GEO 319/PHY 319
Introduction to Geophysics

Professor: F. A. Dahlen

Description/Objectives: Properties of Earth's gravitational and magnetic fields, tides, the rotation of Earth. Seismicity and the mechanism of earthquakes. Seismic body waves, surface waves, free oscillations. Structure and properties of Earth's interior. Heat flow and thermal evolution.

Sample Reading List:
D. Gubbins, Seismology and Plate Tectonics

Reading/Writing Assignments: One term paper due at end of semester.

Requirements/Grading:
Midterm Exam 20%
Take Home Final Exam 30%
Paper(s) 25%
Lab Reports 10%
Problem Sets 15%

Prerequisites: MAT 201 and PHY 104 or permission of instructor.

Other Information: In addition to lectures 3 days/week there will be a series of 3-hour laboratories during the second half of the semester(purpose: working with seismic data).

Schedule:
Lecture: 10:00-10:50 MWF

Geology 319. Introduction to Geophysics

Syllabus

The Earth's gravitational field and the shape of the Earth, the geoid, the theory of the hydrostatic shape, isostasy.

Heat flow, the thermal evolution of the oceanic lithosphere, depth vs. square root of age and the geoid over mid-ocean ridges.

The tidal potential and the tidal response of the Earth and oceans, Love numbers.

Irregularities in the rotation of the Earth, the annual wobble and the Chandler wobble, tidal friction and the evolution of the earth-moon system.

Midterm

Seismology and the physical properties of the Earth's interior, seismic body waves and rays, travel times, seismic velocity distribution within the Earth, Love and Rayleigh surface waves, phase and group velocity measurements, the free oscillations of the Earth, Earth models and the seismological inverse problem, anelasticity and the damping of seismic waves.

Earthquakes, seismicity, the nature of seismic focal mechanism, seismic moments and stress drops, the level of stress in the lithosphere, earthquake prediction, deep focus earthquakes, nuclear test ban verification.

Final

Grading: homework and laboratory exercises 25%, term paper 25%, midterm exam 25%, final exam 25%

GE319. Introduction to Geophysics

References on reserve in SG library

1. Modern introductory textbooks

Bolt, Bruce, Inside the Earth, 1982

Bott, M.H.P., The Interior of the Earth, 1982.

Brown, G. C. and Mussett, The Inaccessible Earth, 1981

Cook, A. H., Physics of the Earth and Planets, 1973

Elder, John, The Bowels of the Earth, 1976

Garland, G. D., Introduction to Geophysics: Mantle, Core and Crust, 1971

Jacobs, J. A., A Textbook on Geonomy, 1974

Jacobs, J. A., Russell, R. D. and Wilson, J. T., Physics and Geology, 2nd Edition, 1974

Officer, C. B., Introduction to Theoretical Geophysics, 1974

Press, F. and Siever, R., Earth, 1974

Stacey, F. D., Physics of the Earth, 1969

Verhoogen, J., Turner, F. J., Weiss, L. E., Wahrhaftig, C. and Fyfe, W. S., The Earth: An Introduction to Physical Geology, 1970

Wyllie, P. J., The Dynamic Earth: Textbook in Geosciences, 1971

2. Modern more specialized treatises

Aki, K. and Richards, P. G., Quantitative Seismology, vols. I and II, 1980

Bullen, K. E., The Earth's Density, 1975

Cathles, L. M., The Viscosity of the Earth's Mantle, 1975

Heiskanen, W. A. and Moritz, H., Physical Geodesy, 1967

Jacobs, J. A., The Earth's Core, 1975

Kasahara, K., Earthquake Mechanics, 1981

Kaula, W. M., An Introduction to Planetary Physics: The Terrestrial Planets, 1968

Lambeck, K., The Earth's Variable Rotation, 1980

Lapwood, E. R. and Usami, T., Free Oscillations of the Earth, 1981

3. Classical treatises and textbooks

Bartels, J. (ed.), Handbuch der Physik, vol. XLVII, Geophysik, 1956

Bullen, K. E., An Introduction to the Theory of Seismology, 3rd Edition, 1963

Coulomb, J. and Jobert, G., The Physical Constitution of the Earth, 1963

Darwin, G. H., The Tides and Kindred Phenomena in the Solar System, 1898, reprinted in 1962.

Gutenburg, B. and Richter, C. F., Seismicity of the Earth, 1949

Jeffreys, H., The Earth, 4th Edition, 1959; 5th Edition, 1970

Munk, W. H. and MacDonald, G. J. F., The Rotation of the Earth: A Geophysical Discussion, 1960

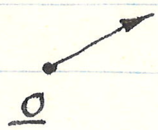
Richter, C. F., Elementary Seismology, 1958

Review of a few mathematical concepts

Familiar concepts:

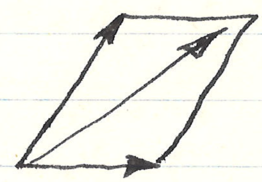
a scalar: $0, -6, \pi$ etc.

a vector: magnitude and direction



written $\underline{x}$ or $\vec{x}$

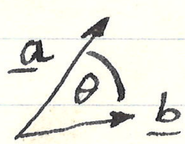
may be added and mult. by scalars



Two common vector products:

dot or inner or scalar product

defn: $\underline{a} \cdot \underline{b} = |\underline{a}|\,|\underline{b}| \cos\theta$



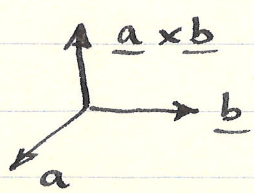
note $\underline{a} \cdot \underline{b} = 0$ iff $\underline{a} \perp \underline{b}$

vector or cross product

defn: $\underline{a} \times \underline{b}$ a vector of

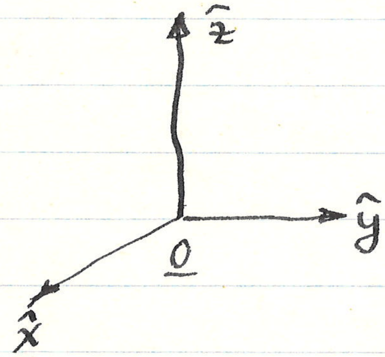
length $|\underline{a}|\,|\underline{b}| \sin\theta$

direction right hand rule



note: defn geometric.
$\underline{a} \times \underline{b}$ not a "pseudovector"

A set of basis vectors
any 3 non-colinear vectors
most useful is a Cartesian basis
or Cartesian axis system



karat $\hat{x}$ denotes
$|\hat{x}| = 1$
unit vectors

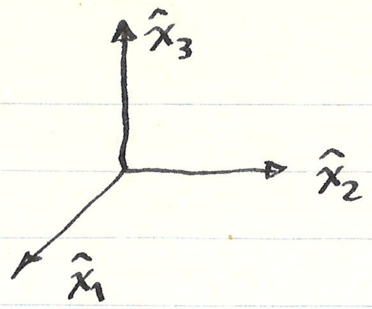
$$\hat{x}\cdot\hat{y} = \hat{x}\cdot\hat{z} = \hat{y}\cdot\hat{z} = 0$$
$$\hat{x}\cdot\hat{x} = |\hat{x}|^2 = 1 \quad , \text{ etc.}$$

Any vector $\underline{a}$ may be expanded
in terms of basis vectors

$$\underline{a} = a_x\,\hat{x} + a_y\,\hat{y} + a_z\,\hat{z}$$

$$\left.\begin{aligned} a_x &= \hat{x}\cdot\underline{a} \\ a_y &= \hat{y}\cdot\underline{a} \\ a_z &= \hat{z}\cdot\underline{a} \end{aligned}\right\} \text{Cartesian components of } \underline{a}$$

A more compact notation is
index notation.



$$\underline{a} = a_1 \hat{x}_1 + a_2 \hat{x}_2 + a_3 \hat{x}_3$$

Einstein or summation convention

single index $\Rightarrow$ 3 equations

sum over a repeated index, e.g.

$$a_i = \hat{x}_i \cdot \underline{a} \Rightarrow a_1 = \hat{x}_1 \cdot \underline{a} \quad \text{etc.}$$

$$\underline{a} = a_i \hat{x}_i \equiv \sum_{i=1}^{3} a_i \hat{x}_i$$

$$\hat{x}_i \cdot \hat{x}_j = \delta_{ij}$$ "Kronecker delta"

9 eqns

Computation of $\underline{a} \cdot \underline{b}$ in terms of components

$$\underline{a} \cdot \underline{b} = a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$|\underline{a}|^2 = a_i a_i = a_1^2 + a_2^2 + a_3^2$$

Cross product, handy rule to remember

$$\underline{a} \times \underline{b} = \det \begin{vmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

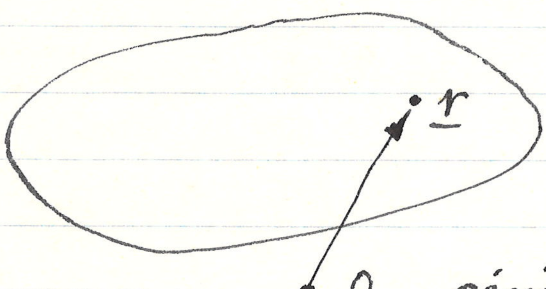
or in index notation

$$(\underline{a} \times \underline{b})_i = \varepsilon_{ijk} \, a_j b_k$$

"alternating symbol"

A key role is played throughout geophysics by the concept of a <u>field</u>.

