Heat — a form of energy — measured in joules

$$1\ J = 1\ \frac{kg\ m^2}{s^2}$$

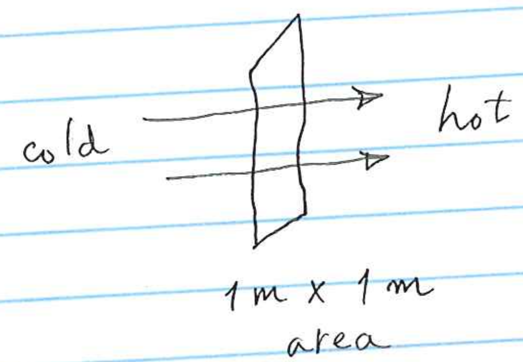
Power — rate at which energy is produced or consumed, etc. — measured in watts

$$1\ W = 1\ \frac{J}{s}$$

Calorie : amount of heat required to raise temperature of 1 gm of H_2O by 1°C.

$$1\ \text{calorie} = 4.184\ J$$

Conductive heat flow within the Earth



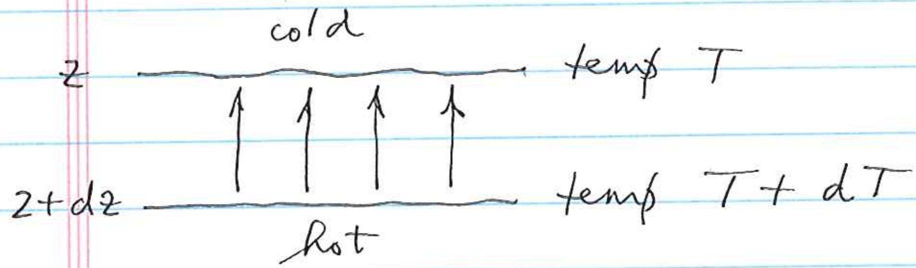
$$\frac{J}{m^2 s} = \frac{W}{m^2}$$

$$1\ \frac{mW}{m^2} = \frac{1}{1000}\ \frac{W}{m^2}$$

Mean surface heat flow : $\bar{q} = 60\ mW/m^2$

$\bar{q} \times$ (surface area of $\oplus$) $= 3 \cdot 10^{13}\ W$

Heat conduction is governed by Fourier's law:



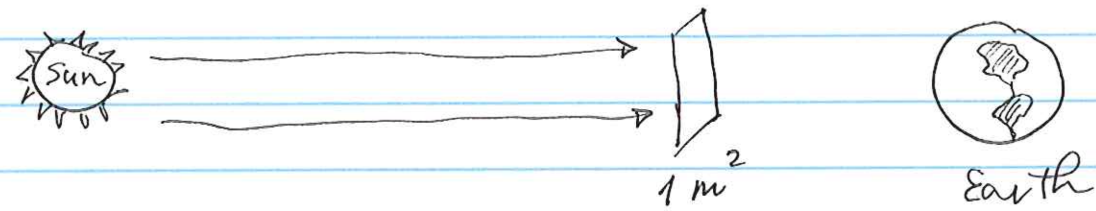
$$q = \kappa \left[\frac{T + dT - T}{z + dz - z} \right] = \kappa \frac{dT}{dz}$$

heat flow = thermal conductivity × temperature gradient

Thermal conductivity κ : $\frac{W/m^2}{°C/m} = \frac{W}{m\,°C}$

material	κ (W/m °C)	
Cu	400	
Al	240	conductors
Fe	80	
dry rock	3	
water	0.6	
soil	~ 0.6	insulators
styrofoam	0.03	

Solar constant: 1370 W/m^2



Averaged over $\oplus$ surface: $\frac{1370}{4} = 340 \frac{W}{m^2}$

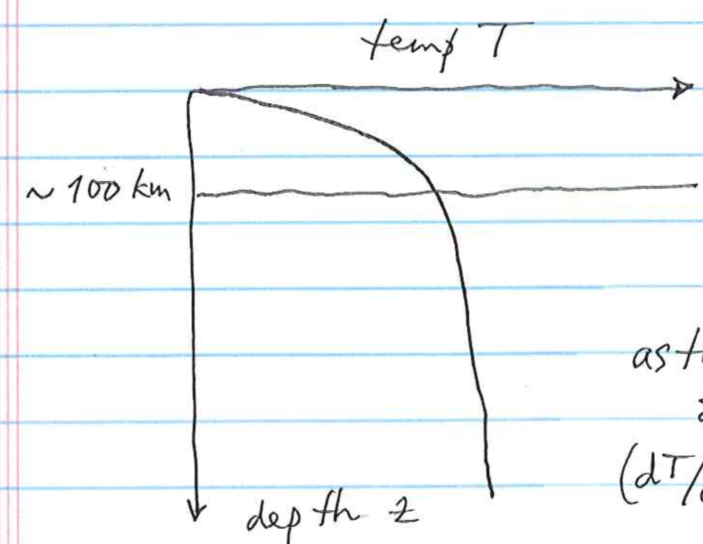
Total solar energy striking the $\oplus$:

$$340 \times 4\pi (\text{radius})^2 = 1.7 \cdot 10^{17} \text{ W}$$

Albedo: 30% reflected back into space

Solar energy incident at $\oplus$ surface
= 4000 × (total conductive heat flow from interior)

Typical geotherm at $\oplus$ surface: 20°C/km



lithosphere: dominated by conduction
$dT/dz \sim 20°C/km$

asthenosphere: dominated by subsolidus convection
$(dT/dz)_{adiabatic} \sim 0.6\,°C/km$

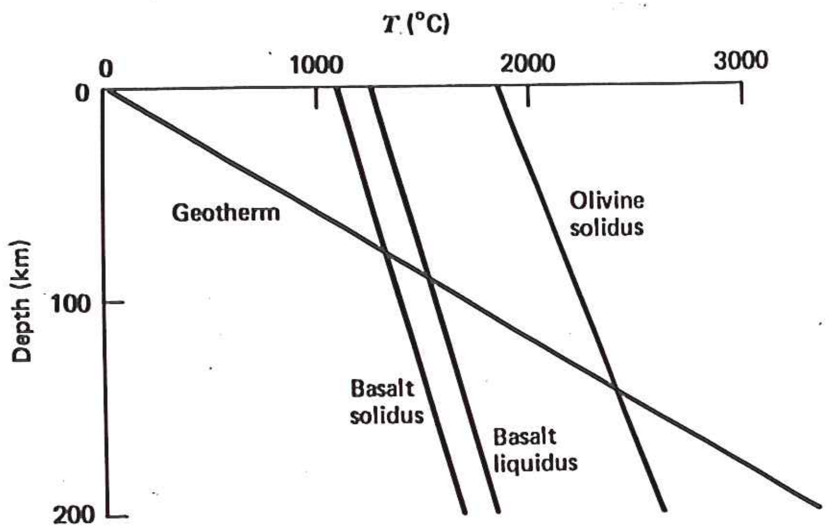
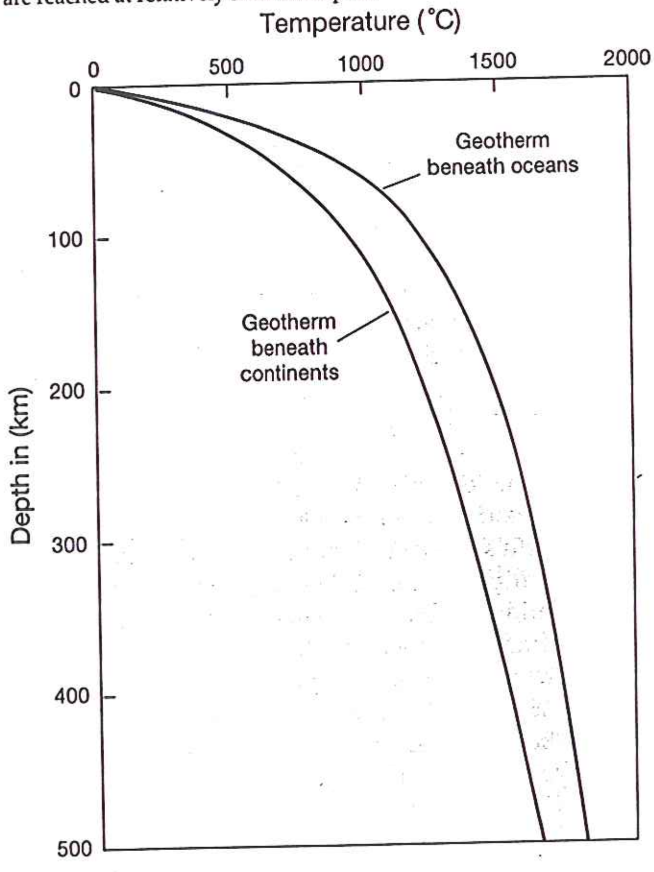
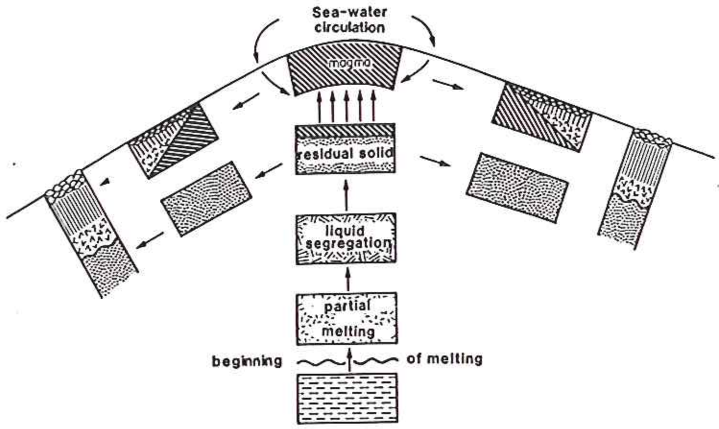


Figure 4-8 Temperature as a function of depth within the earth assuming heat transport is by conduction (conduction geotherm). Also included are the solidus and liquidus of basalt and the solidus of peridotite.

FIGURE 8.12 Estimated average geotherms in continental and oceanic lithosphere. The convecting mantle is nearer to the surface in the ocean basins than under the continents, so high temperatures are reached at relatively shallow depths.





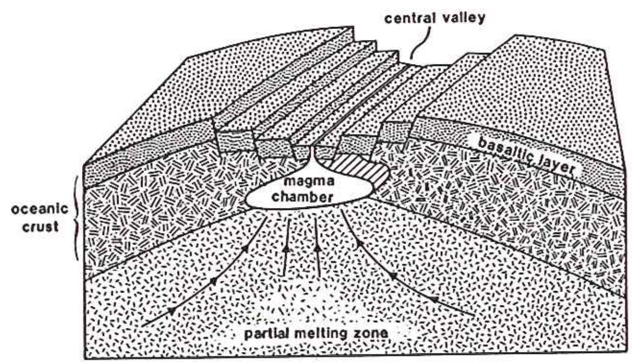


FIGURE 55 The creation of oceanic lithosphere. Material from the mantle rises beneath a ridge axis. During its ascent melting occurs and the liquid thus formed rises faster than surrounding unmelted material. Near the surface the liquid (which forms the crust) cools and interacts with seawater. Then the liquid and residual material spread horizontally away from the spreading axis. On the bottom is a block diagram of a slow-spreading ridge (spreading at, say, a rate on the order of 2 centimeters per year) that shows how the processes of lithosphere creation may occur in a more realistic scenario.

