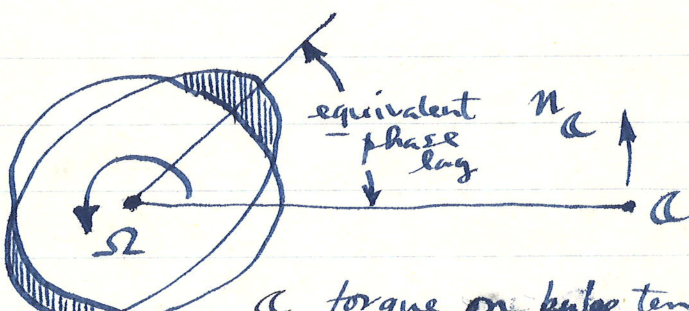
Tidal friction and the ⊕☾ system

Let us make some simplifying assumptions:

1. ignore the presence of the sun ☉
2. assume the ☾ is in a <u>circular</u> <u>equatorial</u> orbit. Actually inclined at ~ 5° to ecliptic which is inclined 23.5° to equator.

really don't need to introduce these 'till later

The simple picture of tidal friction is then ~~[illegible]~~ : ∃ a torque exerted on the lagged tidal bulge.



☾ torque on bulge tends to <u>slow</u> ⊕'s rotation down

Let us see first what we can determine without making <u>any</u> assumptions about the nature of the tidal response since we know the ocean tides in particular do not look like a simple lagged equil. tide.

Let us denote the orbital angular velocity of the ☾ by $n_{☾}$

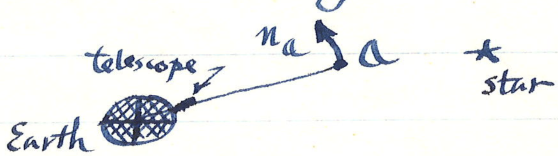
$$[n_{☾}] = \text{rad}/\text{sec}$$

The period $T = 1$ month $= 2\pi / n_{☾}$

The orbital angular velocity is observed to be decreasing. This can be done in several ways:

Note this is completely independent of changes in ⊕'s Ω.

1. telescope observations, esp. of occultations of ★'s by the ☾



Prior to development of atomic time standards this was compared with Ephemeris time. Can now be compared directly with atomic time. The latter method has a short time base but will eventually provide the best results.

The best estimate of $\dot{n}_{☾}$ from all available occultation data, according to Lambeck, is

$$\boxed{\begin{aligned} \dot{n}_{\leftmoon} &= -1.4 \cdot 10^{-23} \ \text{rad}/\text{s}^2 \\ &= -28'' / \text{century}^2 \end{aligned}} \quad \star$$

This is probably uncertain by 10–20%.

2. Lunar laser ranging yields an independent estimate of

$$\dot{n}_{\leftmoon} = -24'' / \text{century}^2 \qquad (\text{LLR})$$

3. Another estimate can be obtained from the perturbations of artificial satellite orbits by the <u>tidal</u> <u>bulge</u> of the $\oplus$. The <u>lag</u> of the $\ell = 2$ bulge can be determined directly in this way. When converted to an <u>equivalent</u> $\dot{n}_{\leftmoon}$, this gives

$$\dot{n}_{\leftmoon} (\text{satellites}) = -1.5 \cdot 10^{-23} \ \text{rad}/\text{s}^2.$$

All methods thus agree fairly well. We shall adopt $\star$.

The present mean motion of the ☾ is

$$n_{☾} = 1''.733 \cdot 10^9 \, / \text{ century}$$

so the <u>fractional</u> rate of change or deceleration is

$$10^9 \cdot \dot{n}_{☾} / n_{☾} = - \frac{24''}{173''} \quad \text{or}$$

$$\boxed{\dot{n}_{☾} / n_{☾} = - 0.14 \, / \, 10^9 \text{ yrs}}$$

At the <u>present</u> rate, in 10^9 yrs would decelerate 14 %.

As ☾ slows down must <u>recede</u> from $\oplus$ since <u>Kepler's third law</u>:

$$n_{☾}^2 R^3 = G(M_{\oplus} + M_{☾})$$

this actually semi-major axis of orbit

$$= \text{constant}$$

Differentiating we find

$$\dot{R} / R = - \frac{2}{3} (\dot{n}_{☾} / n_{☾})$$

$$\boxed{\dot{R}/R = 0.09 / 10^9 \text{ yrs}}$$

If recession of ☾ from ⊕ had been at present rate at all times in past would have been in contact ~ $11 \cdot 10^9$ yr ago

Actually, as we'll see, the rate of recession decreases with time and the time of apparent contact is substantially less, by about 1 order of magnitude.

Since R ~ 60 ⊕ radii the current rate of recession is

$$\boxed{\dot{R} = 3.5 \text{ cm/yr}}$$

We can also calculate the rate at which the ⊕'s rotation is slowing down due to tidal friction. The angular momentum of ⊕ plus ☾ must be conserved (ignoring the ☉).

This leads to

moment of inertia

$$\frac{d}{dt}\left[\, C\Omega + \frac{M_{\leftmoon} M_{\oplus}}{M_{\leftmoon} + M_{\oplus}} R^2 n_{\leftmoon} \,\right] = 0$$

rotational angular mom. of $\oplus$

"reduced mass" $\approx M_{\leftmoon}$ (call this M below)

orbital angular momentum of $\oplus\leftmoon$ system $\approx M_{\leftmoon}$ · velocity · moment arm

All of Ω, R and $n_{\leftmoon}$ vary with time but $\dot{R}$ and $\dot{n}_{\leftmoon}$ are connected by Kepler's 3rd law. We assume C does <u>not</u> vary. Then

$$C\dot{\Omega} = -\, M n_{\leftmoon} (2R\dot{R}) - MR^2 \dot{n}_{\leftmoon}$$

$$= -M\left[n_{\leftmoon}\, 2R \left(-\tfrac{2}{3} R\, \dot{n}_{\leftmoon} / n_{\leftmoon}\right) + R^2 \dot{n}_{\leftmoon} \right]$$

$$\cancel{\dot{\Omega}_{tidal} = \frac{1}{3C} MR^2} \qquad = \frac{1}{3} MR^2\, \dot{n}_{\leftmoon}$$

$$\boxed{\dot{\Omega}_{tidal} = \frac{1}{3}\left(\frac{\frac{M_{☾} M_{\oplus}}{M_{\oplus} + M_{☾}} R^2}{C}\right) \dot{n}_{☾}}$$

The ratio of $\dot{\Omega}_{tidal}$ to $\dot{n}_{☾}$ is thus the constant

$$\frac{1}{3}\frac{MR^2}{C},$$ which is known.

Its value is

$$\frac{1}{3}\frac{(6 \cdot 10^{27}/81 \text{ gm})(3.84 \cdot 10^{10})^2 \text{ cm}^2}{8.04 \cdot 10^{44} \text{ gm cm}^2}$$

$$\approx 45$$

$$\boxed{\dot{\Omega}_{tidal} \approx 45\, \dot{n}_{☾}}$$ — ignoring $\odot$

When account is taken of the $\odot$ the best value for this ratio, according to Lambeck, is

$$\boxed{\dot{\Omega}_{tidal} = (51 \pm 4)\, \dot{n}_{☾}}$$

$\oplus$ is slowing down 50 times faster than ☾.

The rate of deceleration due to tidal friction can thus be deduced from $\dot{n}_{a}$ + cons. of ang. momentum. It is

$$\dot{\Omega}_{\text{tidal}} = -7.1 \cdot 10^{-22} \text{ rad/s}^2$$

The fractional rate of slowing down is ($\Omega = 7.292115 \cdot 10^{-5}$ rad/s)

$$\boxed{\dot{\Omega}_{\text{tidal}} / \Omega = -0.31 / 10^9 \text{ years}}$$

The actual historical variations in the rotation rate of the ⊕ are, as we have seen, dominated by the decade fluctuations of a few parts in 10^8, these make it difficult to measure secular $\dot{\Omega}_{\text{total}}$ directly, but see below

$\boxed{\dot{\Omega}_{\text{tidal}} \text{ corresponds to increase in l.o.d. by 2.7 msec/century}}$

An interesting quantity is the <u>rate</u> of <u>tidal energy</u> <u>dissipated</u>. This also can be deduced directly from <u>only</u> the observation $\dot{n}_☾$ (if the ☉ is ignored)

The k.e. of rotation of the ⊕ is:

$$E_⊕ = \frac{1}{2} C\Omega^2 \qquad \text{so}$$

$$\dot{E}_⊕ = C\Omega\dot{\Omega}$$

The energy associated with the orbital motion is

$$E_☾ = \frac{1}{2}\left(\frac{M_⊕ M_☾}{M_⊕ + M_☾}\right) n_☾^2 R^2 \; - \; \frac{G M_⊕ M_☾}{R}$$

kinetic energy of orbital motion of ⊕, ☾ about c.o.m.