<u>Scalar field</u> : a rule or fcn which assigns a scalar to every pt. $\underline{r}$ within a certain volume V.

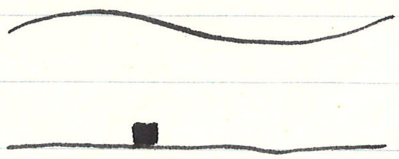


V $\underline{O}$ origin could be $\in V$ or $\notin V$

e.g. $\rho(\underline{r})$ density within the Earth

Could in addition be <u>time-dependent</u>

e.g. $T(\underline{r}, t)$ temp. in this room



$p(\underline{r}, t)$ pressure on ocean bottom varies with ocean tides.

or $p(\underline{r}, t)$ pressure in a sound wave, with which we communicate

<u>Vector field</u> : assigns a <u>vector</u> to pts. $\underline{r} \in V$.

e.g. $\underline{u}(\underline{r}, t)$ velocity of air molecules in this room

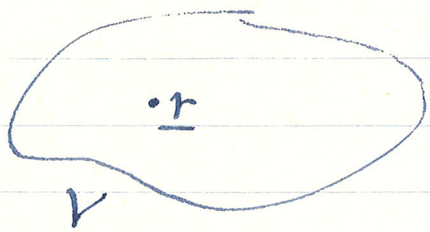
$\underline{B}(\underline{r}, t)$ magnetic field of the Earth.

$\exists$ many other examples of both. Much of geophysics could be said to be the study of $\oplus$'s fields : seek to determine them as fcns of $\underline{r}, t$ and to explain why they are as they are. Some fields change very little with time (e.g. $\oplus$'s $\rho(\underline{r})$), others change a great deal on many different time scales (e.g. $\underline{B}(\underline{r}, t)$ — EM waves due to ionospheric phenomena to reversals on a Myr time scale)

Vector or multidimensional __calculus__.

We shall need only one concept from this subject, the __gradient__ of a scalar field.

Given a scalar field $f(\underline{r})$ in some volume V, the gradient of $f(\underline{r})$ written



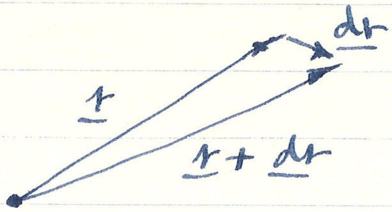
$\underline{\nabla f}(\underline{r})$

called "grad f"

It is a vector field in V.
At every pt. $\underline{r}$ has a magnitude and a direction — this defines it completely.

__direction__ : that in which rate of increase of $f(\underline{r})$ is greatest
__magnitude__ : the rate of increase of $f(\underline{r})$ in the above direction.

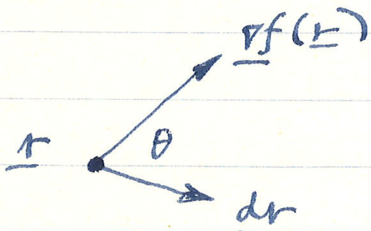
Can find rate of increase in any other direction using dot product.



What is $f(\underline{r} + \underline{dr})$ in terms of $f(\underline{r})$? Answer in terms of gradient.

$$\boxed{f(\underline{r} + \underline{dr}) = f(\underline{r}) + \underline{dr} \cdot \underline{\nabla f}(\underline{r}) + O(|\underline{dr}|^2)}$$

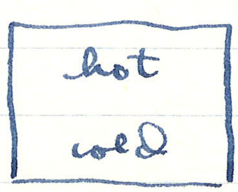
error of second order



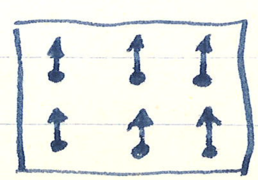
$$\underline{dr} \cdot \underline{\nabla f}(\underline{r}) = |\underline{dr}|\,|\underline{\nabla f}|\cos\theta$$

In every other direction $|\cos\theta| < 1$.

<u>Example</u>: if temp in this room is <u>stratified</u>



then $\underline{\nabla T}$ is a vector field looks like



To compute $\underline{\nabla} f$:

$$f(\underline{r}) = f(x, y, z)$$

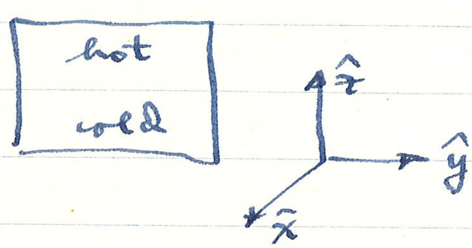
$$\underline{\nabla} f = \hat{x}\,(\partial f/\partial x) + \hat{y}\,(\partial f/\partial y) + \hat{z}\,(\partial f/\partial z)$$

index notation $\quad f(x_1, x_2, x_3)$

$[\underline{\nabla} f]_i = \partial f/\partial x_i$ often written $\partial_i f$.

$$\boxed{\underline{\nabla} f = \hat{x}_i\,\partial_i f}$$

e.g. stratified temperature $\partial T/\partial x = \partial T/\partial y = 0$



$$\underline{\nabla} T = \hat{z}\,\underbrace{(\partial T/\partial z)}_{}$$

this positive if hot near ceiling + cold near floor.

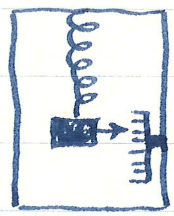
TABLE 2. Normalized Coefficients for Gem 7 and 8

	ℓ	m	Gem 7	Gem 8
C	2	0	-484.1646	-484.1646
C	3	0	0.9588	0.9584
C	4	0	0.5400	0.5400
C	5	0	0.0675	0.0681
C	6	0	-0.1507	-0.1505
C	7	0	0.0941	0.0933
C	8	0	0.0503	0.0501
C	9	0	0.0259	0.0262
C	10	0	0.0542	0.0540
C	11	0	-0.0452	-0.0444
C	12	0	0.0359	0.0377
C	13	0	0.0386	0.0365
C	14	0	-0.0220	-0.0248
C	15	0	0.0078	0.0102
C	16	0	-0.0055	-0.0031
C	17	0	0.0122	0.0108
C	18	0	0.0082	0.0081
C	19	0	0.0021	0.0016
C	20	0	0.0194	0.0181
C	21	0	0.0006	0.0023
C	22	0	-0.0023	0.0008
C	23	0	-0.0226	-0.0241
C	24	0	-0.0006	-0.0026
C	25	0	-0.0026	-0.0026
C	26	0	0.0092	0.0078
C	27	0	0.0039	0.0045
C	28	0	-0.0094	-0.0046
C	29	0	0.0021	0.0031
C	2	1	-0.0031	-0.0001
S	2	1	-0.0009	0.0003
C	3	1	2.0296	2.0317
S	3	1	0.2502	0.2496
C	4	1	-0.5326	-0.5374
S	4	1	-0.4711	-0.4738
C	5	1	-0.0624	-0.0647
S	5	1	-0.0840	-0.0835
C	6	1	-0.0788	-0.0714
S	6	1	0.0215	0.0300
C	7	1	0.2692	0.2716
S	7	1	0.0937	0.0992
C	8	1	0.0295	0.0196
S	8	1	0.0520	0.0421
C	9	1	0.1517	0.1549
S	9	1	0.0272	0.0170
C	10	1	0.0795	0.0901
S	10	1	-0.1259	-0.1201
C	11	1	0.0209	0.0182
S	11	1	-0.0201	-0.0016
C	12	1	-0.0659	-0.0720
S	12	1	-0.0576	-0.0561
C	13	1	-0.0502	-0.0263
S	13	1	0.0543	0.0391
C	14	1	-0.0199	-0.0016
S	14	1	0.0552	0.0372
C	15	1	0.0137	0.0113
S	15	1	0.0058	0.0021
C	16	1	0.0156	0.0136
S	16	1	-0.0039	0.0057

TABLE 2. (continued)