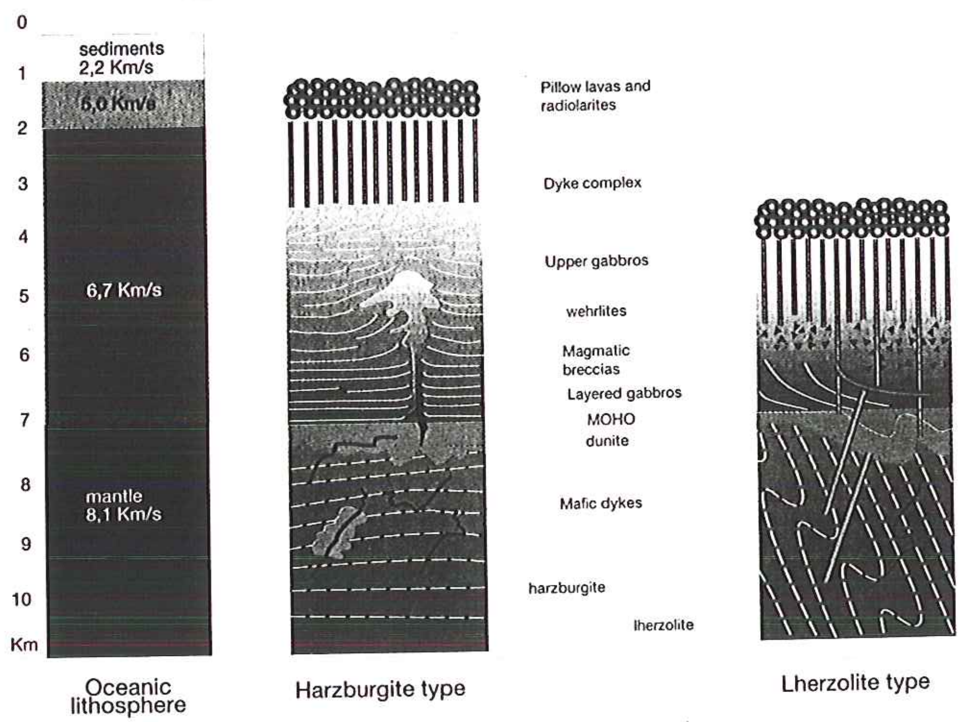


Figure 5.8
Columns comparing the structure of the oceanic crust as defined seismically with the two main types of ophiolites: the harzburgite type as illustrated by the Oman ophiolites and the lherzolite type, by the Trinity ophiolites of California. (After F. Boudier and A. Nicolas 1985, Earth Planet. Sci. Lett., 76, 84–92)

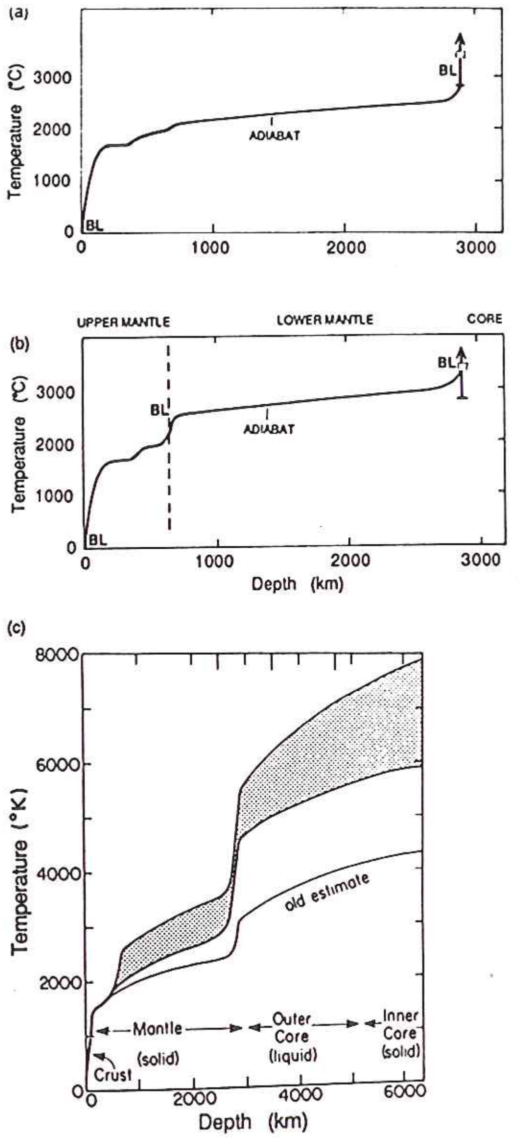


Figure 7.16. Models of temperature profiles in the earth. (a) Mantle adiabat with a thermal boundary layer (BL) at the surface and at the core–mantle boundary. (b) Mantle adiabat with a thermal boundary layer (BL) at both the top and the bottom of the lower mantle. The dashed line indicates a chemical and dynamic boundary between the upper and lower mantle, which are assumed to be separate systems. For (a) and (b) the temperature at the core–mantle boundary is assumed to be in the range 2900–3200°C. (c) An alternative estimate of the temperature in the earth based on high-pressure (over 1000 GPa) and high-temperature (700–6700°C) experiments on iron. The shaded region reflects the uncertainty in the temperature of the core (6600 ± 1000°C at the centre of the earth). The old estimate is similar to profiles shown in (a) and (b) and is typical of temperature profiles proposed prior to 1987. (From Jeanloz and Richter 1979 and Jeanloz 1988.)

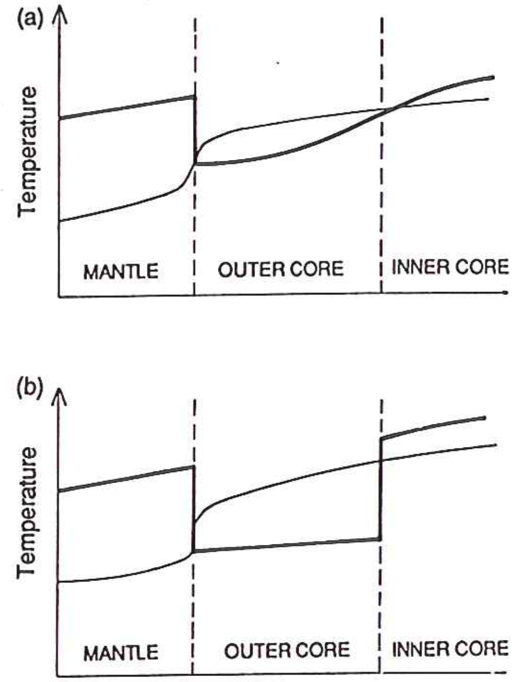


Figure 7.22. Schematic of possible melting temperatures for the mantle and core and the actual temperature profile. Heavy line, melting curve; lighter line, actual temperature profile. (a) Chemically homogeneous core. As the core cools, the inner core grows. (b) The inner and outer core have different chemical compositions and hence different melting temperatures. An outer core composed of an Fe–S or Fe–O alloy would have a much lower melting temperature than a pure iron inner core.

Sources of Earth's internal heat:

(1) kinetic energy of planitesimals during accretion

(2) differentiation of iron core — gravitational potential energy

(3) continuing solidification of inner core

(4) radioactive decay of K, U, Th

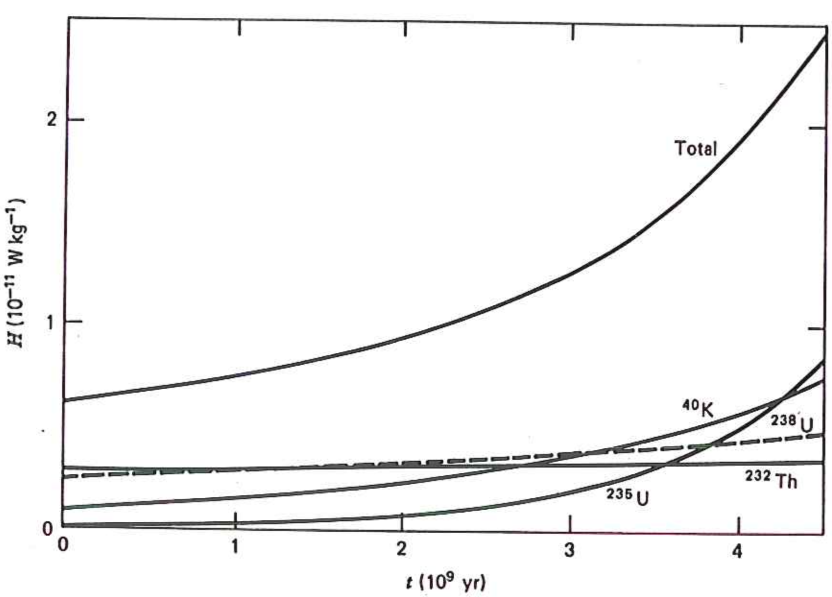


Figure 4-4 Mean mantle heat production rates due to the decay of the radioactive isotopes ^{238}U, ^{235}U, ^{232}Th, and ^{40}K as functions of time measured back from the present.