grav. pot. energy of 2 pt. masses separated by R

but $n_☾^2 R^3 = G(M_⊕ + M_☾)$ so

negative : stable orbit

$$E_a = -\frac{1}{2}\frac{GM_\oplus M_a}{R}$$, half the grav. pot. energy!

$$\dot{E}_a \cong -\frac{1}{3} M_a n_a R^2 \dot{n}_a \left(\frac{M_\oplus}{M_\oplus + M_a}\right)$$

this the neglected term

where $M_a / M_\oplus$ has been neglected.

i.e. this is really just the reduced mass M

Total change

$$\dot{E} = \dot{E}_\oplus + \dot{E}_a$$

$$= C\Omega\dot{\Omega} - \frac{1}{3} M_a n_a R^2 \dot{n}_a$$

$$\dot{E} = (5.9\,\dot{\Omega} - 9.6\,\dot{n}_a)\cdot 10^{40}\ \text{erg/s}$$

Recall $\dot{\Omega}_{tidal} \sim 50\ \dot{n}_a$ so the energy change is dominated by $\dot{E}_\oplus$ ($\dot{E}_a$ is about 4 % of total)

$$\boxed{\text{We find } \dot{E} = -4\cdot 10^{19}\ \text{erg/sec}}$$

Somewhere in $\oplus$☾ system tidal energy must be dissipated at this rate.

Recall total heat flow from $\oplus$ was $4 \cdot 10^{13}$ W or

$$Q_{global} \sim 4 \cdot 10^{20} \text{ erg/s}$$

So by comparison

$$\boxed{|\dot{E}_{tidal\ friction}| \sim \frac{1}{10} \text{ heat flow from } \oplus}$$

tidal energy as a resource exploited in a few places

Aside: it can be shown that an alternate expression for $\dot{E}$ is:

$$\dot{E} = -N(\Omega - n_☾) \approx -N\Omega$$

torque exerted by ☾ on $\oplus$'s tidal bulge

ang. velocity of $\oplus$'s rotation w.r.t. ☾

where the torque $N = -\frac{1}{3} \frac{M_\oplus M_☾}{M_\oplus + M_☾} R^2 \dot{n}_☾$

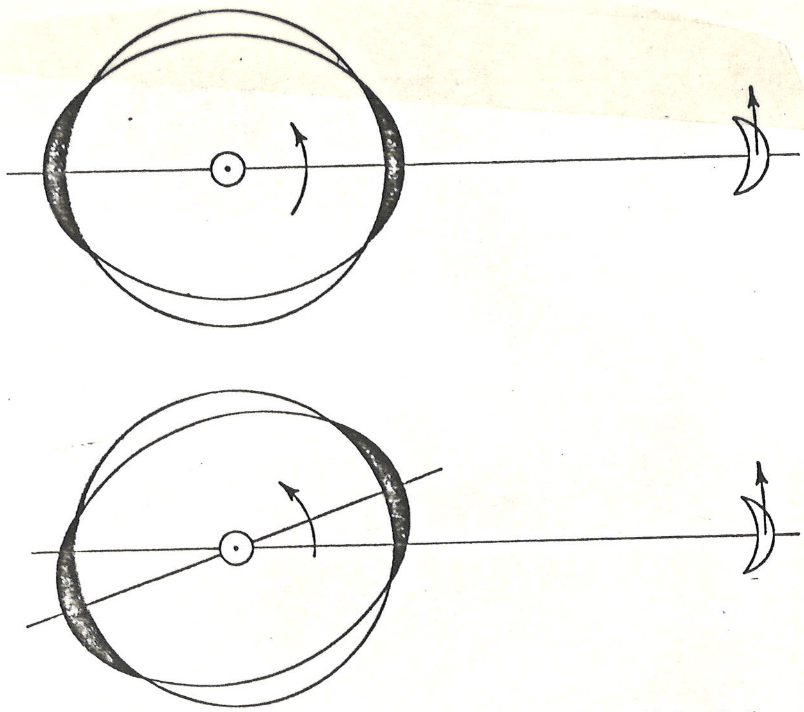


Fig. 11.13. The upper diagram shows the tidal bulge if there is no friction. In the case of friction there is a delay in the time of high tide, and the resulting distortion of the tidal bulge leads to a deceleration in the Earth's rate of rotation, and to an acceleration in the Moon's orbital motion.

Table 10.13. *Summary of tidal accelerations estimated from astronomical, tidal and satellite data. $\delta\dot{\Omega}_T$ is the contribution to $\dot{\Omega}$ from terms other than those related directly to the lunar acceleration (10.5.1)*

	Astronomical estimate	Tidal estimate	Satellite estimate
$\dot{n}_{\leftmoon}$ (10^{-23} rad s^{-2})	-1.35 ± 0.10	-1.49 ± 0.5	-1.33 ± 0.25
$\dot{\Omega}_T\|_{\dot{n}_{\leftmoon}}$ (10^{-22} rad s^{-2})	-5.48	-6.05	-5.40
$\delta\dot{\Omega}_T$ (10^{-22} rad s^{-2})	-1.47	-1.47	-1.47
$\dot{\Omega}_T$ (10^{-22} rad s^{-2})	-6.95	-7.52	-6.87
dE/dt (10^{19} erg s^{-1})	-3.94 ± 0.30	-4.26 ± 0.45	-3.90 ± 0.70

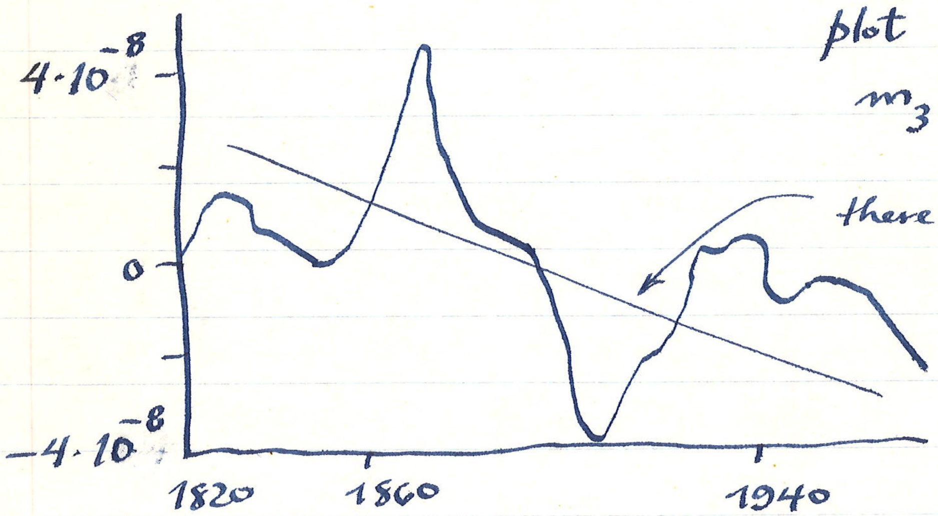
Observations of ancient eclipses

The *inferred* rate of tidal deceleration is $\dot{\Omega}_{tidal} = -7 \cdot 10^{-22} \frac{rad}{s^2}$

or

$$\dot{\Omega}/\Omega = -\,0.31 \cdot 10^{-9} \;/\text{year}.$$

The *observed* changes in l.o.d. from Morrison, since 1820 look like



plot of $m_3 = \delta\Omega/\Omega$

there is a hint of a *secular decrease* in l.o.d. but the decade fluctuations are about 10 times larger.

Ancient eclipse observations provide an *average rate* of *deceleration* over times of order 1000 – 2000 yrs (the decade fluctuations thus averaged

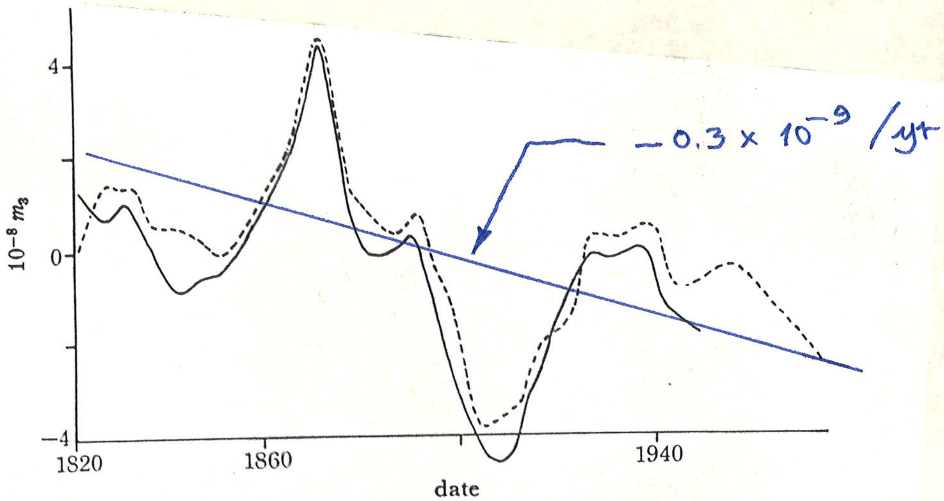


FIGURE 6. Long-period variations observed in m_3 since 1820 as determined by Brouwer (——) and by Morrison (– –).

