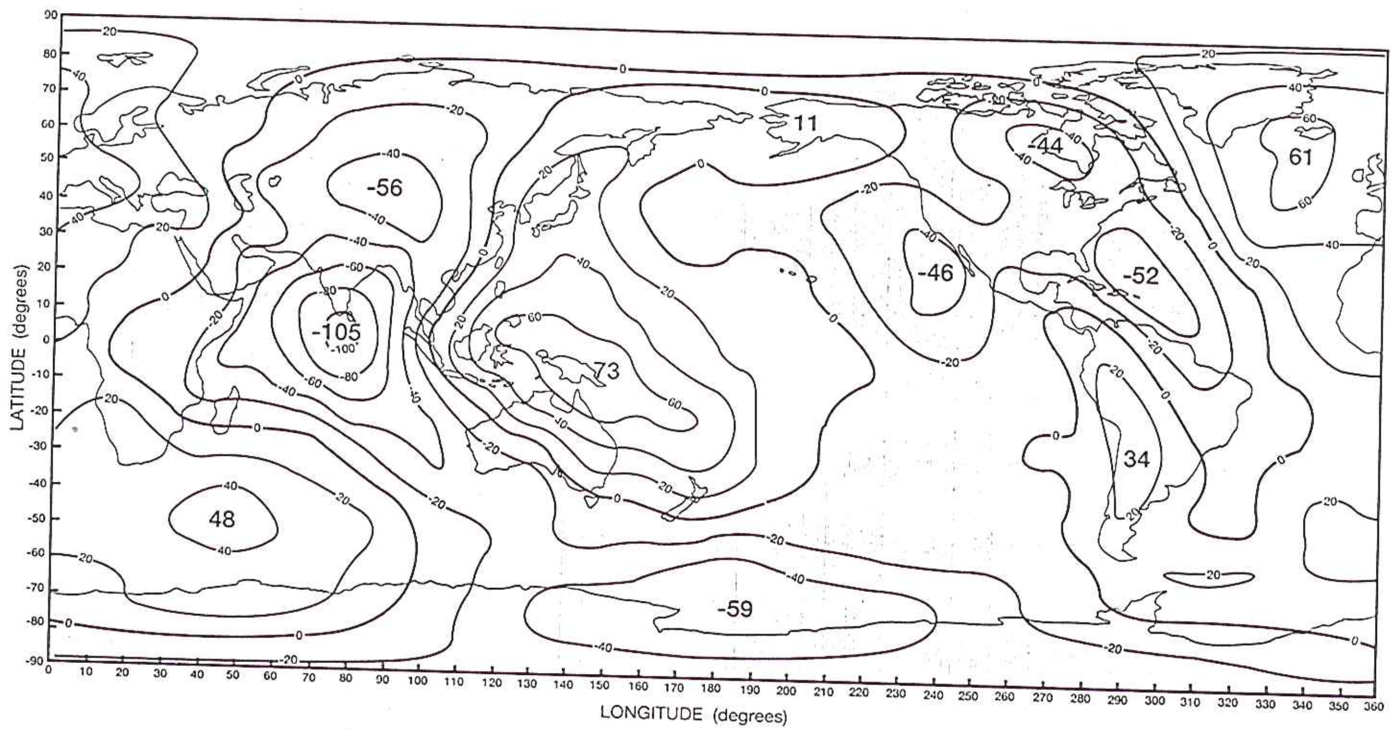


Figure 3.15. Contours of geoid height (in metres), relative to the reference ellipsoid ($f = 1/298.257$) from data by Lerch et al. (1979). This gives the broad scale features of the geoid determined from perturbations of satellite orbits and ignores the finer details that can be added if surface data (or satellite altimetry - Fig. 3.16) are also used.

geoid = mean sea level (in absence of waves & tides)

on land – visualize a network of canals