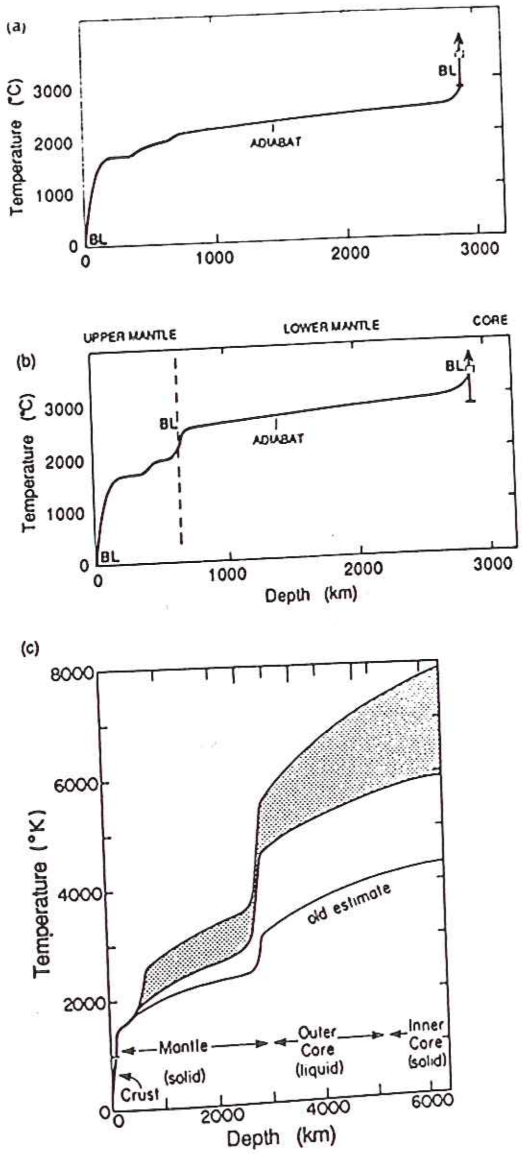
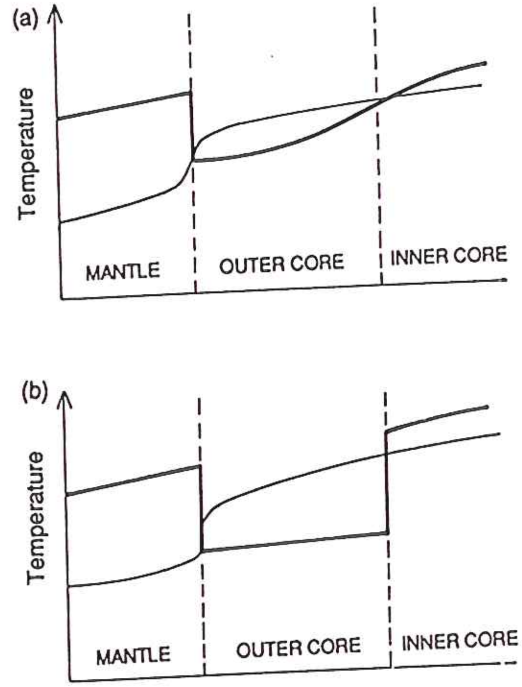


Figure 7.16. Models of temperature profiles in the earth. (a) Mantle adiabat with a thermal boundary layer (BL) at the surface and at the core–mantle boundary. (b) Mantle adiabat with a thermal boundary layer (BL) at both the top and the bottom of the lower mantle. The dashed line indicates a chemical and dynamic boundary between the upper and lower mantle, which are assumed to be separate systems. For (a) and (b) the temperature at the core–mantle boundary is assumed to be in the range 2900–3200°C. (c) An alternative estimate of the temperature in the earth based on high-pressure (over 1000 GPa) and high-temperature (700–6700°C) experiments on iron. The shaded region reflects the uncertainty in the temperature of the core (6600 ± 1000°C at the centre of the earth). The old estimate is similar to profiles shown in (a) and (b) and is typical of temperature profiles proposed prior to 1987. (From Jeanloz and Richter 1979 and Jeanloz 1988.)



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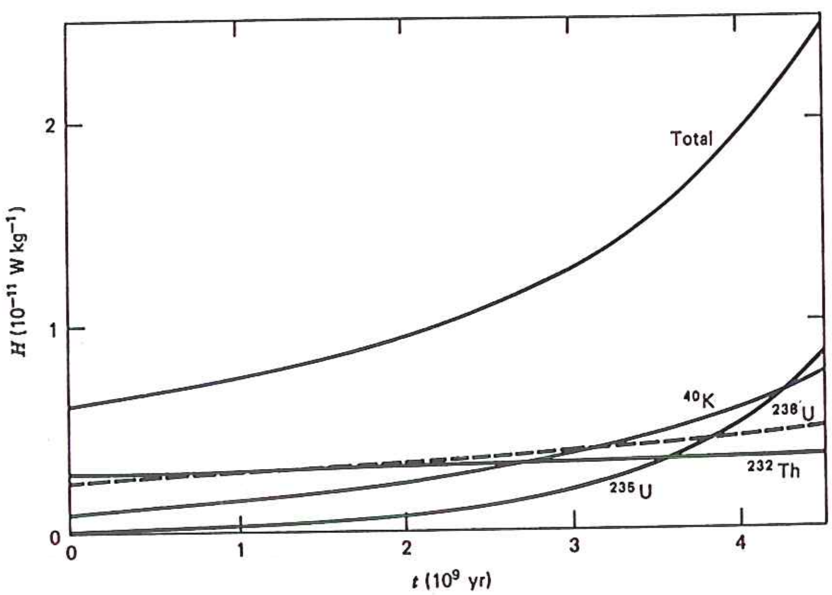
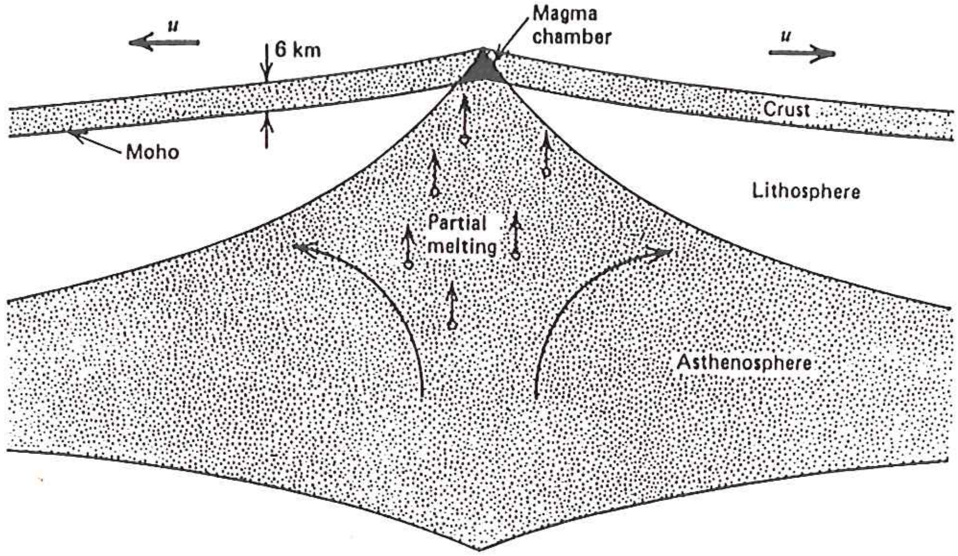
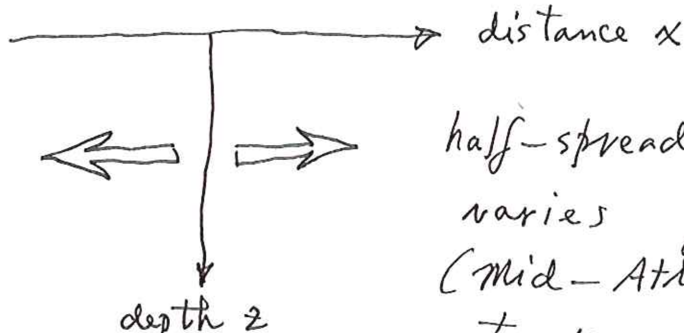


Figure 4-4 Mean mantle heat production rates due to the decay of the radioactive isotopes ^{238}U, ^{235}U, ^{232}Th, and ^{40}K as functions of time measured back from the present.



Idealized model of oceanic lithosphere

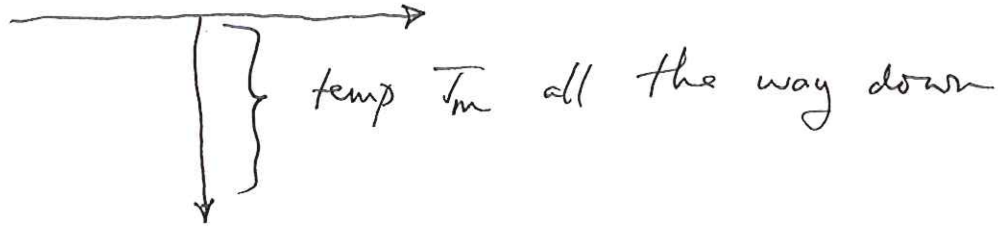


half-spreading rate u varies from 1–2 cm/yr (Mid-Atlantic Ridge) to 10 cm/yr (East Pacific Rise)

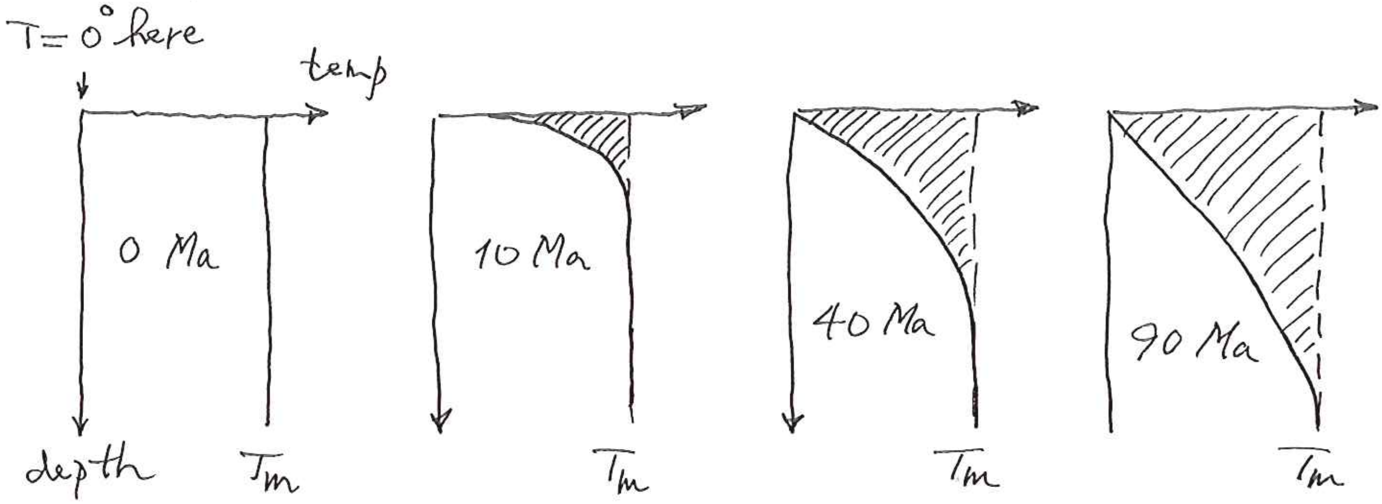
Age of oceanic crust at a distance x from ridge axis:

$$t = \frac{x}{u}$$

The upwelling material just beneath the ridge axis is very hot. For simplicity, say it is at a contant temperature



As the lithosphere spreads it cools off due to thermal conduction out the top. Snapshots of the temperature vs. depth at different times t look like this:



The cooling moves down with time.