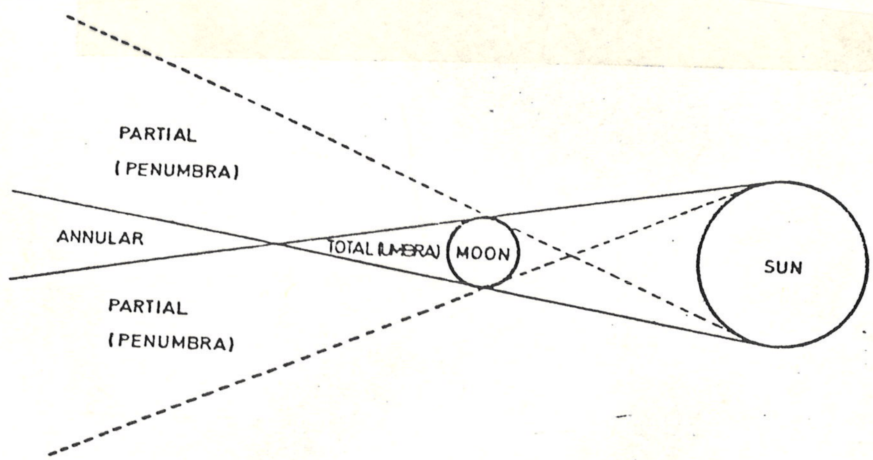
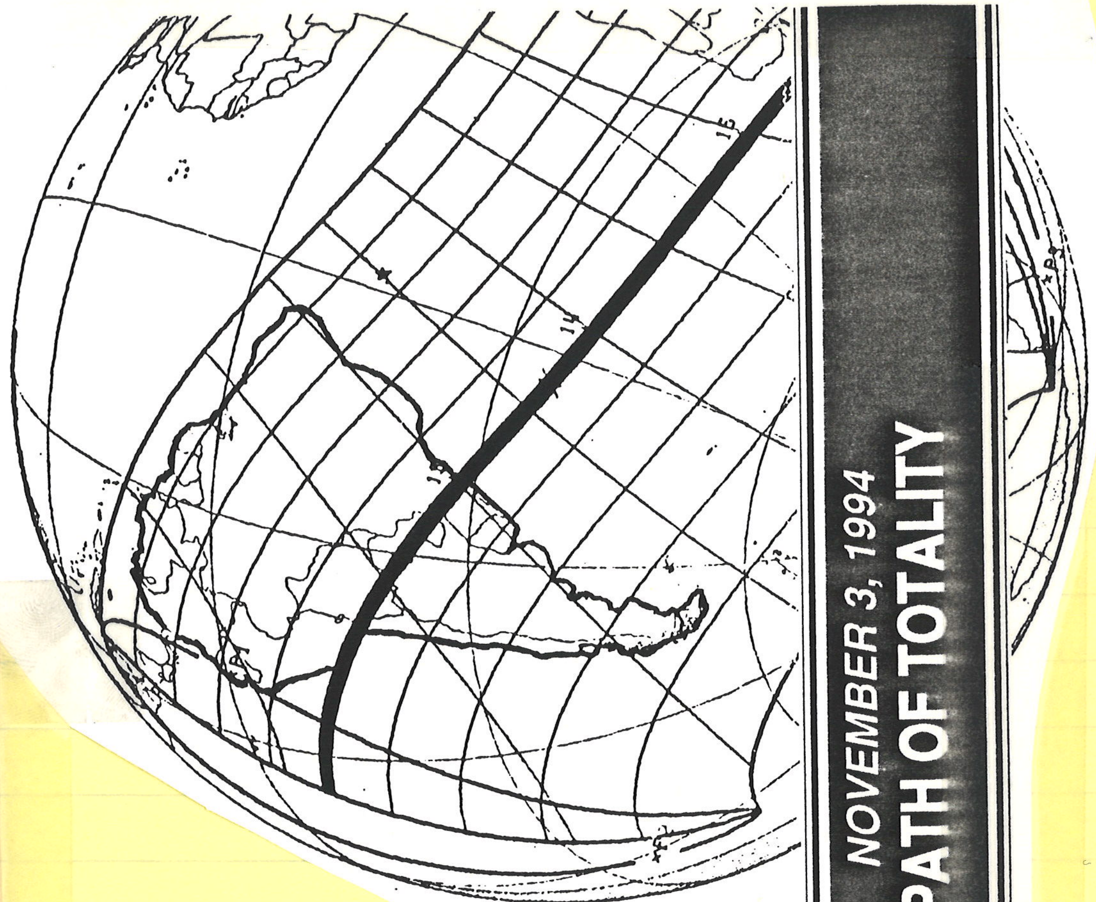


FIGURE 1. Solar eclipse shadow geometry



NOVEMBER 3, 1994
PATH OF TOTALITY

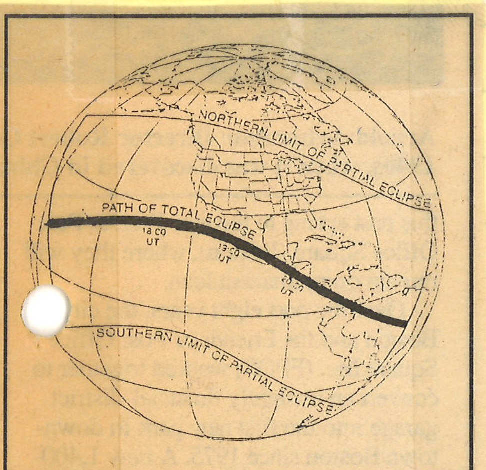
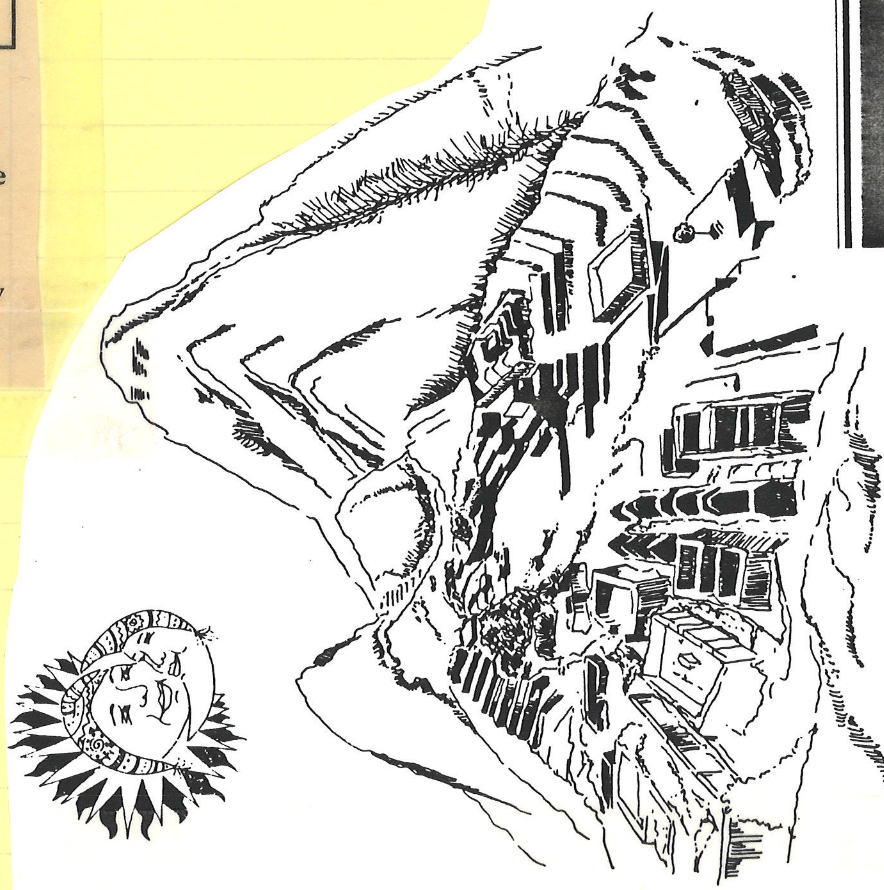


Illustration courtesy of Sky and Telescope magazine/
Fred Espenak, NASA

The total solar eclipse on July 11 will sweep over a narrow strip of the tropics, but the partial eclipse will be visible from a wide swath of the Western Hemisphere, including most of the United States and southern Canada. The event can not be viewed in the New England skies.