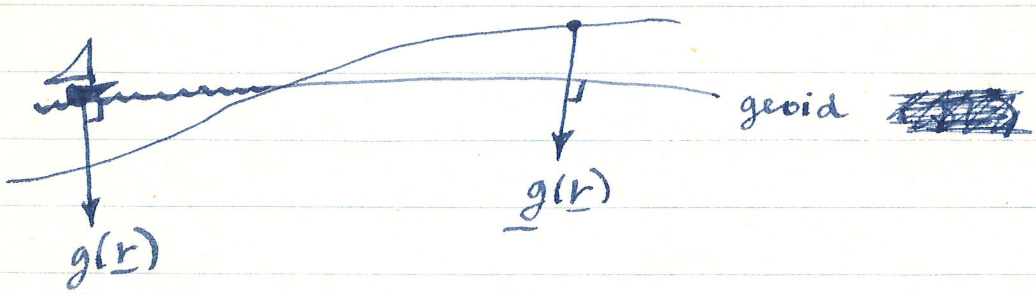
	ℓ	m	Gem 7	Gem 8
C	17	1	-0.0276	-0.0273
S	17	1	-0.0196	-0.0095
C	18	1	-0.0017	-0.0013
S	18	1	-0.0025	0.0024
C	19	1		-0.0398
S	19	1		-0.0115
C	20	1		-0.0158
S	20	1		-0.0360
C	21	1		-0.0052
S	21	1		0.0218
C	22	1		0.0190
S	22	1		0.0096
C	23	1		0.0078
S	23	1		0.0186
C	24	1		-0.0068
S	24	1		-0.0132
C	25	1		0.0189
S	25	1		-0.0032
C	2	2	2.4303	2.4345
S	2	2	-1.3946	-1.3953
C	3	2	0.8972	0.8977
S	3	2	-0.6193	-0.6233
C	4	2	0.3465	0.3473
S	4	2	0.6623	0.6657
C	5	2	0.6622	0.6618
S	5	2	-0.3242	-0.3202
C	6	2	0.0481	0.0524
S	6	2	-0.3612	-0.3640
C	7	2	0.3263	0.3205
S	7	2	0.1002	0.0949
C	8	2	0.0730	0.0736
S	8	2	0.0594	0.0704
C	9	2	0.0213	0.0373
S	9	2	-0.0337	-0.0280
C	10	2	-0.0758	-0.0663
S	10	2	-0.0366	-0.0512
C	11	2	0.0278	0.0140
S	11	2	-0.1044	-0.1119
C	12	2	0.0013	-0.0184
S	12	2	0.0123	0.0293
C	13	2	0.0355	0.0144
S	13	2	-0.0430	-0.0355
C	14	2	-0.0417	-0.0294
S	14	2	0.0085	-0.0027
C	15	2	-0.0053	0.0010
S	15	2	-0.0304	-0.0479
C	16	2	-0.0025	-0.0085
S	16	2	0.0231	0.0324
C	17	2		-0.0356
S	17	2		0.0353
C	18	2		0.0046
S	18	2		0.0167
C	19	2		0.0469
S	19	2		-0.0340
C	20	2		-0.0230
S	20	2		0.0587
C	21	2		0.0044
S	21	2		0.0153

This done with <u>survey</u> <u>gravimeters</u>, basically a mass on a spring



T°C. controlled box – even so prone to drift – can only measure <u>relative</u> gravity variations – must continually close survey circuits to determine rate of instrumental drift.

<u>Gravity</u> $\underline{g}(\underline{r}) = -\nabla U(\underline{r})$ is measured on $\oplus$ surface – with shipboard gravimeters this is on geoid but on land it is not. Shipboard gravimeters must have special filters to null response to ship's pitching, rolling, etc. Early measurements in <u>submarines</u> to get around this problem – used standard land-based instruments.

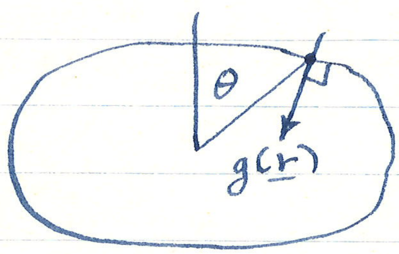


One defines the so-called <u>free-air</u> <u>gravity anomaly</u>.

Geoid variation measured w.r.t. <u>reference ellipsoid</u> defined by GM, Ω, a_e and J_2.

If geoid $\equiv$ reference ellipsoid, $\oplus$ axially symmetric perfect ellipsoid, geoid map $\equiv 0$.

What would a gravimeter then measure on surface of this reference ellipsoid?



gravimeter measures $g(r) \equiv |\underline{g}(r)|$

<u>Answer</u> : clearly depends only on θ.

$$\boxed{\begin{aligned} g_{ref}(\theta) &= g_{eq}\left[1 + \left(\tfrac{5}{2}m - \varepsilon\right)\cos^2\theta + O(\varepsilon^2)\right] \\ g_{eq} &= \frac{GM}{a_e^2}\left[1 + \varepsilon - \tfrac{3}{2}m + O(\varepsilon^2)\right] \end{aligned}}$$

At <u>poles</u> $\theta = 0$ or π

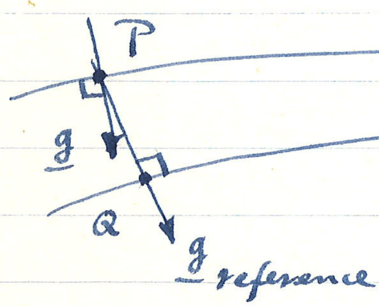
$$g_{poles} = g_{eq}\left[1 - \varepsilon + \tfrac{5}{2}m + O(\varepsilon^2)\right]$$

$g_{poles} > g_{eq}$ for <u>2 reasons</u>: nearer c.o.m. of $\oplus$, not diluted by centrifugal force.

$$\boxed{\frac{g_{poles} - g_{eq}}{g_{eq}} + \varepsilon = \frac{5}{2} m}$$

Clairauts theorem, so-called, gave answer to historical problem of geodesy — can measurements of g on $\oplus$ surface determine shape of $\oplus$?

The above is a reference gravity assoc. with reference ellipsoid.

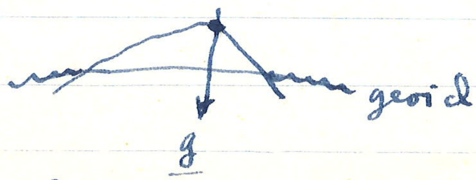


defn: free air gravity anomaly $\equiv g(P)_{\text{on geoid}} - g_{ref}(Q)$

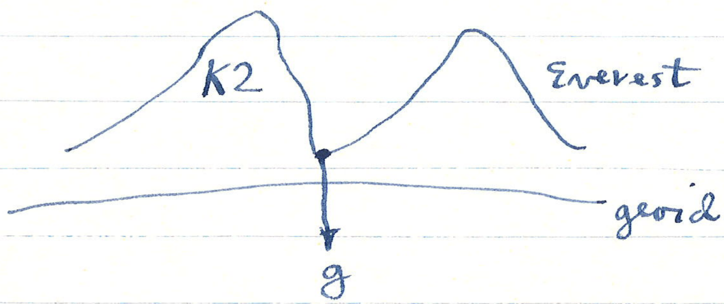
Typically measured in milligals (1 gal = 1 galileo = 1 cm/s^2)

Free air anomalies easy to measure at sea since measurement on geoid.

Not so on land
Must correct measurement to geoid.



Must make so-called free air reduction to correct for elevation above geoid (want g at bottom of canal)
Also in rugged terrain must correct for attraction of topography, e.g. if in valley



JGR vol. 79, no. 35 (Gaposchkin geoid)
Map of geoid w.r.t. best-fitting ellipsoid from Gaposchkin 1974 with $\varepsilon = 1/298.258$.
height of geoid w.r.t. this surface max + 81 m (New Guinea) min −113 m (off S. tip of India).
Very small undulations – geoid very smooth.
Remember in oceans this a map of shape of ocean surface. For comparison GEM8 improved geoid Wagner et al. JGR, 82, #5 (1977).
Also free air gravity map w.r.t. same reference ellipsoid, bumps are ± 40–50 mgal. These maps less smooth since they are essentially a differentiated version of geoid maps since $\underline{g}(\underline{r}) = -\nabla U(\underline{r})$.

mnemonic: Chairman Mao on right conversing with a horse.

The concept of isostasy

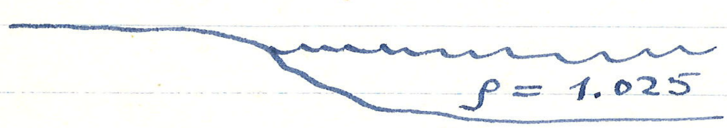
In trying to interpret the "bumps" one of the first things to notice is what they are not due to.

Clearly they are produced lateral density differences w.r.t. the Clairaut ellipsoid.

Most obvious such differences (and the largest and the best known) is the existence of continents and oceans.

Conts. mean ht. above $\underline{\$} \cong$ ref. ellipsoid 840 m.

Oceans mean depth below $\underline{\$}$ 3800 m.



What kinds of "bumps" would this by itself give rise to?

Should be highs over continents with same shape.

This not what is seen at all.
Shape of bumps has <u>nothing</u> to do with conts. and oceans

How big should the cont-ocean bumps be? Maybe they are just too small to see.

One way to answer this. Recall expansion of ext. ⊕ potential

$$V(\underline{r}) = -\frac{GM}{r}\left[1 + \sum_{l=2}^{\infty}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l} P_l^m(\cos\theta)\right.$$
$$\left.(c_l^m \cos m\phi + s_l^m \sin m\phi)\right]$$

Quants. actually measured are c_l^m's and s_l^m's.
Consider total "power" in a degree of wiggliness l.

$$\boxed{\sigma_l^2 = \sum_{m=0}^{l}\left(c_l^{m\,2} + s_l^{m\,2}\right)}$$

Observed power roughly described by

Kaula's "rule of thumb"

$$\boxed{\begin{aligned} \frac{1}{2\ell+1}\,\sigma_\ell^2 &\approx \frac{10^{-10}}{\ell^4} \quad \text{or} \\ \left[\frac{1}{2\ell+1}\,\sigma_\ell^2\right]^{1/2} &\approx \frac{10^{-5}}{\ell^2} \end{aligned}}$$

Plot of this for $\ell \geq 8$ from Gaposchkin (1974).

Other plot normalized w.r.t. the "rule of thumb" using 1971 coefficients.

Easy to compute cont-ocean "bumps" by integration. Plot compares expected and observed power.

Expected power 20 – 100 times greater than observed. Cont-ocean bumps should be 5 – 10 times bigger than observed.

This the key evidence for the phenomenon of <u>isostasy</u>. Continents and other surface topographic features have "roots".