The temperature is given explicitly by

$$T(z,t) = \frac{2}{\sqrt{\pi}} T_m \int_0^{z/\sqrt{4kt}} e^{-u^2}\, du$$

Governed by the parameter $k = \frac{K}{\rho c}$
thermal diffusivity

Here c is the specific heat — the heat required to raise the temp T by a fixed amount:

$$c(H_2O) = 1 \frac{\text{calorie}}{\text{gm } ^\circ C} = 4180 \frac{J}{kg\ ^\circ C}$$

$$c(\text{peridotite}) = 1170 \frac{J}{kg\ ^\circ C}$$

density $\rho(\text{peridotite}) = 3300\ kg/m^3$

units of thermal diffusivity:

$$k = \frac{K}{\rho c} : \frac{J/s}{m\,^\circ C} \times \frac{m^3}{kg} \times \frac{kg\,^\circ C}{J} = \frac{m^2}{s}$$

material	k (cm^2/s)
Cu	1.2
Al	1.0
Fe	0.2
H_2O	0.001
mantle rock	0.008
styrofoam	0.004

At a given time t, what is the depth D to which "significant" cooling has taken place? Let's define "significant" to be $33\frac{1}{3}\%$ cooling, i.e.

$$T = \frac{2}{3} T_m = 900^\circ C$$

Then the "law of cooling" by conduction is:

$$\boxed{D = 1.4\sqrt{kt}}$$

← note that this applies to heating (of skillets, coffee mugs, etc.) also

Using k for mantle peridotite

$$\boxed{D\,(km) = 7\sqrt{t\,(Ma)}}$$

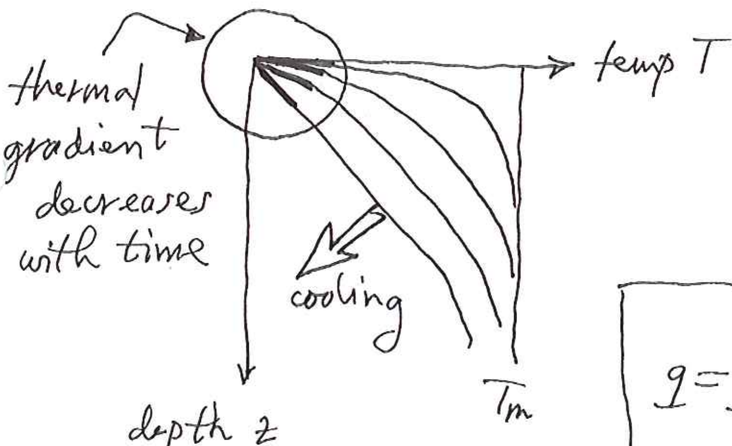
The important thing is the dependence upon $\sqrt{kt}$. Other definitions of "significant" give other numbers in front, e.g.

$$D\,(10\%\ \text{cooling}) = 2.32\sqrt{kt}$$

Thickness of oceanic lithosphere ~ $\sqrt{\text{age}}$

t (age in Ma)	D (km)
0	0
1	7
10	22
50	49
100	70
oldest seafloor → 200	100

The geothermal gradient at the seafloor and thus the surface heat flow decrease with time due to the cooling.



$$q = K\left(\frac{dT}{dz}\right)_{\text{surface } z=0}$$

$$\boxed{q = \rho c T_m \sqrt{\frac{k}{\pi t}} = T_m \sqrt{\frac{\rho c K}{\pi t}}}$$

The surface heat flow varies as $\frac{1}{\sqrt{\text{age}}}$.

$$\boxed{q\,(\text{mW/m}^2) = \frac{470}{\sqrt{t\,(\text{Ma})}}}$$

Finally, our lithospheric cooling model leads to a very simple prediction about <u>seafloor bathymetry</u>.

As the lithosphere cools it <u>contracts</u>.

Governed by coefficient of thermal expansion

α = fractional increase in volume $\delta V/V$ (or decrease in density $\delta\rho/\rho$) due to a 1°C rise in temperature T

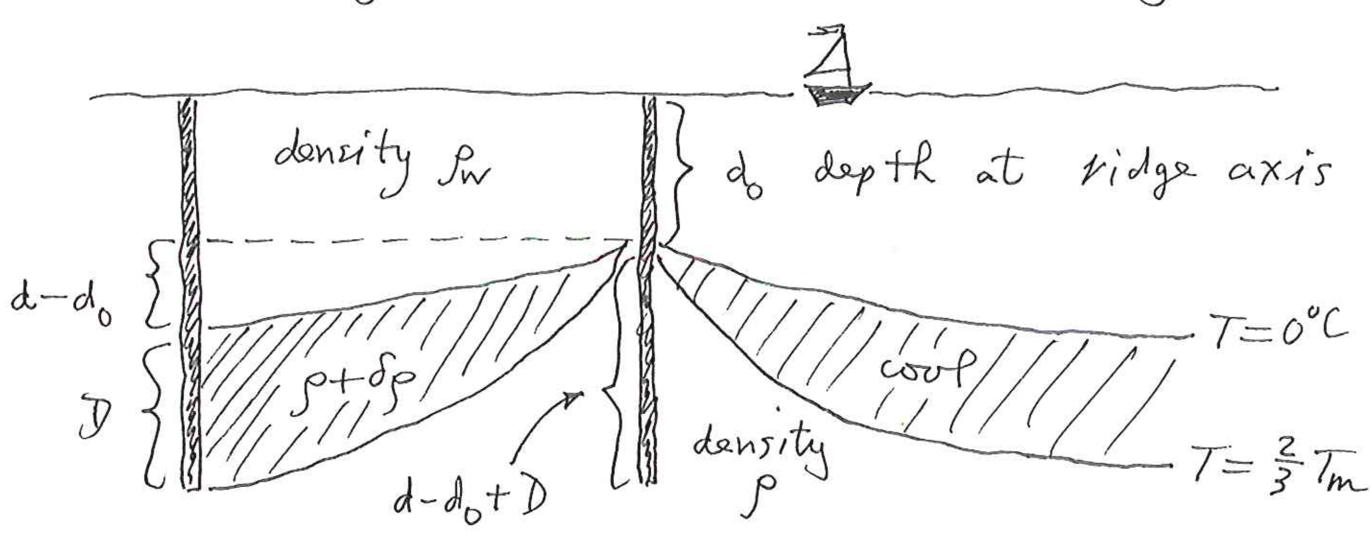
$$\alpha\,(\text{peridotite}) = 3\cdot 10^{-5}\ /°\text{C}$$

Cooling from 1350°C to 0°C <u>increases</u> the density of peridotite by

$$\frac{\delta\rho}{\rho} = (3\cdot 10^{-5})(1350) = 0.04 \quad (4\,\%)$$

Density goes from $3300\ \text{kg/m}^3 \rightarrow 3430\ \text{kg/m}^3$
This slightly denser peridotite sinks into the lighter asthenosphere

To find out how much it sinks we apply Archimedes principle or — as it is known in geology — the principle of <u>isostasy</u>.



Equal mass in equal-depth columns

$$\underbrace{\rho_w d_0 + \rho(d - d_0 + D)}_{\text{ridge}} = \underbrace{\rho_w d + (\rho + \delta\rho) D}_{\text{off-ridge}}$$

Solve for off-ridge depth:

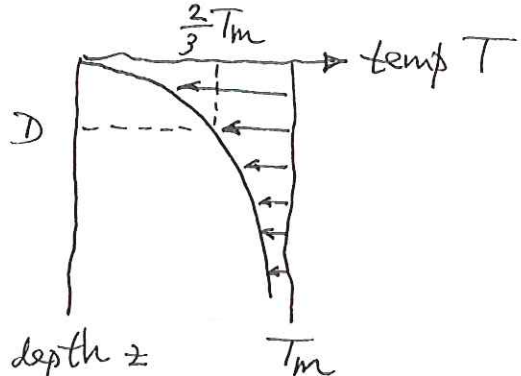
$$\boxed{d = d_0 + \left(\frac{\delta\rho}{\rho - \rho_w}\right) D}$$

$D = 7\sqrt{t}$, $\delta\rho = 130\ kg/m^3$ (4% contraction)

$\rho_w = 1000\ kg/m^3$

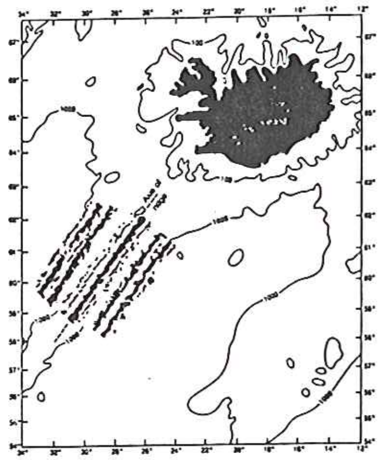
$$\boxed{d(m) = d_0(m) + 350\sqrt{t\,(Ma)}} \leftarrow \star$$

The above derivation assumes that entire thickness D cools from 1350°C to 0°C. Actually, each layer cools by a different amount

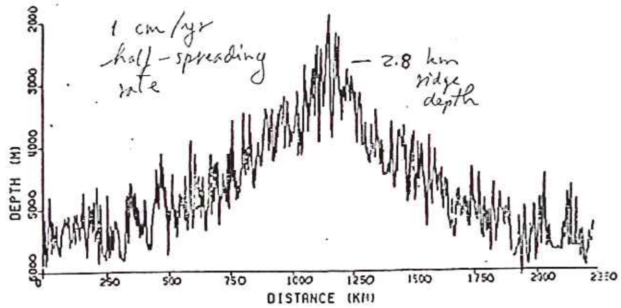


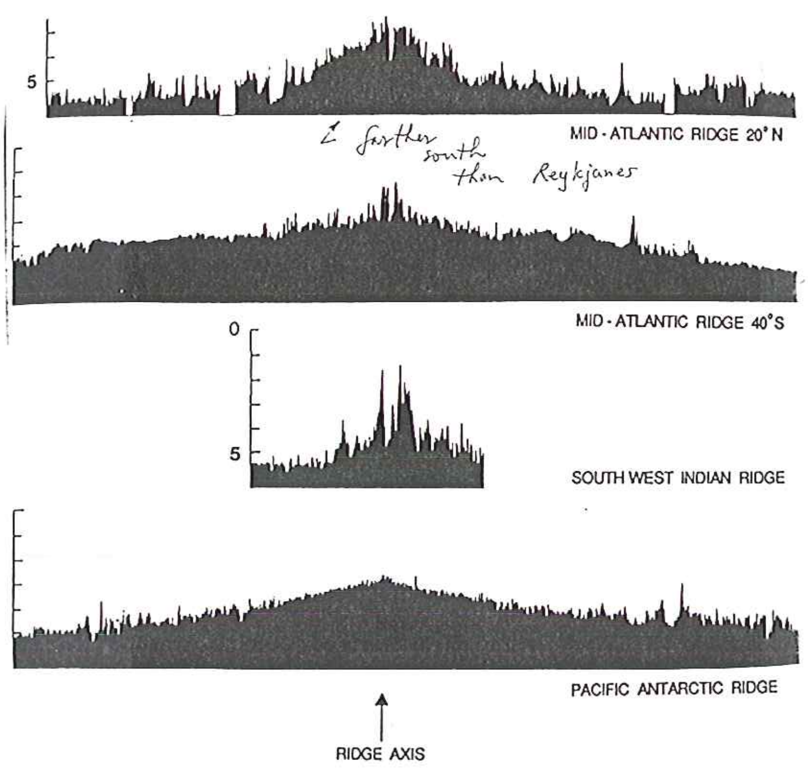
A more careful analysis, which accounts for all the layers — by integration — gives the same result $\star$