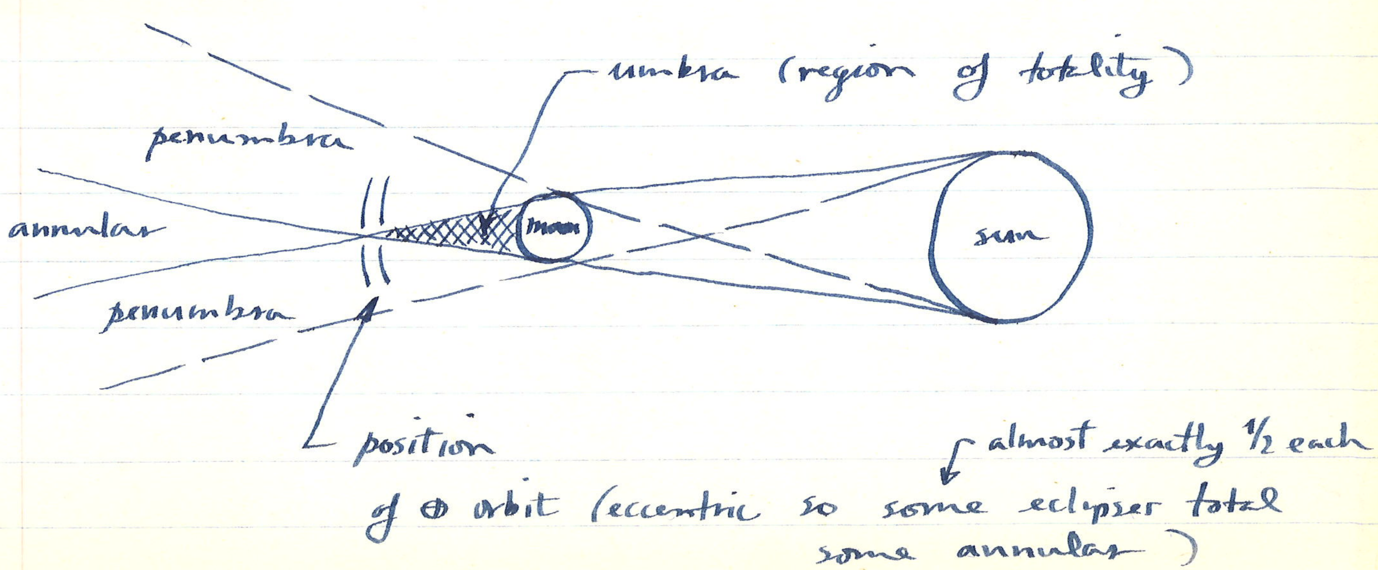
out).

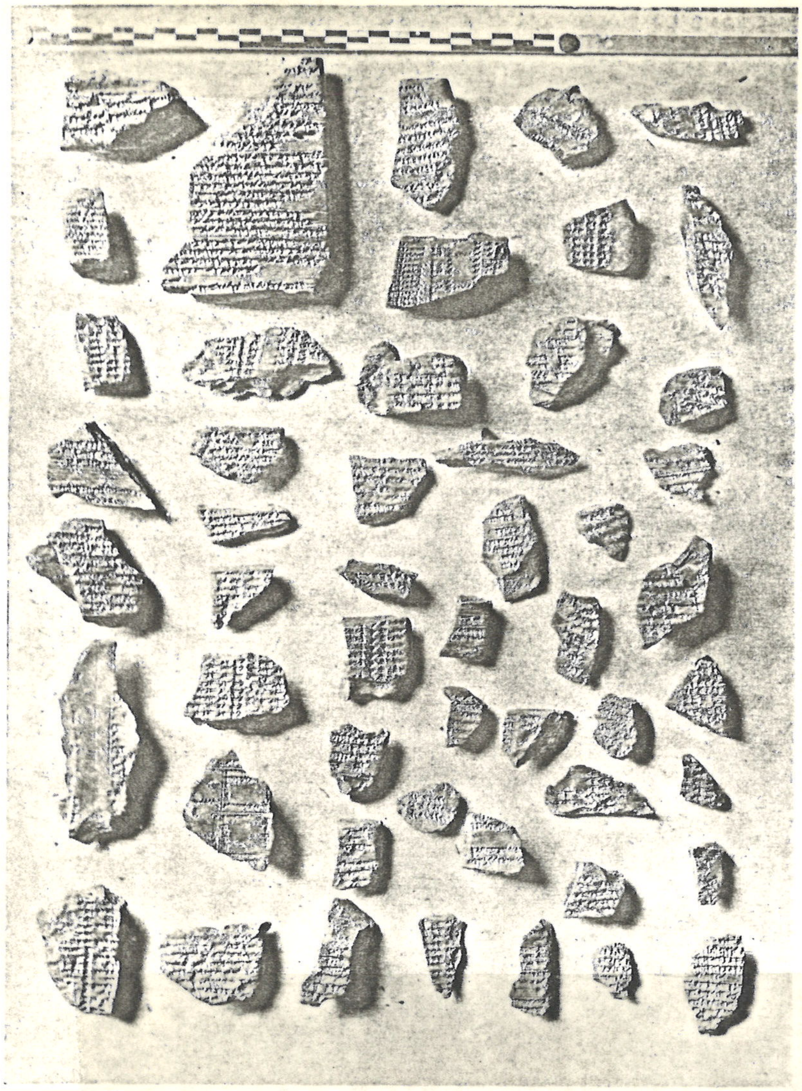
Several kinds of observations have been employed, e.g. reports of the times of eclipses before or after sunset or sunrise.

The best data thought to be simply the reported location of total solar eclipse. The date not even required, except roughly.

Total eclipses are very memorable events, many recorded instances in antiquity.

The paths of totality on ⊕'s surface very narrow, about 100 km. Reason





'Daytime of the 28th the north wind blew. Daytime of the 29th, 24 *uš* after sunrise, a solar eclipse beginning on the south west side . . . Venus, Mercury and the Normal Stars (i.e. the stars which were above the horizon) were visible; Jupiter and Mars, which were in their period of disappearance (i.e. between last and first visibility) were visible in that eclipse . . . (the shadow) moved from south west to north east. (Time interval of) 35 *uš* for obscuration and clearing up (of the eclipse). In that eclipse, north wind which . . .'

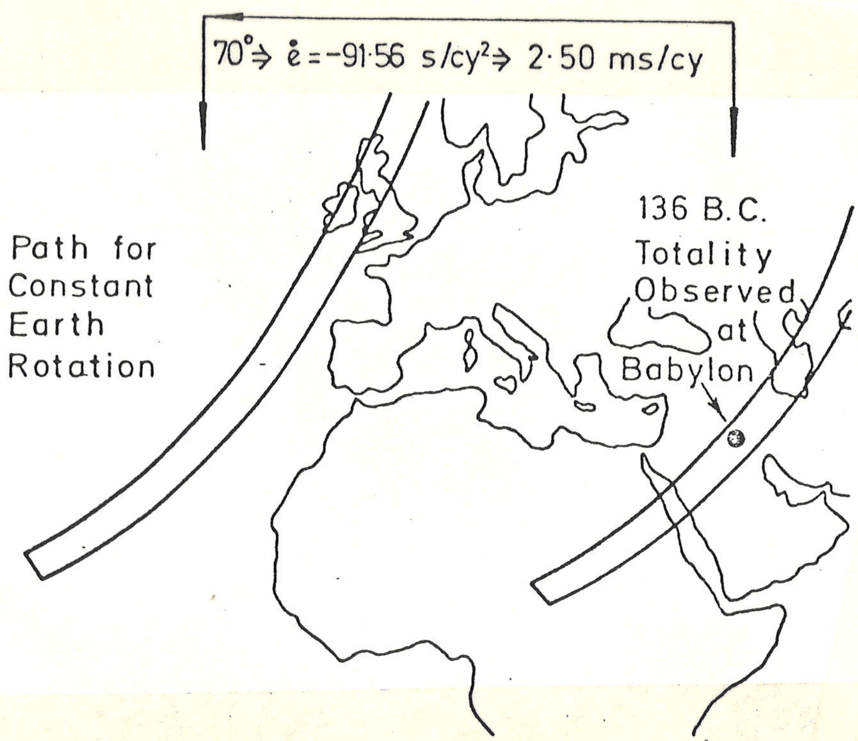


FIGURE 6. Earth's rotational displacement

Principal of technique illustrated by eclipse of 136 B.C. 15 April observed at Babylon, recorded in cuneiform on clay tablets

Date given in text Addaru II.
"On the 29th day there was a solar eclipse beginning on the southwest side. After 18 <u>uš</u>... it became complete (<u>til</u>) such that there was complete night at 24 <u>uš</u> after sunrise."
Venus, Mercury, Jupiter and Mars were all observed and recorded — gives confidence in date as calculations show they would have all been visible.