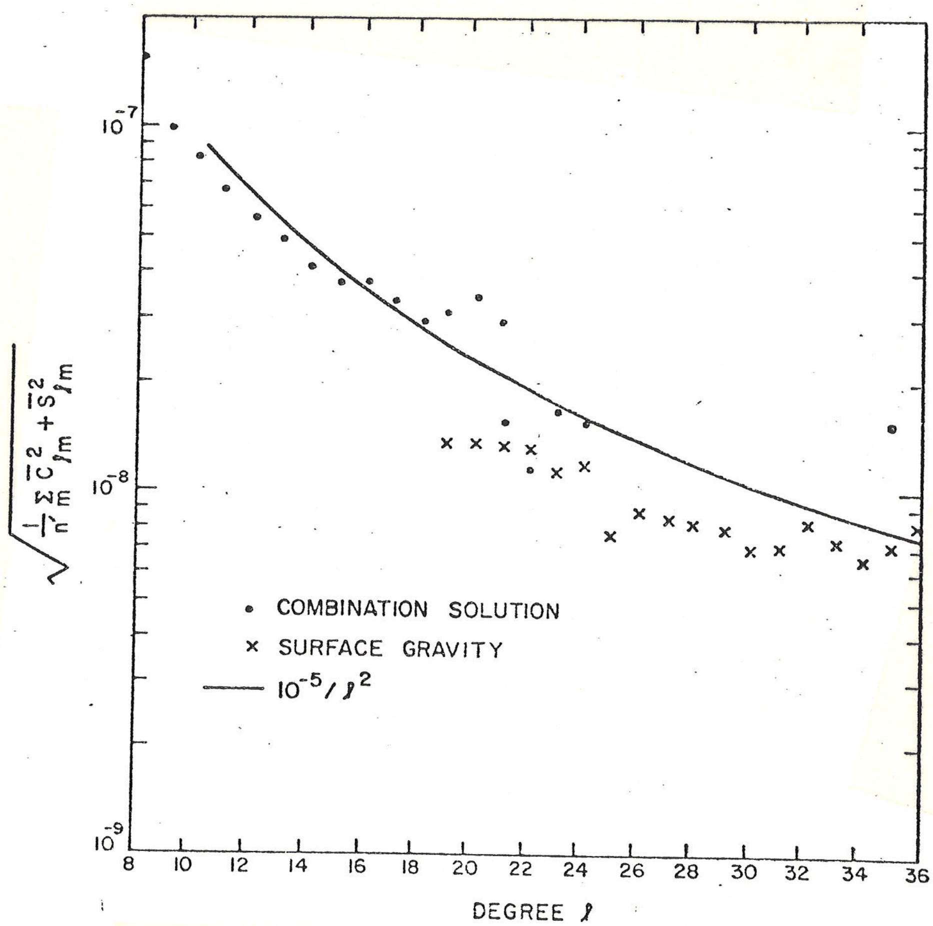
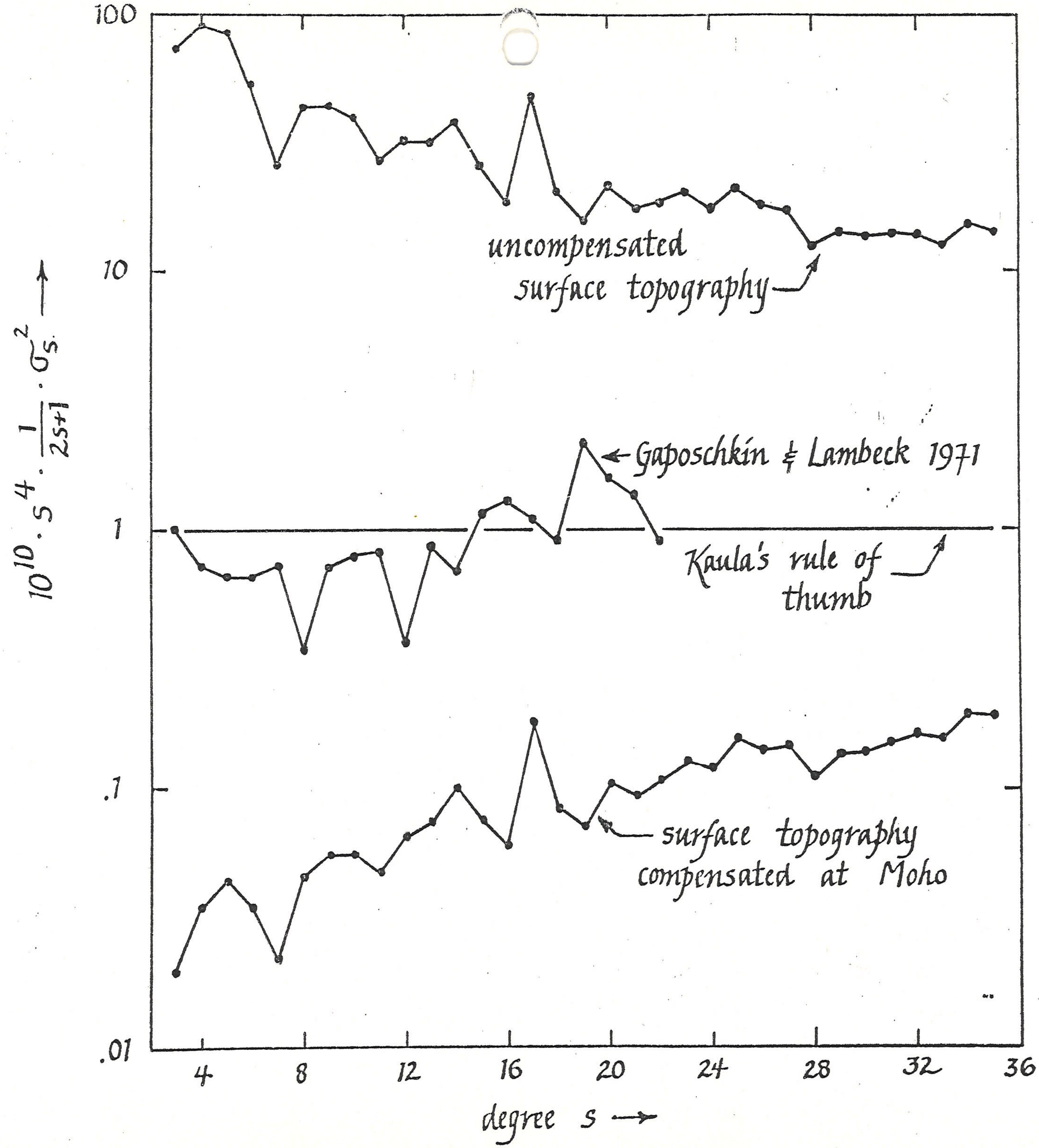
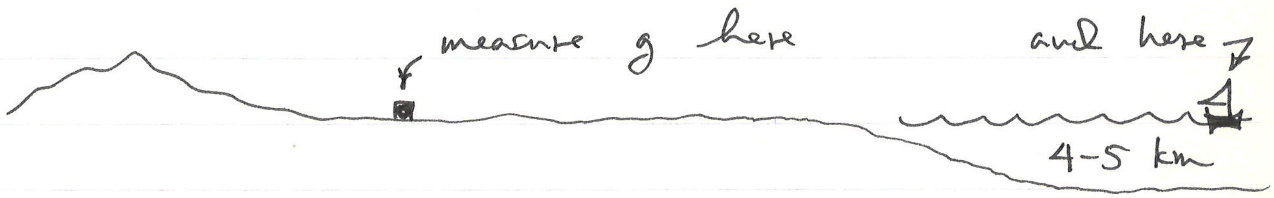


Fig. 9. Mean potential coefficient by degree; n' is the number of coefficients in the sum.


100
10
1
.1
.01
4
8
12
16
20
24
28
32
36
degree s →
10^10 · s^4 · 1/(2s+1) · σ_s^2 →
uncompensated
surface topography
Gaposchkin & Lambeck 1971
Kaula's rule of
thumb
surface topography
compensated at Moho

Another way of seeing same thing: Continents stand 4–5 km above ocean basins



If a long way from coasts the free-air gravity anomaly on land relative to that aboard ship will be given by the Bouguer formula (homework problem)