$$\boxed{\text{The depth of off-ridge seafloor} \sim \sqrt{\text{age}}}$$



Reykjanes Ridge
South of Iceland





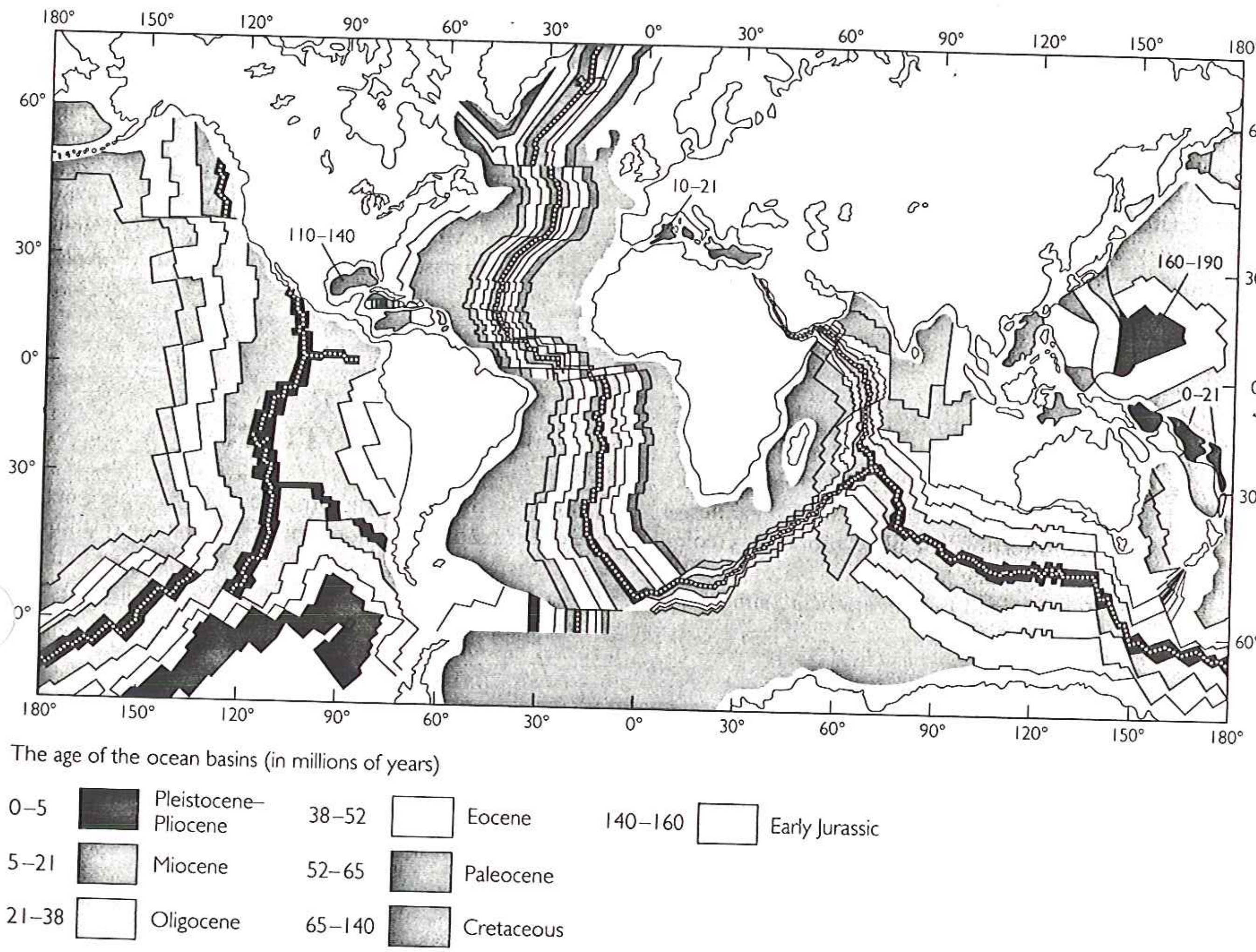


FIGURE 20.11 The worldwide pattern of seafloor spreading is revealed by the isochrons (contours separating the bands of different colors and textures) that give the age of the seafloor in millions of years since its creation at ridges. Mid-ocean ridges, along which new seafloor is extruded, coincide with the youngest seafloor (red). The Atlantic Ocean is symmetrical about the Mid-Atlantic Ridge. Asymmetry of the pattern in the Pacific is caused partly by subduction in the Aleutian Trench south of Alaska, in the Peru-Chile Trench along the west coast of South America, and in many trenches in the western Pacific. (After map prepared by J. Sclater and L. Meinke.)

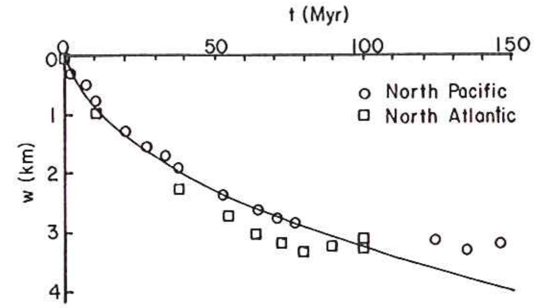


Figure 4-45 Ocean depth relative to the depth of the ridge crest as a function of age of the seafloor. Data from B. Parsons and J. G. Sclater. An analysis of the variation of ocean floor bathymetry with age, *J. Geophys. Res.* **82**, 803–827, 1977.

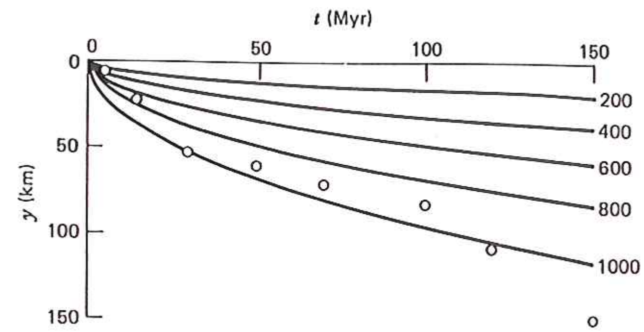
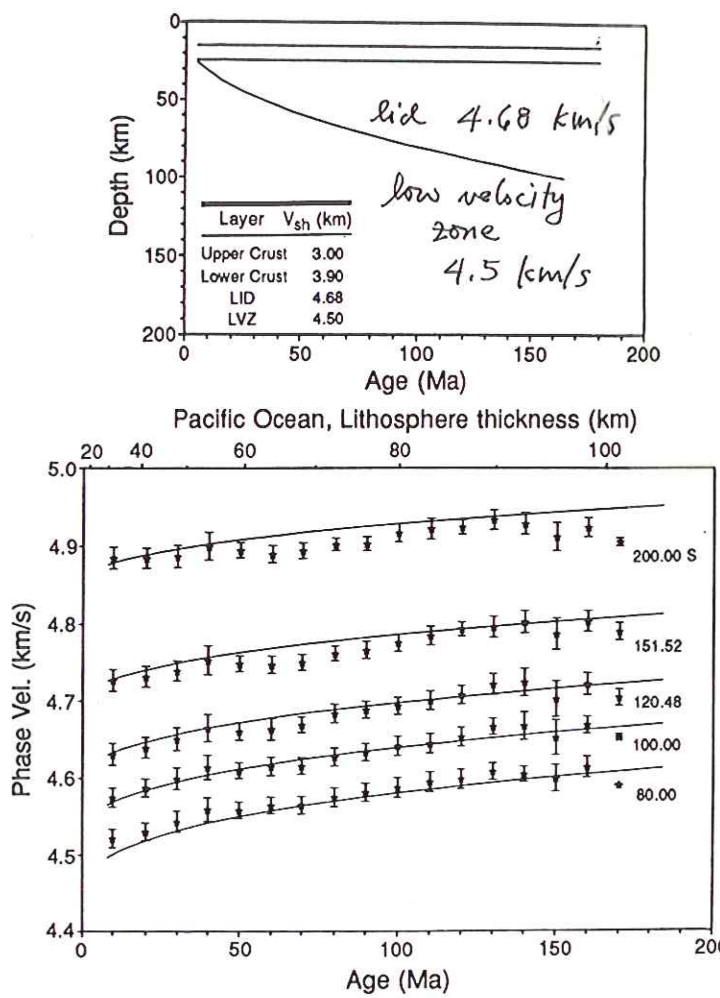


Figure 4-24 The solid lines are isotherms, $T - T_s$ (°K), in the oceanic lithosphere from Equation (4-125). The data points are the thicknesses of the oceanic lithosphere in the Pacific determined from studies of Rayleigh wave dispersion data. (From A. R. Leeds, L. Knopoff, and E. G. Kausel, Variations of upper mantle structure under the Pacific Ocean, *Science*, **186**, 141–143, 1974.)



(Top) A model for lithospheric evolution of shear velocities under the Pacific Ocean. (Bottom) Comparison of the model predictions (solid lines) with observed Love-wave phase velocities with periods from 80 to 200 s plotted as a function of plate age. The observations were obtained by a tomographic inversion of Love waves traversing the Pacific. (From Zhang, 1991.)

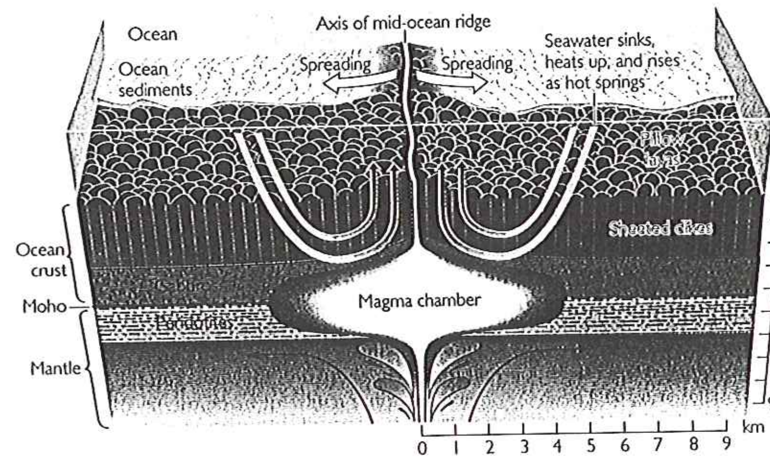


FIGURE 20.15 Oceanic crust forms at an ocean spreading center. (After G. I. Bass, "Ophiolites," *Scientific American*, August 1982, p. 122.)