See fig. 6 of Muller + Stephenson for method. This eclipse gives <u>average</u> rate between 136 B.C. and now 2.50 ms/century.

All in all this a very difficult subject, many authorities disagree on dates, interpretations, etc.

Eclipse of Hipparchus 129 B.C.

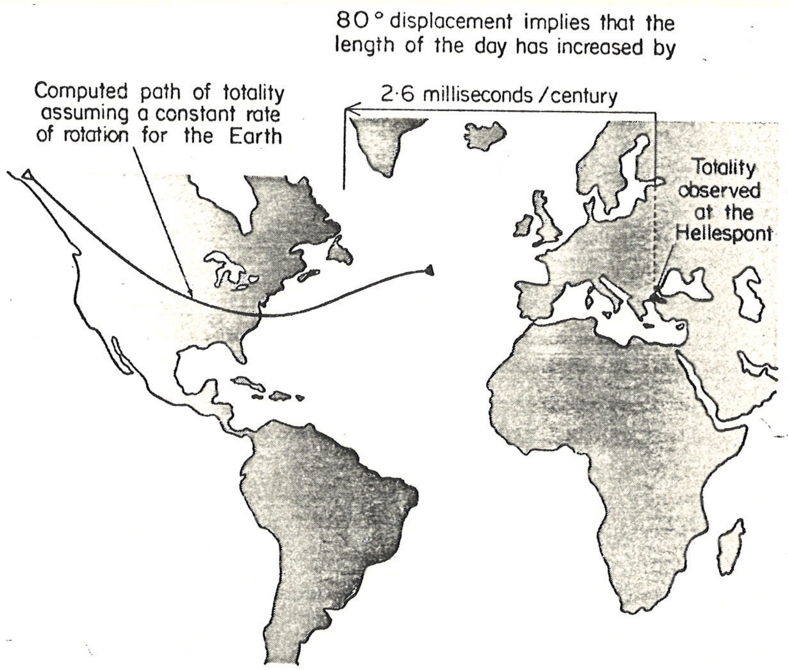


FIGURE 3. Illustration of how the long-term deceleration in the rotation of the Earth is deduced from the observed occurrence of a total solar eclipse

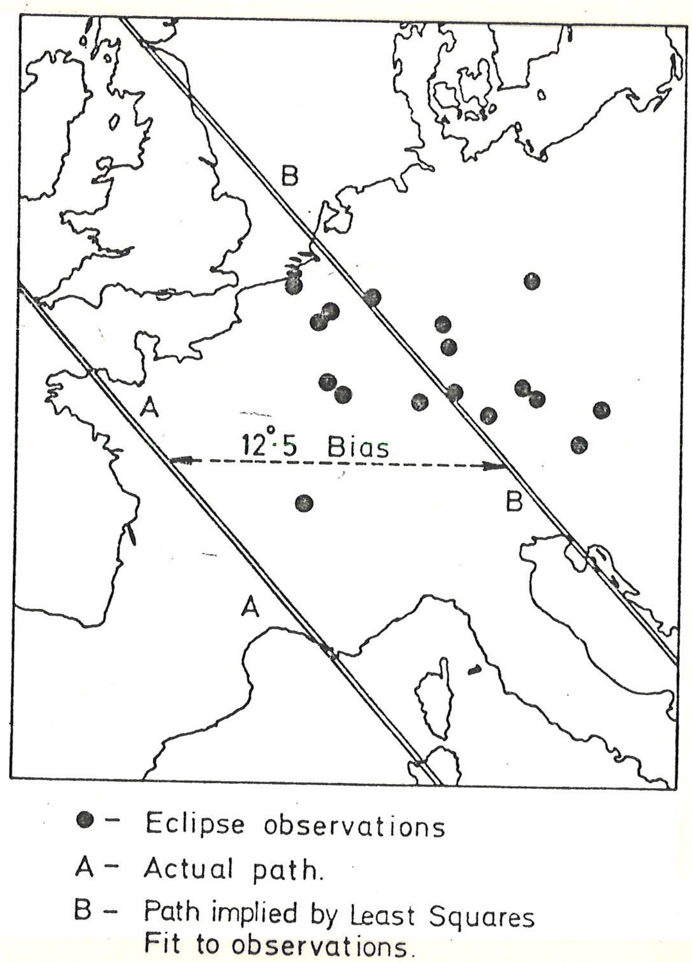
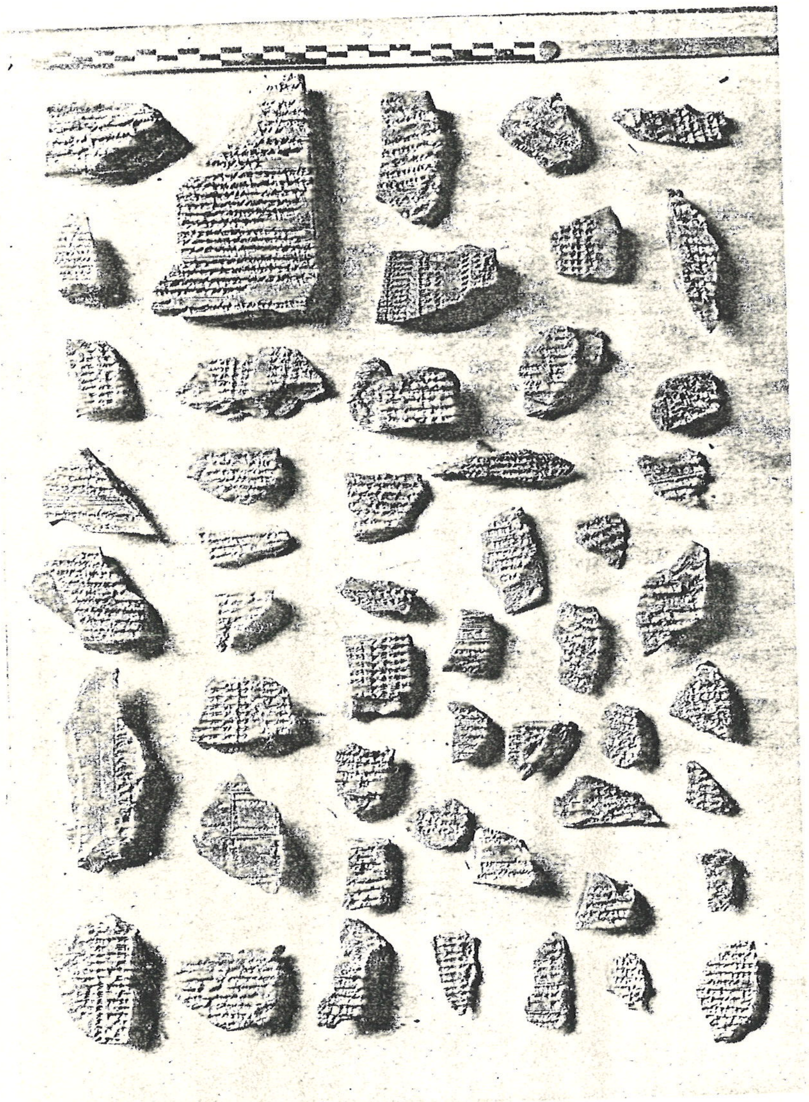


FIGURE 4. Population bias effects for the annular solar eclipse of 22 August A.D. 1039



Tidal friction (see Chapter 11). From Neugebauer (1957), Plate 6: Warka fragments.

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arises, theoretically, from the tidal couple between the Moon and rotating Earth. Figure 5 indicates this effect schematically. The Earth rotates faster (on its axis) than the Moon does (in its orbit) around the Earth. The Earth therefore drags the gravitationally induced oceanic tidal bulges ahead of the Moon (on the nearer Earth hemisphere), and behind the Moon (on the farther Earth hemisphere). This 'phase lag' is small, around 3°. The nearer leading bulge to the Moon tends to accelerate the Moon in its orbit, while the further trailing bulge tends to retard the Moon. The former is larger, and the net effect is to accelerate the Moon in its orbit. If energy is thereby added to the Moon in its orbit, it will move farther from the Earth, slow down, and thereby fall behind its unperturbed position as time progresses. The net tidal torque on the Moon gives rise to a negative acceleration in lunar longitude.