$$\Delta g = 2\pi G \left(\rho_{rock} - \rho_{water}\right) h$$

$$= 2\pi \left(6.67 \cdot 10^{-8}\right)\left(2.7 - 1.0\right) 5 \cdot 10^{5}$$

$$= 0.356 \ \mathrm{cm^2/sec}$$

$\Delta g_{expected} \sim .350$ mgal, positive gravity anomaly over continents

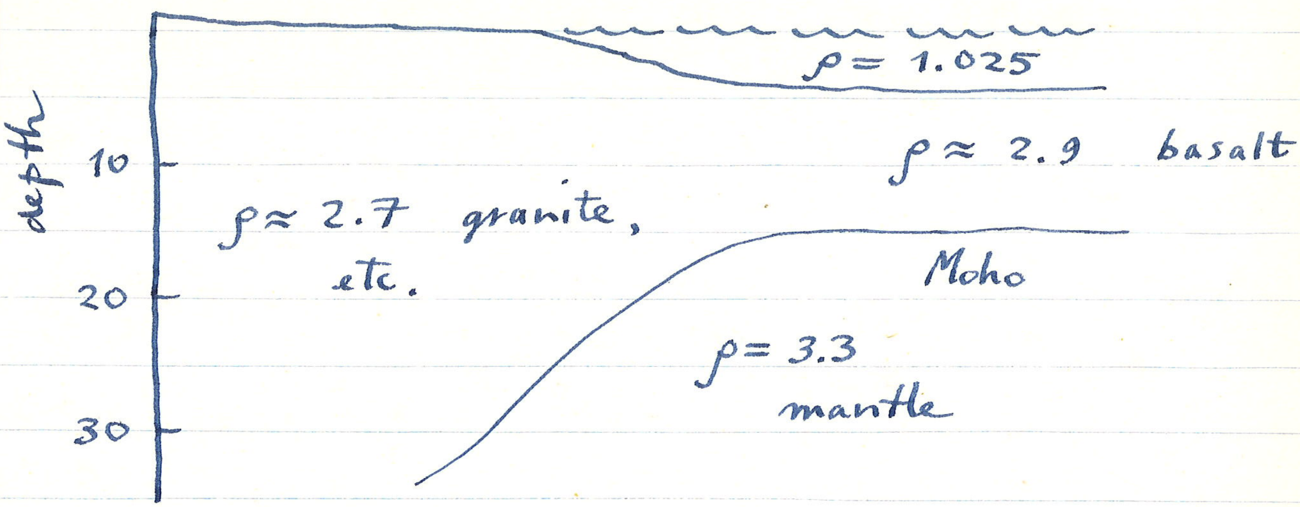
Actual anomalies about 10 times smaller, no visible correlation with cont–ocean basins. This key evidence for isostasy.

This is not a precise "law" of geophysics, merely a strong tendency or a good rough approximation.

Mass excesses associated with high topography tend to be ~~[illegible]~~ compensated by mass deficiencies beneath.

Seismic measurements reveal that $\exists$ a profound and well-known "discontinuity" in the P-wave velocity $\alpha(r)$ typically from ~ 6.7 km/s to ~ 8.1 km/s. This called the Moho or Mohorovicic discont., used to demarcate crust from mantle. Compensation associated with cont-ocean structure conventionally taken to occur mainly at Moho. Natural to expect a discont. in $\rho(r)$ if $\exists$ one in $\alpha(r)$.

A rough current view of cont. vs. oceanic crust looks like this:

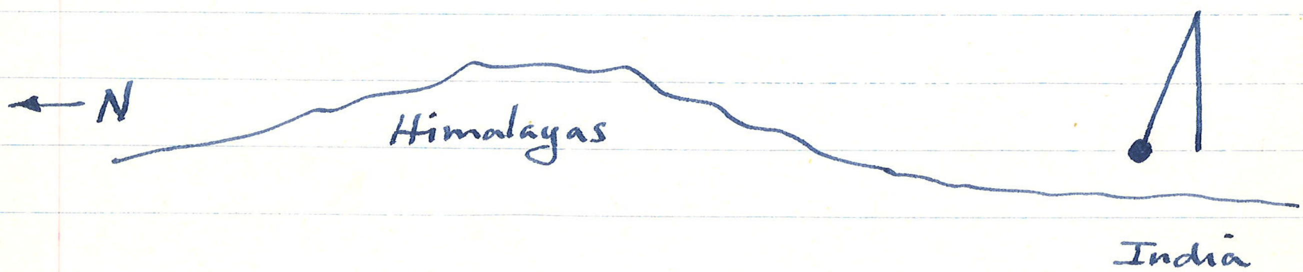


If one assumes cont-ocean topography is completely compensated at Moho the expected "bumps" then would have a power 10-100 times less than observed. This would explain lack of correlation with geography and would imply the source of the bumps is something else, probably deeper.

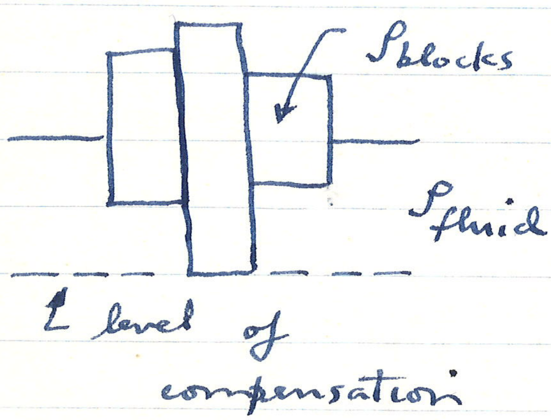
<u>Isostasy</u> an old idea: first quantitative study mid 19th century following British survey of Northern India. Two methods of surveying compared, one using plumbobs and astronomy, one using triangulation.

1855 measurements reduced by Pratt, the Anglican archdeacon of Calcutta.

Accounted for attraction of Himalayas on plumbobs. Found actual attraction less.

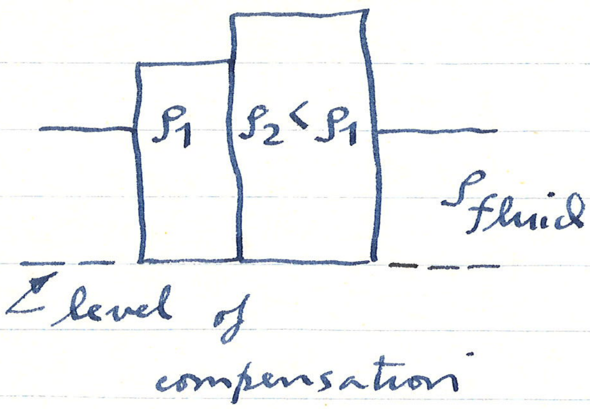


Airy, the to-be astronomer royal of Britain explanation 1855 envisioned crust of ⊕ as lighter than fluid substrate on which it is floating, like blocks or icebergs in water



Airy mechanism of isostasy

1859 Pratt advanced alternative "explanation" : density of mtns less than "normal crust"



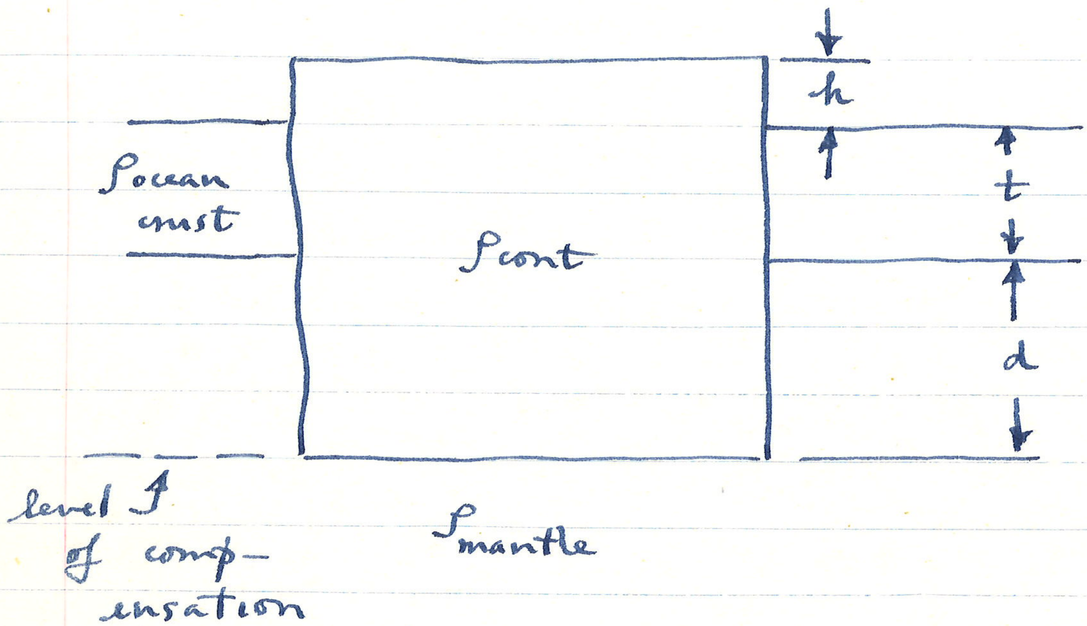
Pratt mechanism of isostasy.

Actual mechanism cannot be distinguished using gravity data alone — another example of non-uniqueness of gravity inverse problem.

The compensation of continents has elements of both but is more like Airy isostasy than Pratt. The Moho (bdry between $\rho \sim 2.7 - 2.9$ and $\rho \sim 3.3$) is deeper under continents — this an observed seismological fact.

How much deeper would we expect it to be according to the Airy mechanism? Ignoring the ocean water consider the model

<u>Extremely</u> crude model just to see if about right



$\rho_{\text{cont}} \sim 2.7$ and $\rho_{\text{ocean crust}} \sim 2.8 - 2.9$

Airy : take them to be the same, say $\rho = 2.8$ vs. 3.3 in mantle.

If conts stick up h above ocean floor and thickness of ocean crust is t what is root d?

By <u>Archimedes principle</u> mass / cm^2 in any two columns must be the same ; in geology this called principal of <u>local isostatic balance</u>

Letting $\rho_c = \rho_{crust}$, $\rho_m = \rho_{mantle}$, we have

$$\rho_c t + \rho_m d = \rho_c (h + t + d) \quad \text{or}$$

$$\boxed{d = \frac{\rho_c}{\rho_m - \rho_c} h}$$

The mean elevation of conts. above seafloor is $h \sim 4.5$ km.

$$d \sim \left(\frac{2.8}{0.5}\right)(4.5) \sim 25 \text{ km}$$

Thickness of ocean crust $t \sim 10-15$ km. Thus thickness of cont. crust should be about 35 – 40 km. This agrees well with observed location of seismic Moho.