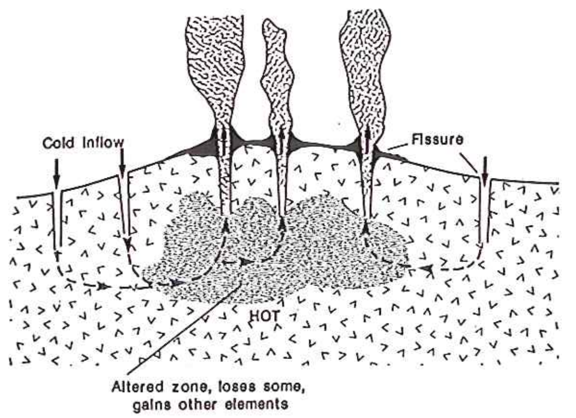


Figure 14.3. The hot crust of the mid-ocean ridges is cooled partly by conduction of heat to the seafloor. Almost as much heat is carried away by cold ocean water that enters fissures and circulates through the hot crust. Once heated, the water returns to the ocean as hotsprings, with exit temperatures that range from a few degrees to 350 °C. While passing through the hot crust the seawater leaves some constituents behind and extracts others, thereby altering the rocks substantially. At the vents, manganese and iron oxides and hydroxides (black) are deposited that may contain valuable amounts of such metals as silver and copper.

not boiling because of high pressure

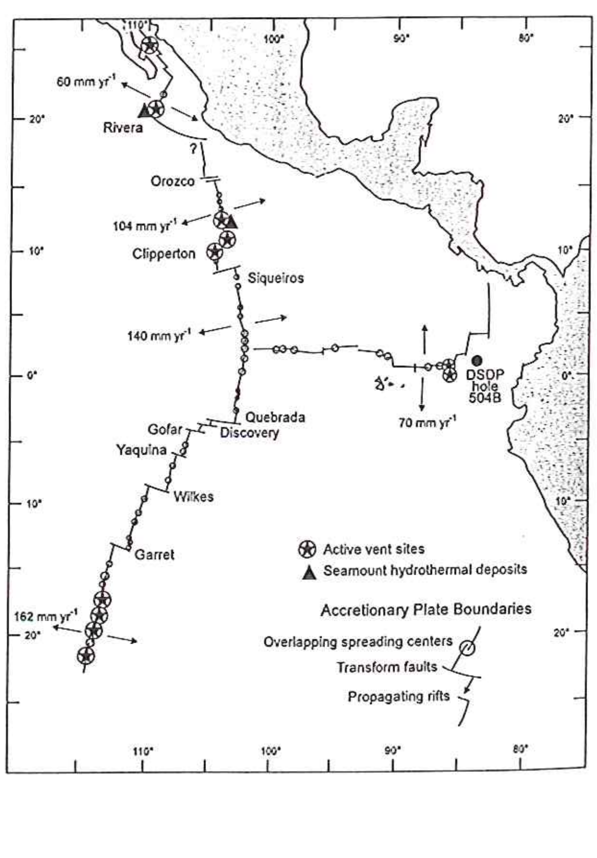


Figure 7.29. Distribution of known sites of active hydrothermal venting along the East Pacific Rise and Galapagos Rift. Solid stars mark sites of active venting; solid triangles mark locations of off-axis seamounts associated with hydrothermal mineral deposits. Solid circle shows location of DSDP Hole 504B. (Haymon 1989)

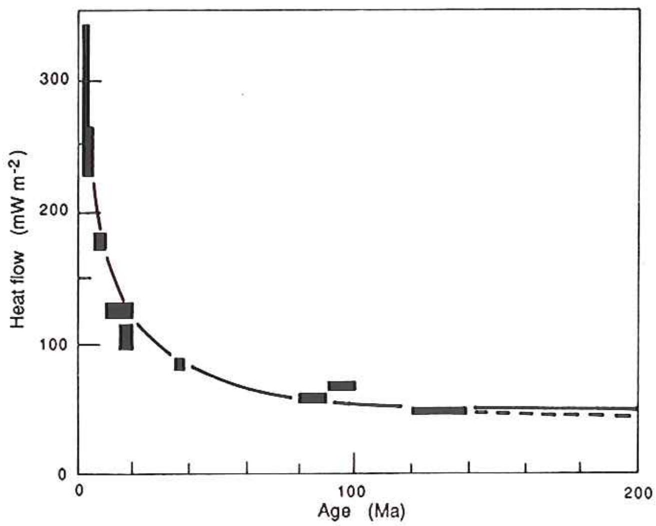


Figure 7.7. Mean heat flow for well-sedimented areas in the North Pacific and North Atlantic plotted against age. Solid curve is the heat flow predicted by the plate model; dashed curve, the heat flow predicted by the boundary layer model. (After Sclater et al. 1980.)

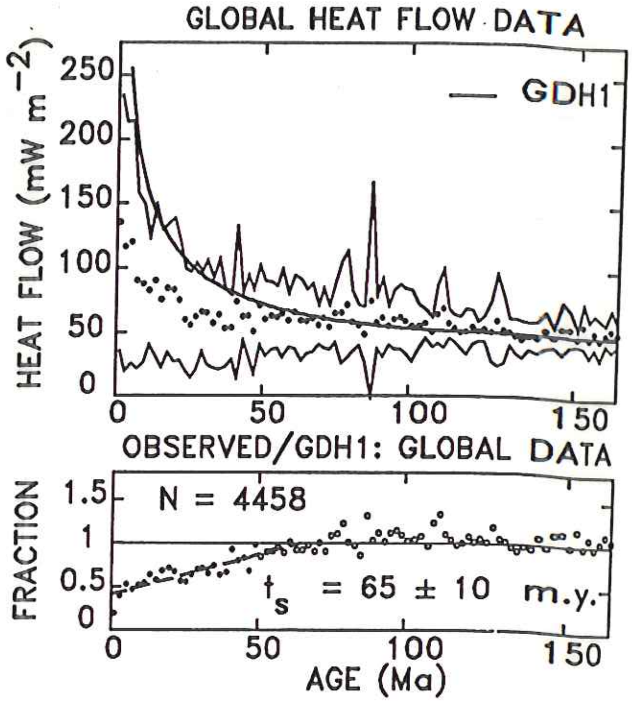


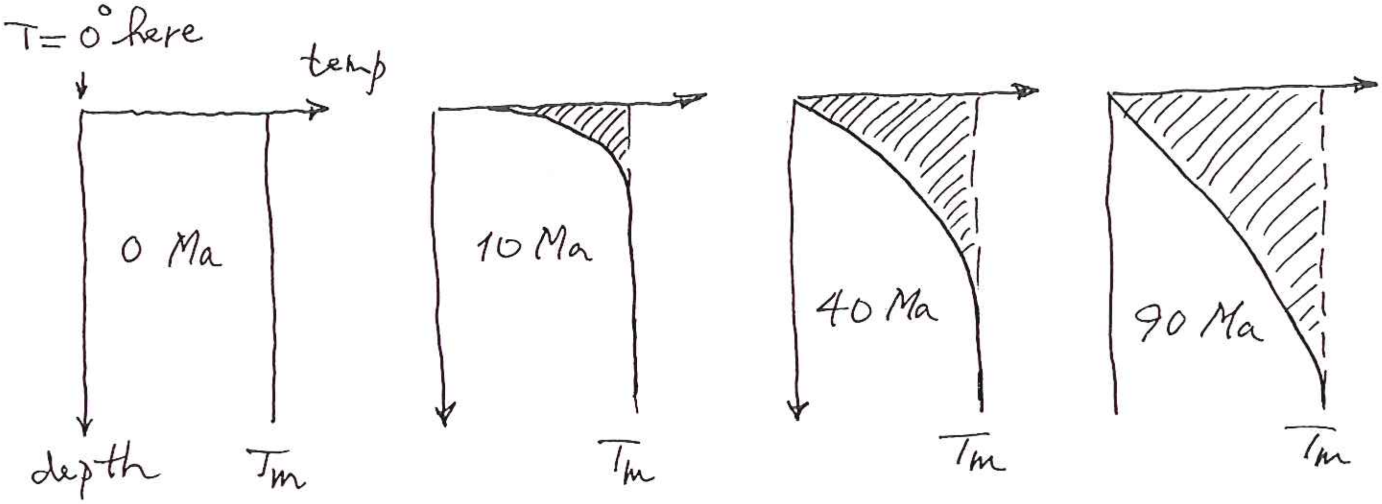
Fig. 2. Observed heat flow versus age for the global data set from the major ocean basins and predictions of the GDH1 model, shown in raw form (*top*) and fraction (*bottom*). Data are averaged in 2-m.y. bins. The discrepancy for ages < 50-70 Ma presumably indicates the fraction of the heat transported by hydrothermal flow. The fractions for ages < 50 Ma (closed circles), which were not used in deriving GDH1, are fit by a least squares line. The sealing age, where the line reaches one, is 65 ± 10 Ma [107].

Table 7.3. *Heat loss and heat flow from the earth*

	Area ($10^6\ km^2$)	Mean heat flow ($10^{-3}\ W\ m^{-2}$)	Heat loss (10^{12} W)
Continents (including volcanoes)	149		8.8
Continental shelves	52		2.8
Total continents and continental shelves	201	57	11.6
Deep oceans	282		27.4
Marginal basins	27		3.0
Conductive contribution		66	20.3
Hydrothermal contribution		33	10.1
Total oceans and basins	309	99	30.4
Worldwide total	510	82	42.0

Note: Estimate of convective heat transport by plates is ~65% of total heat loss; this includes lithospheric creation on oceans and magmatic activity on continents. Estimate of heat loss as a result of radioactive decay in the crust is ~17% of total heat loss. Estimate of the heat loss of the core 10^{12}–10^{13} W; this is a major heat source for the mantle.
Source: Sclater et al. (1980, 1981).

As the lithosphere spreads it cools off due to thermal conduction out the top. Snapshots of the temperature vs. depth at different times t look like this:



The cooling moves down with time.