From conservation of momentum, we can see that the Earth must be retarded in its rotation by this torque. The length of day gradually increases, and the value of ET–UT therefore must change as time passes, because the 'Earth clock' is running ever slower. The rotation of the Earth is deaccelerated by this torque, and the effect is termed the 'lunar tidal part' of the (negative) secular acceleration of the Earth's rotation. There are two tides raised by the Sun, an oceanic tide

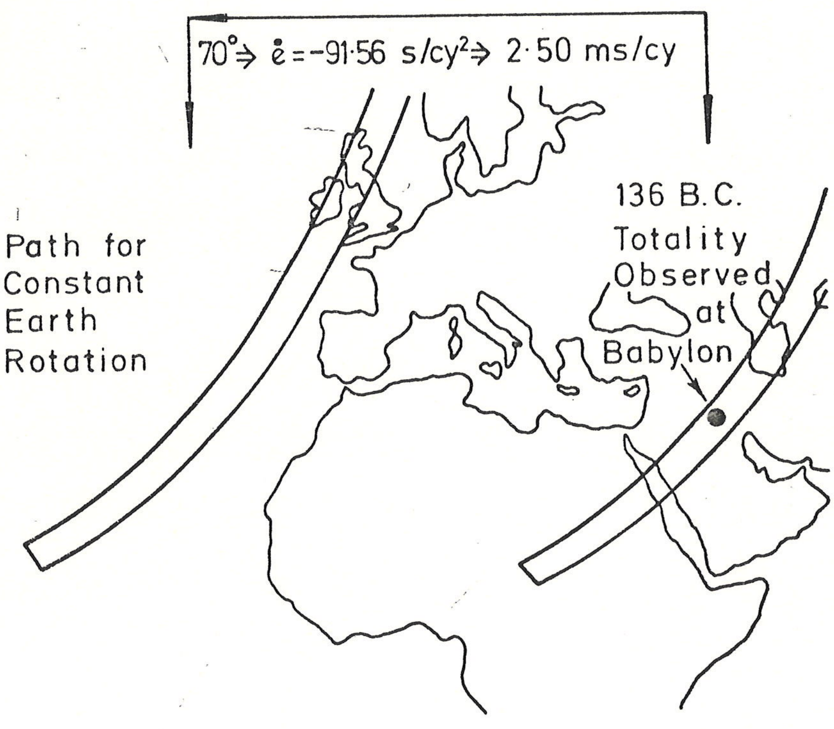

FIGURE 6. Earth's rotational displacement

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and the latter yields a small p
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contributions is the actual Ea
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The assumption of constant a
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The data examined so far has included Babylonian tablets, Persian and Chinese chronicles, chronicles of medieval European monasteries, and finally classical (Greek) records.

A well-known classical observation is that of <u>Hipparchus</u>: "Once when it (the $\odot$) was observed to be completely eclipsed at the Hellespont, at Alexandria it was eclipsed with the exception of a fifth of its diameter".

The totality + place are certain, but in this case the <u>date</u> is not. Could be anywhere between founding of Alexandria 332 B.C. and death of Hipparchus 120 B.C. Some authors give it zero weight, others considerable weight, e.g. Morrison Fig. 3 assigns it the date 129 B.C. following Fotheringham — this implies 2.6 ms /century.

<u>Oldest</u> viable eclipse record Babylonian tablet from city of Ugarit "On the day of the new moon in the month

of Hiyar, the Sun was put to shame and went down in the daytime, with Mars in attendance."

Is this an eclipse? Is it total? If so when did it occur?

Muller + Stephenson If we could be certain that the Ugaritic calendar at this period ≡ Babylonian and Hebrew calendars, then certainly date would be 3 May 1375 B.C. But we can't be certain.

Fig. 4 shows population bias effect: heavy concentration of monasteries in what is now Germany

Lambeck says that best value of $\dot{\Omega}_{total}$ from all such evidence is

this seeks to use not completely total eclipses

$$\boxed{\dot{\Omega}_{total} = -5.5 \cdot 10^{-22} \text{ rad}/\text{sec}^2}$$

corresponds to 2.2 ms/century

There is according to this a slight non-tidal secular acceleration

$$\dot{\Omega}_{\substack{non\\tidal}} = [-5.5 - (-7.1)] \cdot 10^{-22}$$

$$\boxed{\dot{\Omega}_{\substack{non\\tidal}} = 1.6 \cdot 10^{-22} \text{ rad}/\text{sec}^2}$$

or 0.6 ms/century

This the most uncertain quantity of all being the small difference between two nearly equal numbers both uncertain by 10–20%. Muller and Lambeck both conclude that the present evidence does not <u>require</u> any acceleration at all.

If it does ∃, there are many suggested mechanisms:

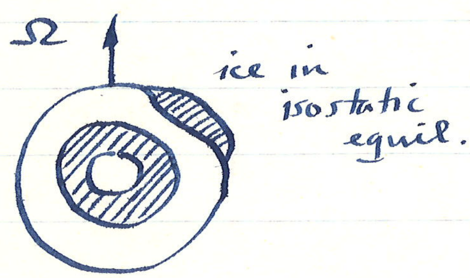
1. change in G (non-Einstein relativity)
2. isostatic rebound after deglaciation
3. EM coupling core-mantle
4. growth of core with time
5. changes in depth of upper mantle transition zones.

Assuming the cause is response of ⊕ to Pleistocene deglaciation there have been attempts by O'Connell, Dicke and others to infer mean ν of mantle.

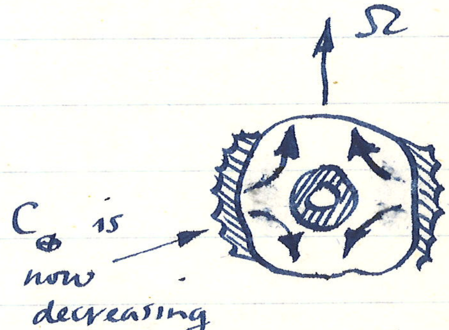
The general idea is this:

18 k.y. ago the Wisconsin glaciation was at its height. There was a large ice load on $\oplus$ surface in northern latitudes, presumably in isostatic equilibrium.

It melted fairly quickly filling the oceans and they then began to sink. They are still sinking so $C_\oplus$ is decreasing and $\therefore$ Ω is increasing.



18,000 years ago



isostatic response to infilling of tropical oceans (the net load shifted toward equator)

If this is the cause of $\dot{\Omega}_{\substack{\text{non}\\\text{tidal}}}$ can use to infer ν.

To change $C_\oplus$ the viscous flow is $l=2$ $\Rightarrow$ the ν estimated is for as whole including <u>lower</u> mantle.

For a <u>very rough</u> estimate we use as before

$$\tau = \frac{19}{2}\frac{\nu}{\rho g a}$$

time scale for $l=2$ response of uniform viscous sphere

Clearly τ must be about 18,000 years, when the whole process started. If τ were much greater, the isostatic adjustment would not have started, if much less it would be over. To allow for the fact that melting was not instantaneous suppose $\tau \sim 10{,}000$ yrs.

Then we find that $\underline{\nu \sim 10^{23} \text{ poise}}$, about 10 times greater than ν inferred for upper mantle from post-glacial rebound, probably not sufficiently greater to prohibit lower mantle convection.

whether to believe this argument or not is a matter of opinion as there is no shortage of other possible causes for $\dot{\Omega}_{\text{non tidal}}$.

The dissipation of tidal energy

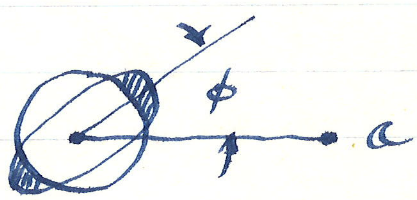
We have seen that $4 \cdot 10^{19}$ ergs are dissipated per second.
Dissipated means that either the ⊕ or ☾ are heated up.

What is the *mechanism of dissipation* and where does it occur?

We first consider dissipation in the *bodily tide*. *Assumptions*:

1. ⊕ has *no oceans*
2. ⊕ is incompressible and homogeneous $k(r) = \infty$, $\rho(r) = \text{const}$, $\mu(r) = \text{const}$
3. solid ⊕ tide is a lagged equilibrium tide

okay for an order of magnitude answer



ϕ = lag angle

Suppose the equilibrium tidal potential is on $r = a$, ⊕ surface (consider M_2 only)

ampl. at equator

$$\pi = \pi_0 \sin^2\theta \cos \sigma t$$

Then it may be shown subject to above assumptions that dissipation rate (rate at which the tidal force derivable from $\mathcal{U}$ does work) is (Munk and MacDonald pp. 207 – 208)

$$\boxed{\begin{aligned}\dot{E} &= -\frac{16\pi G}{15}\, \rho g h\, (1+k) \left(\frac{\mathcal{U}_0}{g}\right)^2 \sigma\, a^2 \cdot \sin 2\phi \\ &= -2.4 \cdot 10^{20} \sin 2\phi \ \text{erg/sec}\end{aligned}}$$

Love numbers

height of equil. tide

radius

$\frac{2\pi}{\sigma} = 12.42$ hrs

(for M_2 tide only)

A rough estimate of the order of magnitude of the expected phase lag of the tidal bulge is

$$\boxed{\tan 2\phi \sim 2\phi \sim \sin 2\phi \sim \frac{1}{2Q}}$$

where $2\pi Q^{-1}$ is the fractional energy lost per cycle during an elastic oscillation. Q can be estimated from the rate of decay of the ⊕'s free oscillations.