The level of compensation below which the stress is hydrostatic is everywhere very shallow compared to $a = 6371$ km, radius of ⊕.

Permissible in discussing isostasy to regard ⊕ as locally flat for this reason.

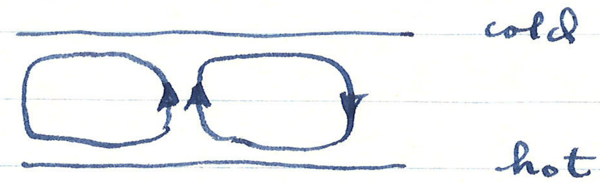
Interpretation of bumps : a brief qualitative discussion

Geoid bumps w.r.t. ε_{hydro} have a direct physical significance. One of 2 assumptions violated :

1. not hydro :

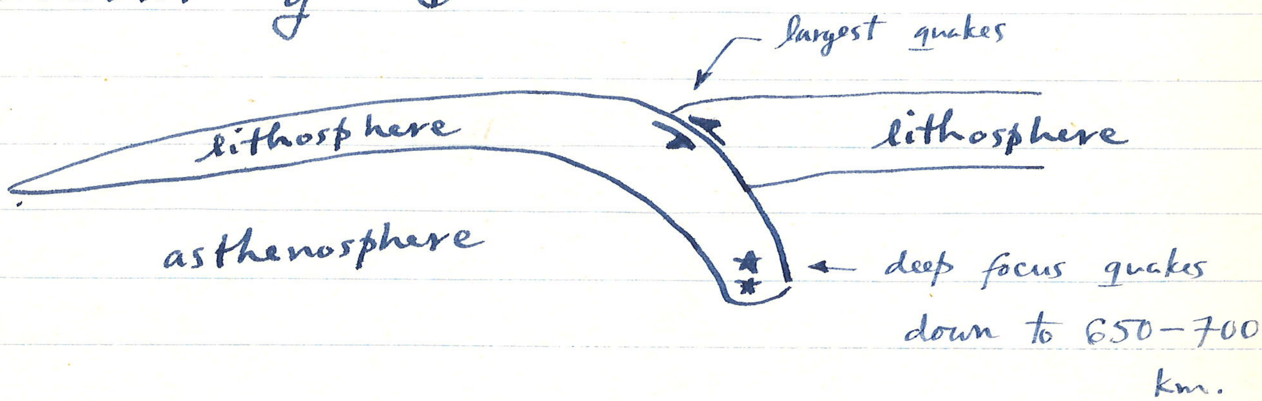
supported by finite long-term shear strength. Lateral variations in density $\Rightarrow$ shear stresses.
If long-term shear strength great enough this explanation viable.

2. not static :
e.g. a convecting fluid



geoid will be high over hot and therefore less dense thermal plume and low over cold descending sinking plume.

From a wide variety of evidence we have obtained the following schematic picture of long-term rheological behavior of ⊕



Surface consists of plates of <u>lithosphere</u> 50 – 100 km thick. The lithosphere not a seismic concept, not same as crust. Its the only part of ⊕ with sufficient long-term strength to support on geological time scales density contrasts large enough to give rise to obs. geoid bumps.

Property best characterizing long-term strength is <u>viscosity</u> $\nu(r)$, one of most poorly known physical properties of ⊕. Lab experiments of the creep of olivine and other substances show that ν is strongly temp-sensitive

$$\boxed{\begin{gathered}\nu \sim \nu_0 \exp(-E/kT) \\ E \text{ called activation energy}\end{gathered}}$$

$E_{\text{olivine}} \sim 125$ kcal / mole.

As $T \nearrow$, ν decreases, a common observation true e.g. for road asphalt too. Exponential dependence on T.

Sketch a plot of T and ν versus depth

Temp profile in $\oplus$ (geotherm) near 0°C at surface, increases to about 1300°C at a depth about 75 km under oceans, greater under continents. A typical geothermal gradient is of order ~~16°/km~~ 15°/km to 30°/km near surface of conts.

As a result the viscosity of the cold lithosphere is $\gg$ than that of the asthenosphere. This responsible for its long-term strength.

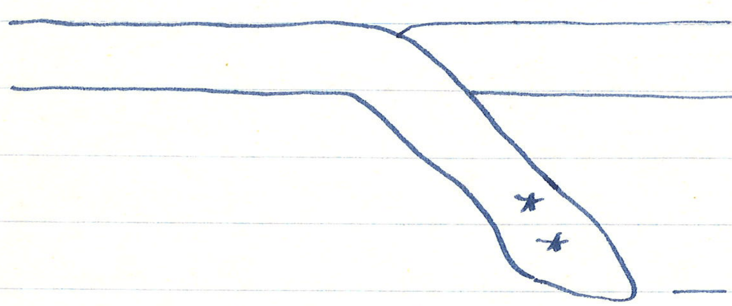
Moral for gravity interpretation: only lithosphere can statically support bumps in geoid. This the opinion of most geophysicists, those who believe in fossil bulge would say lower mantle (below 650 km discont) could perhaps too.

General picture: hydro violations

associated with lithosphere, static violations associated with thermal convection in asthenosphere.

An interesting correlation noted recently by Crough + Jurdy 1980.

Largest lithospheric density differences associated with <u>subducted slabs</u>. Being colder than surrounding asthenosphere they are more dense.



Their locations known from deep seismicity patterns.
Note geoid map has highs (as expected) in many major subduction zones: coast of SA, New Guinea, Indonesia, Japan.

The thermal and density structure can be modelled and removed.

Residual has 2 broad elliptical highs surrounded by lows. Still no ocean - cont correlation.

But there is a correlation with location of so-called hotspots, areas of recent intraplate volcanic activity, e.g. Hawaii, Yellowstone, Iceland, Azores, etc.

Thus appears that broad-scale features of geoid due to mass anomalies assoc. with slabs and hotspots. Both give rise to highs, lows are where neither are present. There are still, however, unexplained features, e.g. the low off India.

Discuss Hager's results briefly

Relevant reading in Stacey for next 2 weeks: Sections 10.1 and 10.2 on plate tectonics, lithosphere, viscosity. Sections 7.1 and 7.2 on heat flow and temperature distribution in the Earth.

In Garland Chs. 24, 25 on heat flow, 27 on geodynamics + rheology.

Heat flow on continents and q vs. A

Reliable continental heat flow measurements require rather deep drill holes (> 100 meters). This, among other things, to get down below transient effects of last ice age. A number of other phenomena can influence the measurements, e.g. local topography and hydrology.

Also there are effects from local vegetative cover, e.g. hole in Blodgett forest, California drilled in small clearing in a forest of virgin pine, less shade produces local heating (annual mean temp ~ 0.5°C higher than average for area), thermal anomaly down to 150-200 m. At Loomis, California the cooling effect of a small stream produces a similar disturbance of opposite sign. Moral: drill sites should be chosen with care.

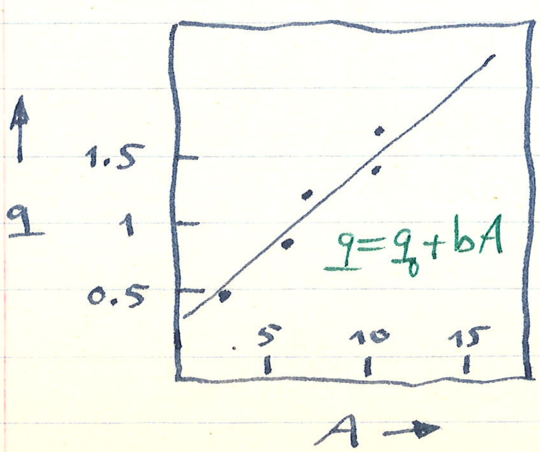
All these kinds of problems avoided on ocean floor, very stable thermal environment, temp ~ 2° C.

Perhaps most interesting finding of cont. heat flow measurements to date is the q vs. A correlation (Birch + coworkers 1968).

If one goes to a single geological province, e.g. (in this context) the eastern U.S. or the Southern Rocky mountains and plots surface heat flow q vs. surface heat productivity A (in cal/cm³-sec) a striking correlation is observed.

A is determined by chemical analysis (concentration in rocks of U, Th, K)

Examples: Fig. 10 of Sclater + Francheteau 1970 plutons of New England + 3 other regions. Fig. 14 of Roy et al. 1972 Southern Rockies



q in 10^{-6} cal/cm²-sec (HFU)

A in 10^{-13} cal/cm³-sec

A simple explanation of this relationship would be the following.