The temperature is given explicitly by

$$T(z,t) = \frac{2}{\sqrt{\pi}} T_m \int_0^{z/\sqrt{4kt}} e^{-u^2} \, du$$

Governed by the parameter $k = \frac{K}{\rho c}$
thermal diffusivity

Here c is the specific heat – the heat required to raise the temp T by a fixed amount:

$$c(H_2O) = 1 \frac{\text{calorie}}{\text{gm °C}} = 4180 \frac{J}{\text{kg °C}}$$

$$c(\text{peridotite}) = 1170 \frac{J}{\text{kg °C}}$$

$$\text{density } \rho(\text{peridotite}) = 3300 \text{ kg/m}^3$$

units of thermal diffusivity:

$$k = \frac{K}{\rho c} : \frac{J/s}{m\,^\circ C} \times \frac{m^3}{kg} \times \frac{kg\,^\circ C}{J} = \frac{m^2}{s}$$

material	k (cm^2/s)
Cu	1.2
Al	1.0
Fe	0.2
H_2O	0.001
mantle rock	0.008
styrofoam	0.004

At a given time t, what is the depth D to which "significant" cooling has taken place? Let's define "significant" to be 33 1/3 % cooling, i.e.

$$T = \frac{2}{3} T_m = 900^\circ C$$

Then the "law of cooling" by conduction is:

$$\boxed{D = 1.4 \sqrt{kt}}$$

← note that this applies to heating (of skillets, coffee mugs, etc.) also

Using k for mantle peridotite

$$\boxed{D(km) = 7 \sqrt{t\,(Ma)}}$$

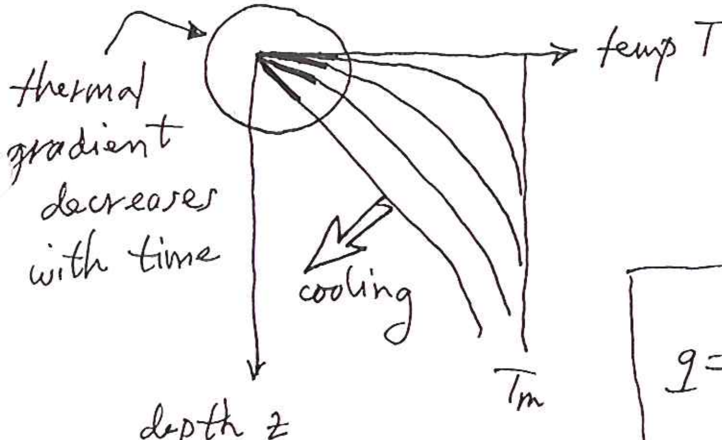
The important thing is the dependence upon $\sqrt{kt}$. Other definitions of "significant" give other numbers in front, e.g.

$$D(10\%\ \text{cooling}) = 2.32 \sqrt{kt}$$

Thickness of oceanic lithosphere $\sim \sqrt{\text{age}}$

t (age in Ma)	D (km)
0	0
1	7
10	22
50	49
100	70
oldest seafloor → 200	100

The geothermal gradient at the seafloor and thus the surface heat flow decrease with time due to the cooling.



$$q = k\left(\frac{dT}{dz}\right)_{\text{surface } z=0}$$

$$\boxed{q = \rho c T_m \sqrt{\frac{\kappa}{\pi t}} = T_m \sqrt{\frac{\rho c k}{\pi t}}}$$

The surface heat flow varies as $\frac{1}{\sqrt{\text{age}}}$.

$$\boxed{q\,(\text{mW}/\text{m}^2) = \frac{470}{\sqrt{t\,(\text{Ma})}}}$$

Finally, our lithospheric cooling model leads to a very simple prediction about seafloor bathymetry.

Equal mass in equal-depth columns

$$\underbrace{\rho_W d_0 + \rho(d - d_0 + D)}_{\text{ridge}} = \underbrace{\rho_W d + (\rho + \delta\rho) D}_{\text{off-ridge}}$$

Solve for off-ridge depth:

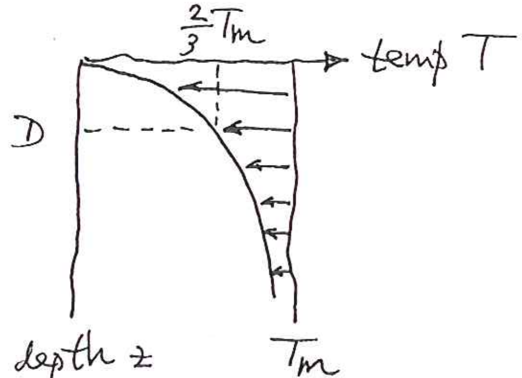
$$\boxed{d = d_0 + \left(\frac{\delta\rho}{\rho - \rho_W}\right) D}$$

$D = 7\sqrt{t}$, $\delta\rho = 130\ \mathrm{kg/m^3}$ (4% contraction)

$\rho_W = 1000\ \mathrm{kg/m^3}$

$$\boxed{d\,(m) = d_0\,(m) + 350\sqrt{t\,(Ma)}} \leftarrow \star$$

The above derivation assumes that entire thickness D cools from 1350°C to 0°C. Actually, each layer cools by a different amount



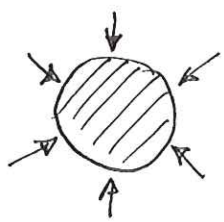
A more careful analysis, which accounts for all the layers — by integration — gives the same result $\star$

$$\boxed{\text{The depth of off-ridge seafloor} \sim \sqrt{\text{age}}}$$

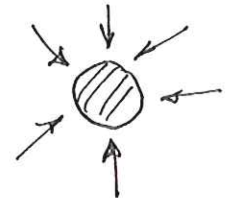
A seismological model of the Earth specifies the radial dependence of three parameters:

1. density $\rho(r)$: kg/m^3

2. incompressibility $\kappa(r)$



sample volume V
under pressure P



increase pressure
$P \to P + dP$
$V \to V + dV$

fractional change in volume

$$dP = -\kappa \left(\frac{dV}{V} \right)$$

makes κ positive

units of κ : $\frac{\text{force}}{\text{area}}$ Pa

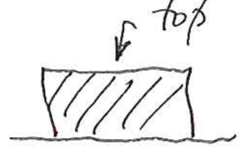
pascal

$1 \text{ Pa} = 1 \text{ N/m}^2$

$1 \text{ atm} = 10^5 \text{ Pa} = \frac{1}{10} \text{ MPa}$

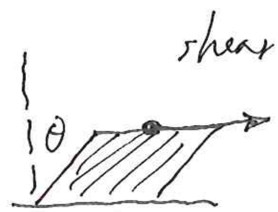
3. rigidity $\mu(r)$

top area A



glue block to table

shear force F

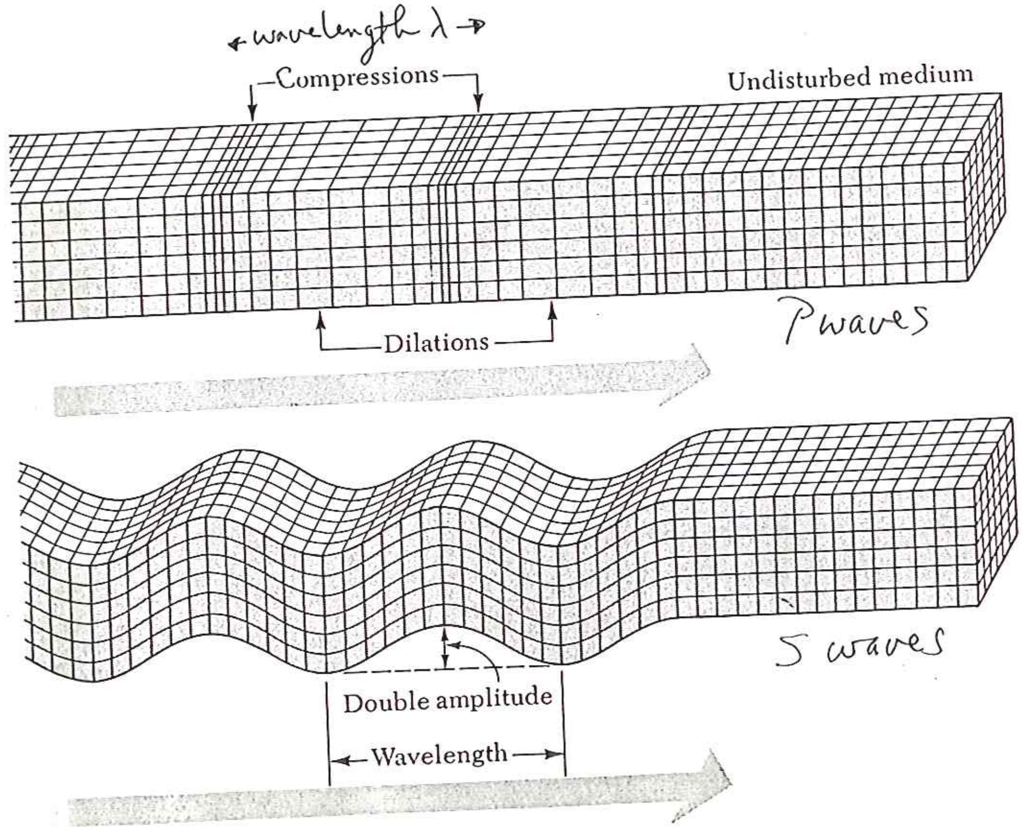


$$\frac{F}{A} = 2\mu\theta$$

shear angle

material	ρ (kg/m^3)	κ	μ
air	1.2	0.14 MPa	0
water	1000	2 GPa	0
granite	2700	55 GPa	20 GPa
peridotite	3300	130 GPa	80 GPa

Two types of seismic waves can propagate in a solid elastic material



P: primary or compressional waves

speed: $\alpha = \sqrt{\dfrac{\kappa + \frac{4}{3}\mu}{\rho}}$

S: secondary or shear waves

speed $\beta = \sqrt{\mu/\rho}$

material	α (km/s)	β (km/s)
air	0.34	0
water	1.5	0
granite	6.0	3.8
peridotite	8.0	4.6

Typically in a solid $\alpha \approx \sqrt{3}\,\beta$ or $\kappa \approx \frac{5}{3}\mu$

The variation in time t and space s of particle displacement in a P or S wave is

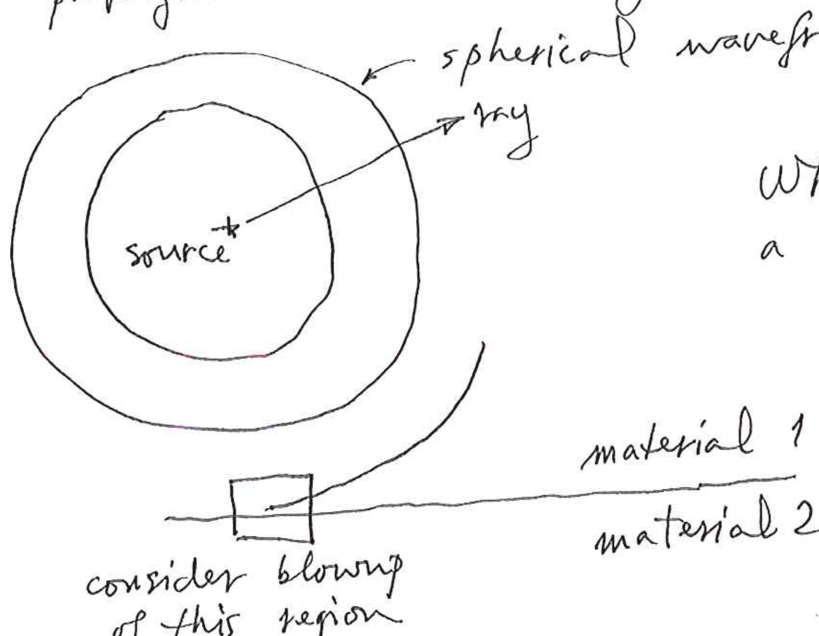
$$A \cos 2\pi \left(\frac{t}{T} - \frac{s}{\lambda} \right)$$