A suitable value in this context would be the Q of the ${}_0S_2$ mode (also $l = 2$ deformation), not well determined but of order $Q \sim 500$.

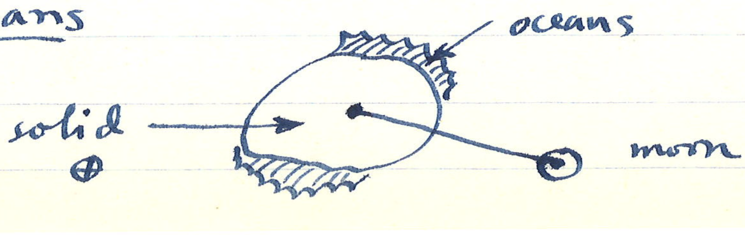
This implies the phase lag is of order $\phi \sim 0.03°$, very small, cannot be detected, obscured by load tides (actually <u>instrumental</u> phase lags are different but are of this order).

Thus:

$$\dot{E}_{\text{solid } \oplus} \sim -2.4 \cdot 10^{17} \text{ erg/sec}$$

less than 1% is dissipated in solid $\oplus$ by anelasticity, even less is dissipated in ☾

The only remaining sink is the <u>oceans</u>



Can this be verified?

The existence of <u>numerical tidal models</u> makes this possible. These give the theoretical ocean tide ζ at all points in the oceans. The rate at which the tidal forces are doing work to produce these tidal variations can be calculated by integration, either over the oceans as a whole or merely along coasts, depending on nature of tidal model.

Estimates of $\dot{E}$ from various tidal models are:

	tide	$\lvert\dot{E}\rvert$ (10^{19} erg/sec)
Accad + Pekeris	M_2	2.6 + (includes self-grav, others dont)
Zahel	M_2	4.3
Hendershott	M_2	3.5
Bogdanov + Magarik		3.5
Accad + Pekeris	S_2	0.5 + (also includes self-grav.)

Accad + Pekeris $M_2 + S_2$ combined gives

$$\boxed{\dot{E} = -3 \cdot 10^{19} \text{ erg/sec}}$$

General feeling is that remaining tides including diurnal can account for remainder and that essentially all tidal energy is dissipated in oceans.

The exact mechanism is unclear. Some must be by bottom drag in shallow seas. This fraction can be independently estimated by measuring tidal hts + tidal currents at inlets. Allows one to compute net flux of energy into shallow seas. This done by Miller (1965). See his figure 1. Finds total

$$\boxed{\dot{E}_{\substack{\text{shallow}\\\text{seas}}} = -1.7 \cdot 10^{19} \text{ erg/sec}}$$

this thought by many to be an upper bound.

About 1/4 in Bering Sea + Sea of Okhotsk.

TABLE 1. A Comparison of Jeffreys' and Heiskanen's Estimates of Frictional Dissipation with Miller's Estimate of the Flux of Energy (Units are 10^{17} ergs/sec.)

Location	Jeffreys	Heiskanen	Miller	
Andaman Sea to 21°N		1.9		
Antarctica			0.1*	
Australia to Lesser Sunda Islands		15.	15.	
Barrier Reef	0.0	5.	2.4*	
Bay of Biscay	0.0	1.1	0.4	
Bay of Fundy	2.3	2.	2.3	
Bering Sea	75.	48.	24.	note
Bristol Channel		1.8	2.8	
Chile	0.0	4.	0.4	
Davis Strait and North		2.	2.	
East Africa	0.0	3.9	0.0	
Eastern Greenland	0.0	0.0	0.0*	
England, Scotland, Ireland (other)	0.5	3.4	0.8	
English Channel	5.	5.	5.	
Florida to Trinidad	0.0	0.0	0.3	
Formosa to Luzon	Small	3.7	0.5	
Gulf of California		2.5	4.	
Gulf of Panama to Gulf of California	0.0	0.9	0.6	
Gulf of St. Lawrence		0.0	0.8	
Hudson Strait	8.	1.1	12.	
Irish Sea	2.	2.	3.2	
Japan Islands	0.0	6.5	4.3	
Java to Sumatra	0.0	0.0	0.0	
Malacca Strait	7.	.6	7.	
Mediterranean		0.0	0.0	
Mindanao to N. Guinea		0.0	2.	
N. E. Coast of S. America	0.0	7.	5.	
North Sea	4.5	2.1	4.5	
Northern Bay of Bengal	0.0	3.2	6.0	
Norway to Svalbard		1.2	3.2	note
Okhotsk Sea	4.	0.0	21.	
Oman–Persian Gulf	0.0	1.85	1.6	
Philippines	0.0	0.0	0.1	
Red Sea	0.0	0.0	0.2	
Ryukyu Islands	5.5	10.8	6.	
S. E. Coast of S. America		10.	13.	
Southern Alaska		5.	5.	
Southern India	0.0	10.3	4.	
Torres Strait		0.0	0.3	
U. S. East Coast	0.0	0.0	0.0*	
Vancouver, Juan de Fuca		1.5	1.6	
West Africa		0.0	0.0*	
West S. America, equator to 42°S		0.0	0.0	
Western Australia		0.0	4.2*	
Western Norway	0.0	0.0	0.0*	
Western Spain		0.0	0.0*	
Western U. S. to Baja California	0.0	3.	0.4*	
Total	1.1×10^{19}	1.9×10^{19}	1.7×10^{19}	

* Values based on estimates of frictional dissipation.

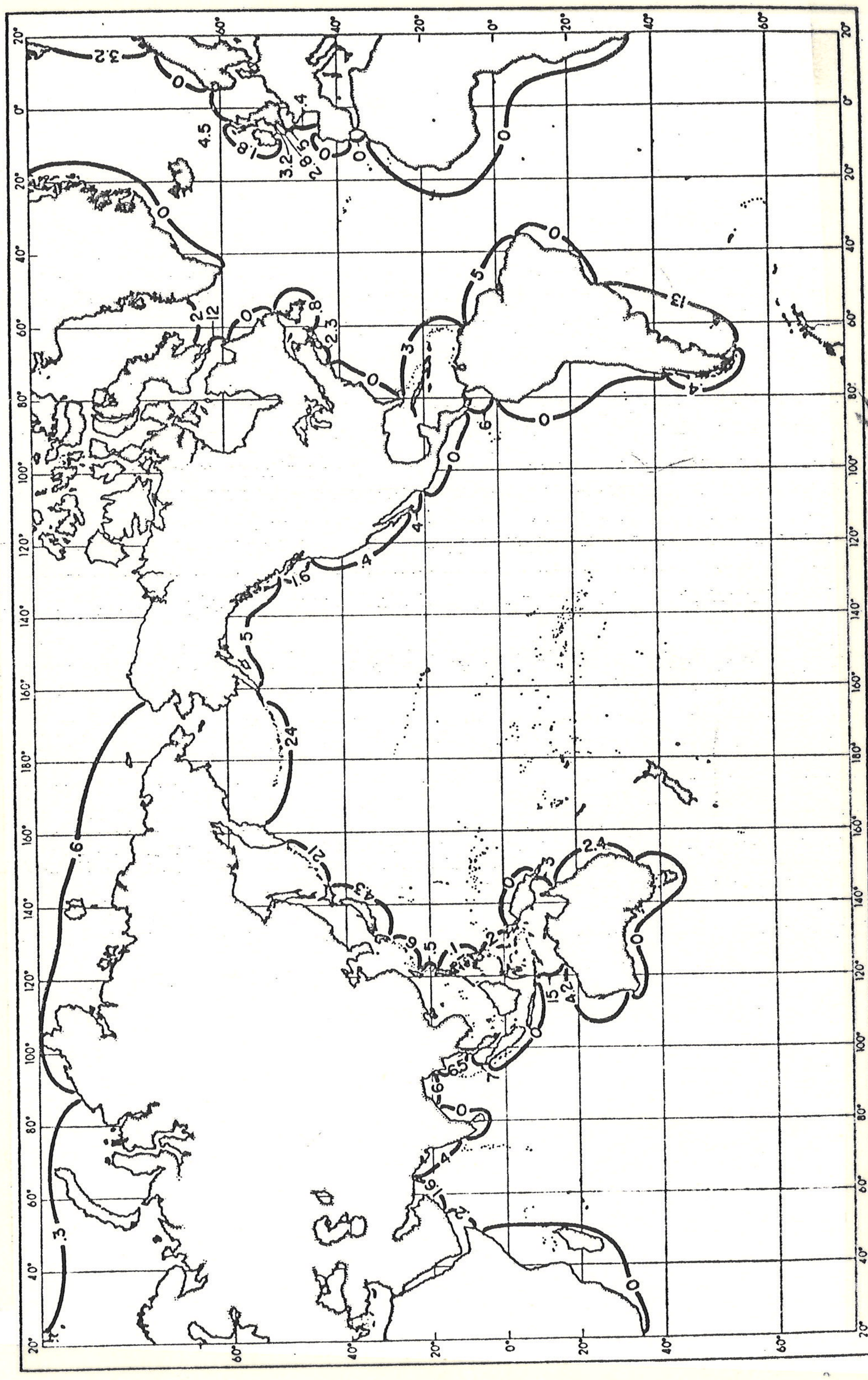


Fig. 1. Lunar tidal energy flux out of the deep ocean in units of 10^{17} ergs/sec.