Heat flow equation

previously we called this h

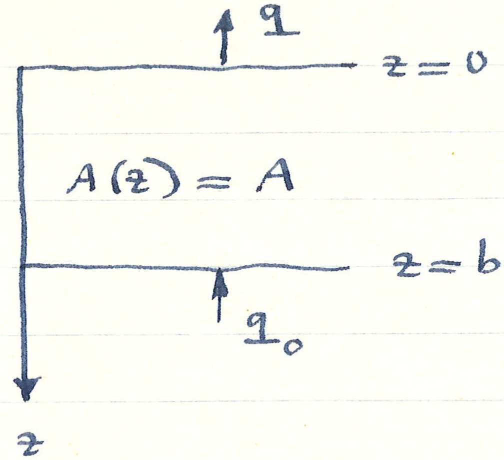
$$\rho c_p \frac{dT}{dt} = k \frac{d^2 T}{dz^2} + A$$

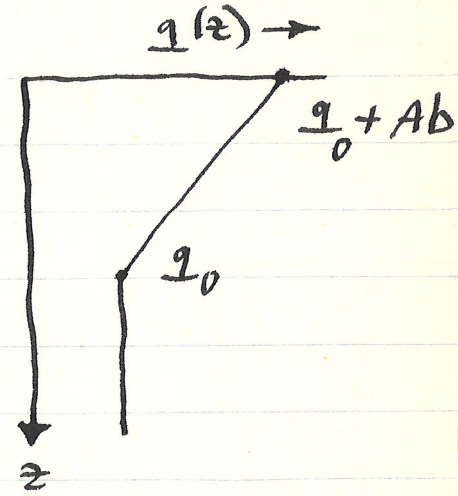
Suppose the temp. distribution is <u>steady state</u> no active cooling occurring. This a good approx. if the last thermal "event" which reached the surface was several 100 m.y. ago. Then $\partial T/\partial t = 0$ and eqn. becomes

$$-\frac{d}{dz}\left(k \frac{dT}{dz}\right) = A \quad \text{or}$$

$$\boxed{-\frac{dq}{dz} = A \quad *}$$

Consider the following model: suppose the radioactive elements are <u>uniformly</u> concentrated in a layer at the surface.





Solution to * is $q(z) = \text{const} - Az$
Let the layer be of thickness b
and let q_0 be the heat flowing
into the base from below. Then

$$q(b) = q_0 = \text{const} - Ab$$
$$\text{const} = q_0 + Ab$$

$$\boxed{q(z) = q_0 + A(b-z) \quad \text{heat flow in steady state as a function of depth}}$$

see plot above

Decreases with depth because of overlying heat production.

$$\boxed{\text{Heat flow at surface } q = q(0) = q_0 + Ab \quad \text{This the form of the correlation.}}$$

Interpretation of q_0 (zero A intercept) and b (slope of q vs. A) thus:

q_0: called <u>reduced</u> heat flow is heat flowing into base, <u>not</u> due to radioactivity in crust, it is heat flow <u>from mantle</u>.

b: <u>thickness</u> of radioactive layer.

In the <u>eastern U.S.</u> measured q_0 and b are $q_0 = 0.8$ HFU and $b = 7.5$ km, much thinner than continental crustal roots under plutons.

Southern Rockies: $q_0 = 1.3$ HFU and $b = 10$ km

actually varies from ~ 4.5 to 16 km

In general b is almost always about 10 km but q_0 varies from 0.6 HFU in old shield areas to 1.4 HFU in Basin and Range province of western U.S., a region of active tectonism, many hot springs, seismic activity, active slow rifting, recent volcanism.

The observed relationship q vs. A is very fundamental.

Problem with above interpretation: relationship observed to hold over geol. provinces of considerable horizontal extent. Such provinces have suffered differential erosion often of several km, a substantial fraction of b.

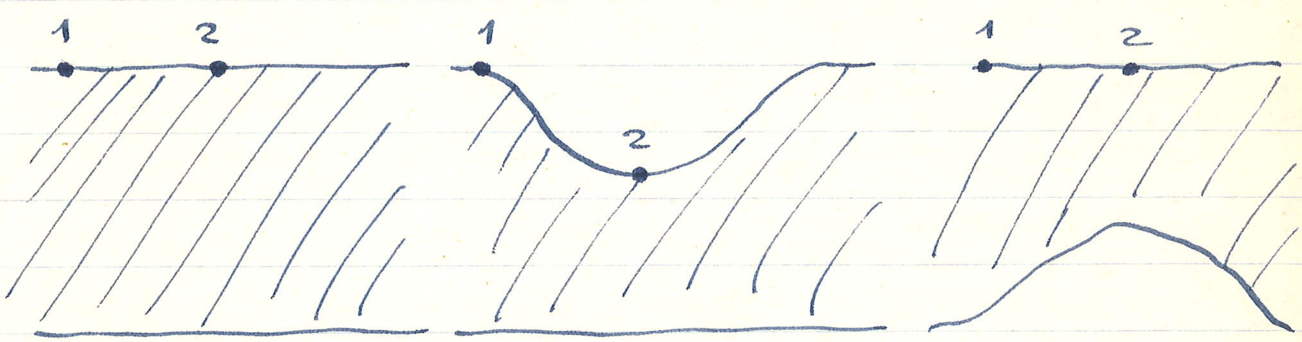
How would we have ever gotten

A 1   2

b const

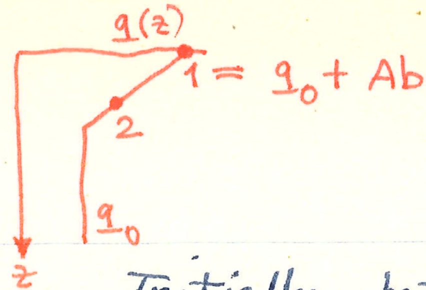
$A_1 < A_2$ in the first place?

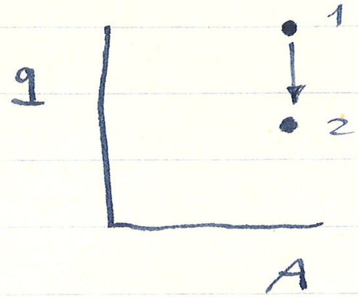
The above model would not preserve q vs. A during differential erosion. To see this consider hypothetical instantaneous erosion:



initial situation A const to depth b | erosion occurs | after isostatic uplift relief no longer evident

Heat flow at points 1 and 2:

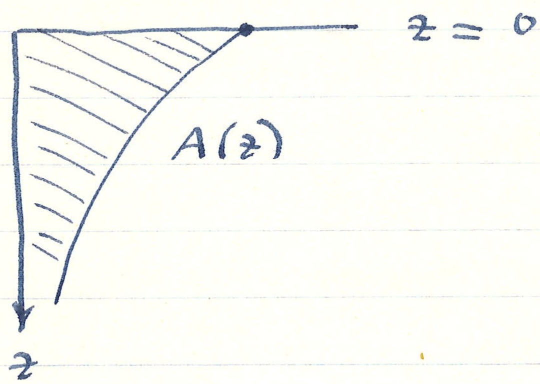




Initially both at 1.
After erosion both still same A at surface but q at 2. will be less because of removal of overlying material.

Let us ask : is there a productivity distribution $A(z)$ which would preserve q vs. A during differential erosion ?

Suppose initially surface at $z = 0$ and productivity $A(z)$ some decreasing function of z. We assume $A(\infty) \to 0$.



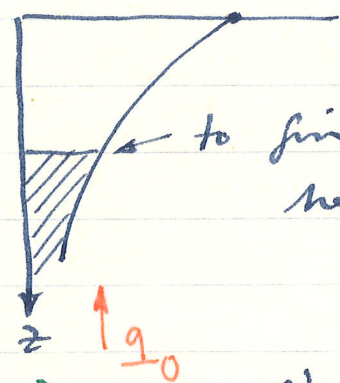
Consider the steady state heat flow equation

$$-\frac{dq}{dz} = A(z)$$

Solution is $\quad q(z) = q(\infty) + \int_z^\infty A(z)\, dz$

Let $q(\infty) = q_0$, the heat coming up from the mantle below.

$$\boxed{\begin{array}{l}\text{Solution for } \underline{\text{arbitrary}}\ A(z): \\[1em] \quad q(z) = q_0 + \int_z^\infty A(z)\,dz \\ * \end{array}}$$



to find $q(z)$ here integrate the heat productivity below you and add the heat from below that, q_0. No contribution from productivity above.

Note: if the overlying stuff is eroded then $q(z)$ is the surface heat flow.

Suppose now erosion occurs down to various depths in various places and suppose that differential erosion is responsible for the different q's and A's in different places

here is a reason for the variability

erosion

followed by isostatic uplift

The supposition thus is that a plot of $q(z)$ vs. $A(z)$ will be of the form

$$\boxed{q(z) = \overset{q_0}{\cancel{\text{const}}} + bA(z)}$$

It is assumed that the <u>transient</u> effects of erosion are negligible.