T = period , λ = wavelength

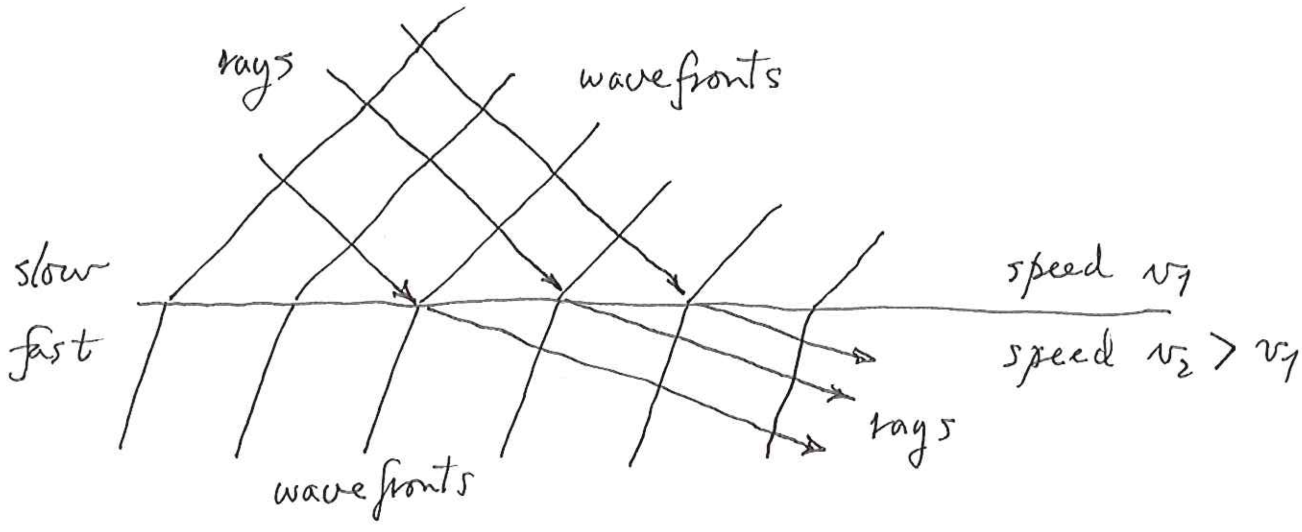
s = distance measured along a seismic ray

Speed v (either α or β) $= \frac{\lambda}{T}$

The waves emitted by a point source in a <u>homogeneous</u> medium (constant ρ, α, β) propagate outward along straight rays

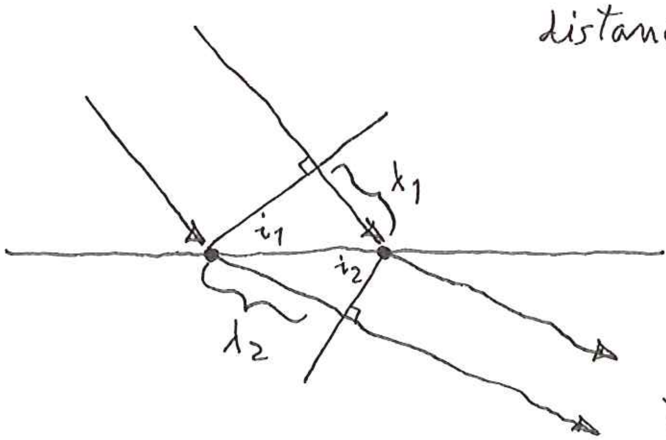


What happens when a wave encounters a boundary between two materials?



A wave is refracted <u>upward</u> upon entering a faster medium

distance $= \dfrac{\lambda_1}{\sin i_1} = \dfrac{\lambda_2}{\sin i_2}$



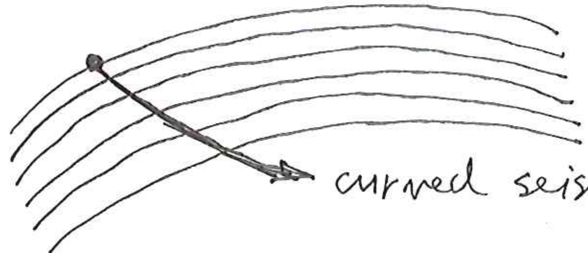
$$\frac{\sin i_1}{v_1 T} = \frac{\sin i_2}{v_2 T}$$

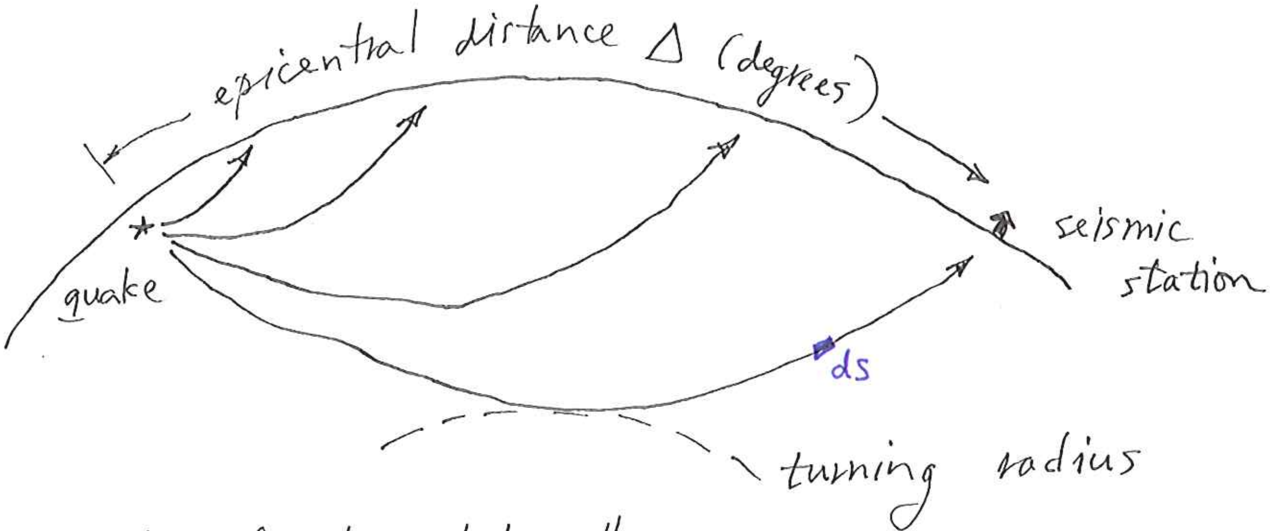
same period

$$\boxed{\text{Snell's law of refraction or reflection} \qquad \frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2}}$$

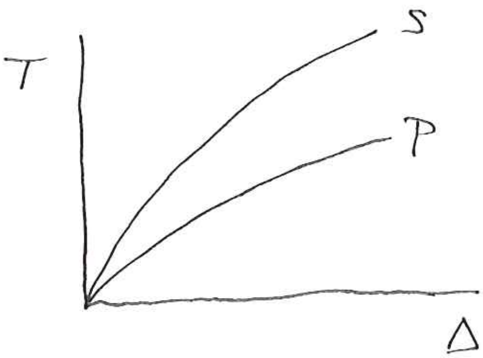
The P and S velocity variation within the $\oplus$ increases gradually with depth

Can regard this as limit of many thin layers





Conventional to plot the travel time of a seismic wave versus epicentral distance

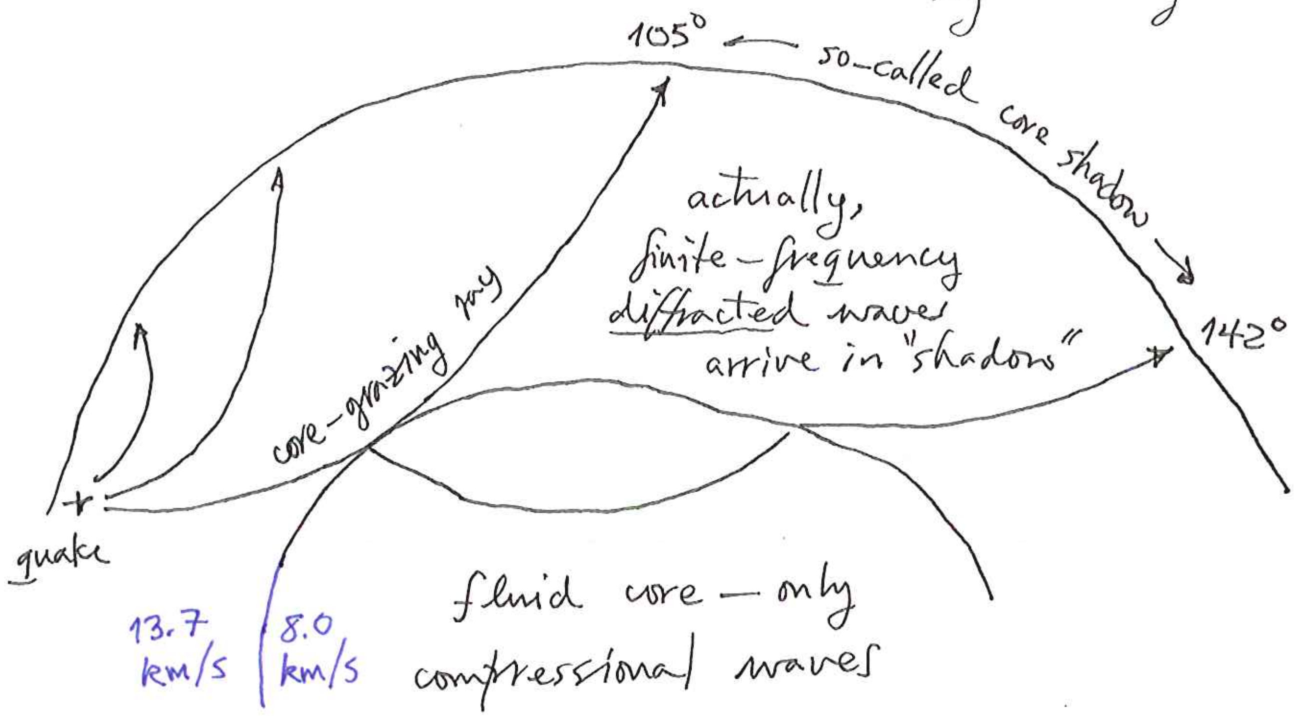


In any little layer

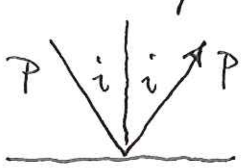
$$dT = \frac{ds}{v} \leftarrow \text{distance travelled}$$

Total travel time

$$T = \int_{\text{ray}} dT = \int_{\text{ray}} \frac{ds}{v}$$

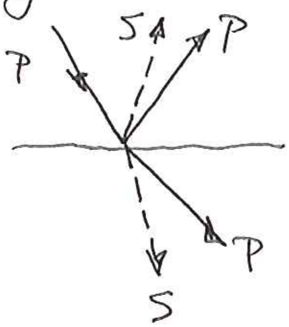


Snell's law governs reflections also:



angle of incidence = angle of reflection

Incoming P also generates S and vice-versa



This leads to a myriad of wave arrivals after an earthquake

Nomenclature for the zoo of seismic arrivals

P : P wave in mantle
S : S wave in mantle
K : P wave in fluid outer core
I : P wave in solid inner core
c : core-mantle boundary reflection
i : inner-core boundary reflection