Equating this to $\star$:

$$\underset{q_0}{\cancel{\text{const}}} + bA(z) = q_0 + \int_z^\infty A(z)\,dz$$

Differentiating :

$$dA/dz = -b^{-1}A(z)$$

Solution an exponential

$$\boxed{A(z) = A_0 e^{-z/b}}$$

initial concentration at surface

Then $q(z) = q_0 + \int_z^\infty A_0 e^{-z/b}\,dz$

$$= q_0 + bA_0 e^{-z/b} = q_0 + bA(z)$$

so the const $= q_0$

$$\boxed{q(z) = q_0 + bA(z)}$$

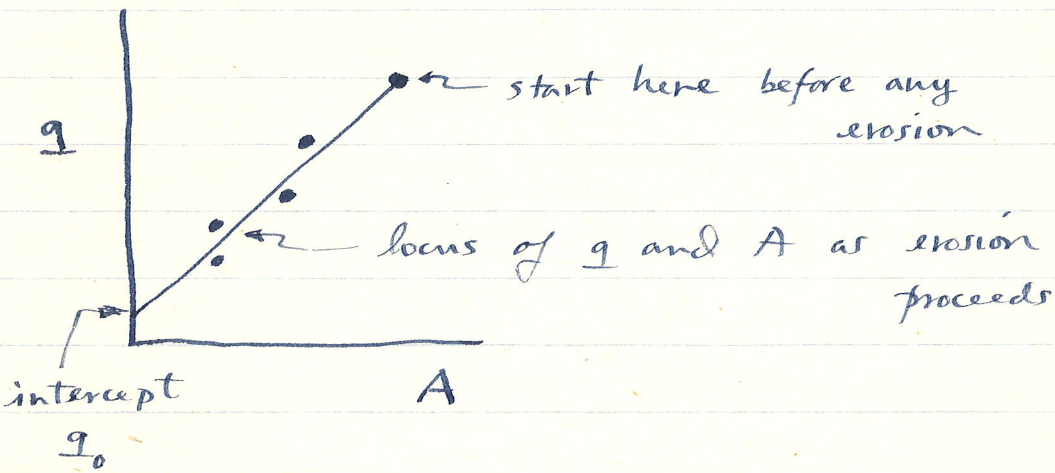
Further differential erosion will preserve this relationship. Only an exponential $A(z)$ will have this property. This first pointed out by Lachenbruch (1970)

The radioactive species K, Th, U are exponentially concentrated upward in the continental crust. The e^{-1} falloff depth is the slope b of q vs. A about 10 km.

The reduced heat flow or intercept q_0 still has the same interpretation as the heat flow from the mantle (several e^{-1} depths below the surface).

The geochemical or petrological reason for the exponential concentration in the uppermost 10 or so km is not known.
Actually h varies from ~ 4.5 km (WAS) to 16.0 (EW).

Implications of model: the more deeply eroded a particular place is the lower should be both q and A.



The mean erosion level of 4 batholiths in the Colo. front range has been estimated geologically and there does appear to be an exp. relationship ($\log_{10} A$ vs. erosion depth is linear) but the value of b obtained from the slope is 1–2 km, not 10. Reason for this discrepancy not known. See Fig. 11.

Reason for variation in reduced or mantle heat flow q_0: thought to be primarily a function of thermal "age" or time since last major thermal "event", e.g. passage over a hotspot as suggested by Crough (1979).

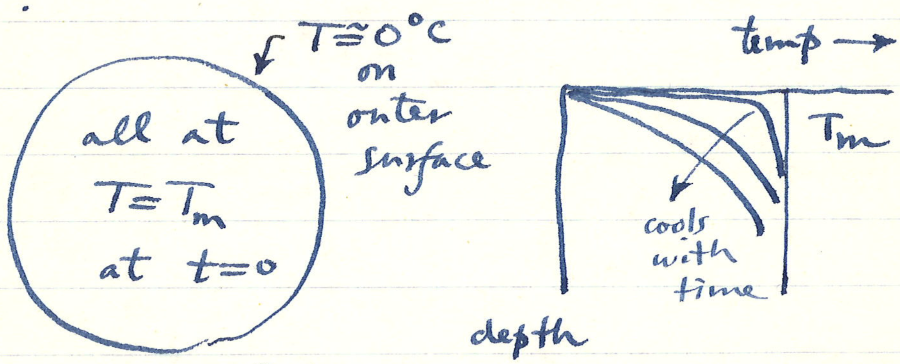
He has suggested that mantle heat flow is localized at hotspots. When a region passes over a hotspot its thermal age is "reset" and so is q_0. The fact that $d - d_0$ in the oceans continues to vary as $t^{-1/2}$ when regions near hotspots are excluded is consistent with the above suggestion. Parsons + Sclater model implies, on other hand, a spatially uniform mantle heat flow.

"Thermal" age difficult to determine since resetting q_0 does not require as much heat input as resetting of radiometric ages. One possibility is fission track dating.

Baltic and Canadian shields, according to this argument have a low q_0 ($\sim$ 0.6 HFU) because it has been some time since they passed over a hotspot. Basin and Range province may on other hand have done so quite recently (or may be rifting + thinning assoc. with subduction of Kula plate).

Lord Kelvin and the age of the ⊕

In the 1800's Lord Kelvin estimated the age of the ⊕ from a heat flow argument. Suppose, he said, the ⊕ started out as a sphere of constant temp. slightly below the melting temp. of rock. Then how long would it take for the heat flow to be reduced to its present value?



What is $T(r, t)$?

He solved the same problem for a sphere we have solved for a half-space. We can reconstruct his argument, to a first approx., with the half-space model. We found heat flow at surface:

$$q(t) = a t^{-1/2}$$

$$a \sim 500 \text{ mW/m}^2 \text{ (m.y.)}^{1/2}$$

[actually, $a = \rho_m c_p T_m (k/\pi)^{1/2}$]

decreases with time like $t^{-1/2}$

The temp. gradient could be measured down in coal mines, etc. so a rough value for the heat flow was known.

Using $q \sim 80-110 \ mW/m^2$ Kelvin found t to be $20-40$ m.y. "and probably much nearer 20 than 40".

Kelvin was the leading physicist of his day and his opinion carried authority. But his estimate in conflict with the geological record. Led to considerable controversy.

His ~~[illegible]~~ math was right but he overlooked a then unknown physical phenomenon; the discovery of <u>radioactivity</u> was made by Henri Becquerel in 1896. As we have seen this acts to heat up the $\oplus$. Given $A(z)$ the <u>steady state</u> (not cooling off) heat flow at $z=0$ is

$$q(z) = q_0 + \underbrace{\int_0^\infty A(z)\, dz}_{\text{heat flow due to radioactivity.}}$$

First quantitative treatment of effect of radioactivity on heat flow by Lord Rayleigh in 1906. He measured radioactivity content of a suite of different igneous rocks (he isolated the radium in the rocks).

As we have seen radioactivity content of oceanic basalts is quite low and Kelvin's argument for oceans is not bad. In fact his estimate $t = 20-40$ m.y. in right ball park for <u>age</u> of <u>oceans</u> because oceanic heat flow is dominated by <u>cooling</u> of initially hot material as he assumed.

Ever since this episode there has been a contingent amongst geologists unwilling to accept the authority of arguments by physicists, e.g. those who would have the ⊕ expand by a factor of 2.

Global heat flow : a summary

Assuming $\underline{q}$ in the oceans varies as $t^{-1/2}$ and knowing the age t everywhere we can infer the total heat escaping out of the ocean floor

$$\boxed{\begin{array}{l} q(t) = a t^{-1/2} \\ \text{the data imply } a \sim 500 \\ \qquad (mW/m^2)(m.y.)^{1/2} \end{array}}$$

The total heat flow in the oceans (including the hydrothermal part) is, just from the area vs. age curve,

$$\boxed{\bar{q}_{oceans} = 95 \pm 10 \quad mW/m^2}$$

Recall 1 HFU = 41.84 mW/m^2

Best estimate of continental heat flow uses empirical q_0 vs. "tectonic age" relationship to extrapolate to regions where data is scarce, e.g. China.
This is crude but yields

$$\boxed{\bar{q}_{conts} \approx 55 \pm 5 \text{ mW/m}^2}$$

Average heat flow out of oceans about 70% greater than continental.

Previously thought to be much more nearly equal but this before contribution from oceanic hot springs known.

Total heat escaping ⊕'s surface

$$\boxed{\begin{array}{l} Q_{global} = 4.1 \pm 0.4 \cdot 10^{13} \text{ W} \\ q_{ave} = 80 \pm 8 \text{ mW/m}^2 \end{array}}$$

SJG (1980) find 70 mW/m², but this must not count the hot springs.

About 20% of this total thought to be due to hot springs in oceans

$$\boxed{Q_{hydrothermal} \sim 0.8 \cdot 10^{13} \text{ W} \sim .20\ Q_{global}}$$

Previous ocean-cont equality puzzling because of known low heat productivity of tholeites ~ 30 × less than granites. Now known this "equality" not really so equal.

The near "equality" is simply coincidental. The processes afecting q are very different in conts and oceans. In oceans, cooling of a spreading thermal boundary layer. In conts, extra contr. from surface radioactivity, concentrated exponentially in uppermost 10 – 20 km of granitic crust.

Best estimate of heat flow out of mantle is the average of all reduced heat flow determinations q_0 in cratons. About eight regions: Niger, Baltic, Canadian shields, W. Australia, C. Australia, eastern US, Zambia, Basin + Range, average

$$\boxed{\begin{array}{l} \langle q_0 \rangle = 31 \text{ mW/m}^2 \\ \text{inferred } Q_{\text{hot spots}} = 1.6 \cdot 10^{13} \text{ W} \\ \qquad\qquad \text{about } 40\% \text{ of } Q_{\text{global}} \end{array}}$$

Comparison with other "energy sources"

$$Q_{global} = 4 \cdot 10^{13} \text{ W} \sim 10^{28} \text{ erg/yr}$$

Total energy released in quakes (Kanamori) $\sim 10^{25}$ erg/yr, mostly in 1–2 largest quakes

quakes $\sim$ 0.1 % heat flow

this the efficiency of $\oplus$ as a heat engine, roughly speaking.

human electrical production $5 \cdot 10^{26}$ erg/yr
$\sim$ 5 % heat flow

But to exploit geothermal energy must go to a region where heat flow $\gg$ average.

solar influx (solar constant) = 1400 W/m^2 compared to heat flow = 80 mW/m^2

solar influx $\sim$ 18,000 x heat flow
total flux $2 \cdot 10^{32}$ erg/yr

Clearly a significant future resource.