Evolution of the ⊕☾ system:

The implications of tidal friction for the evolution of the ⊕☾ system and other planet-satellite systems is of some interest.

At the present time the ☾ is receding from the ⊕ at

$$\dot{R}_0 = 3.5 \text{ cm/yr}$$

If it had receded at that rate forever it would have been in contact 11 b.y. ago, i.e. at $t = -(R_0/\dot{R}_0)$ in the past. But the rate of recession was much more rapid in the past. The extent to which $\dot{R}$, the rate of recession, depends on R depends on the nature of the frictional process. Three cases may be distinguished:

Let us denote the amplitude of the tide by $\mathcal{G}$. We know that

Variation of tidal force with distance

$$\boxed{\mathcal{G} \sim R^{-3}}$$

$\boxed{\dot{E} \approx \dot{E}_\phi = \frac{d}{dt}\left(\frac{1}{2} C\Omega^2\right)}$

nature of friction	dependence
	this assume phase lag ϕ is const.
linear	$\dot{E} \sim \mathcal{G}^2 \Omega$
"unsaturated" quadratic	$\dot{E} \sim \mathcal{G}^3 \Omega$
"saturated" quadratic	$\dot{E} \sim \mathcal{G}^{3/2} \Omega$
	freq. of tide

best way to present: $\dot{E} \cong C\Omega\dot{\Omega}$ and $C\dot{\Omega} = \frac{1}{3} M_{\mathbb{C}} R^2 \dot{n}_{\mathbb{C}}$ so $\dot{E} \sim \Omega R^2 \dot{n}_{\mathbb{C}}$ $\sim \Omega \dot{R} / R^{1/2}$

Thus $\dot{E} \sim \Omega R^{-j}$, $j = 4.5, 6, 9$ for sat. quad., linear, unsat. quad. friction

Now from $\boxed{\begin{aligned} \dot{E} &= -N\Omega \\ N &\approx -\frac{1}{3} M_{\mathbb{C}} R^2 \dot{n}_{\mathbb{C}} \end{aligned}}$

$\boxed{\dot{E} \sim \Omega R^{-6} \text{ for const. } \phi}$

and $n_{\mathbb{C}}^2 R^3 = \text{const}$, Thus

$\dot{n}_{\mathbb{C}} \sim n_{\mathbb{C}} (\dot{R}/R)$ and $\boxed{n_{\mathbb{C}} \sim R^{-3/2}, \text{ so } \dot{n}_{\mathbb{C}} \sim \dot{R}/R^{5/2}}$

$\dot{E} = \frac{1}{3} M_{\mathbb{C}} R^2 \dot{n}_{\mathbb{C}} (\Omega - \dot{n}_{\mathbb{C}}) \approx \frac{1}{3} M_{\mathbb{C}} R^2 \dot{n}_{\mathbb{C}} \Omega$

we find that $\boxed{\dot{R} \sim \Omega^{-1} R^{1/2} \dot{E}}$

write this as $\dot{E} \sim \Omega R^{-1/2} \dot{R}$

$$\boxed{\dot{R} \sim R^{1/2 - j} \quad \text{with } j = 4\tfrac{1}{2}, 6 \text{ or } 9 \text{ for sat. quad., linear, and unsat. quad. friction}}$$

$\boxed{\dot{R} \sim R^{-11/2}, \text{ linear friction.}}$

Thus the <u>weakest</u> dependence is

$$\dot{R} \sim R^{-4}$$

and the <u>strongest</u> $\dot{R} \sim R^{-17/2}$

For <u>linear</u> friction $\dot{R} \sim R^{-11/2}$ or $R^{11/2}\dot{R} = \text{const.}$

In any case a very strong dependence, rate of recession much more rapid in past.

In the linear friction case, then, we know that $R^{11/2}\dot{R} = \text{const.}$

Integrating we find

check: $\frac{13}{2} R^{11/2} \dot{R} = \frac{13}{2} R_0^{11/2} \dot{R}_0$

$$R^{j+1/2} = R_0^{j+1/2} + (j+1/2)\, \dot{R}_0\, R_0^{j-1/2}\, t$$

R_0 = present distance

$R^{13/2} = R_0^{13/2} + \frac{13}{2} \dot{R}_0 R_0^{11/2} t$ $\dot{R}_0$ = present rate

for linear friction case (<u>present</u> refers to $t=0$)

$R = 0$ when $t = -\frac{1}{j+1/2}\left(\frac{R_0}{\dot{R}_0}\right)$ $\frac{2}{13}$ this the time at const. rate $= 11$ b.y.

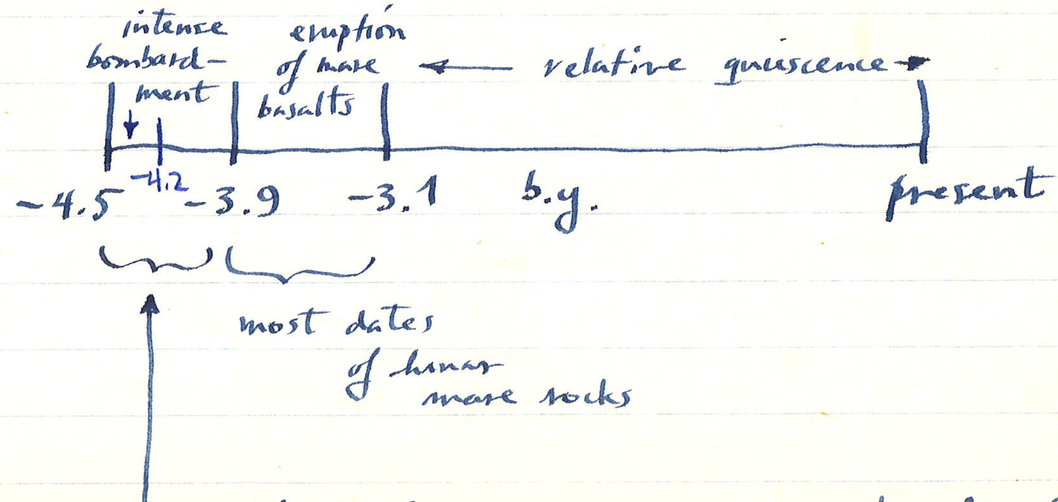
linear

$= -2.3$ b.y., $\boxed{-1.8 \text{ b.y.}}$, -1.2 b.y. for three cases

A straightforward extrapolation of tidal friction into the past thus predicts that the ⊕ and the moon would have been in close approach 1–2 b.y. ago.

There is an abundance of <u>geological evidence</u> which belies this. Particularly persuasive is the geological history of the ☾, now known from dating of lunar samples.

The moon of about same age as ⊕ $\sim 4.5 \cdot 10^9$ years. For first $\sim 0.3 \cdot 10^9$ years underwent large scale differentiation, formed crust. lunar highlands
Intense bombardment continued from $\sim 4.5 \cdot 10^9$ to $\sim 3.9 \cdot 10^9$.

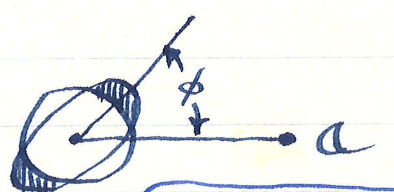


xtallization ages of lunar highland rocks can be found in this range.

This is good evidence that ⊕ and ☾ have not been terribly close during last 3 m.y.

This discrepancy is known as the "<u>time-scale</u>" problem. Numerous people have done the extrapolation, notably Gerstenkorn and Goldreich, and all find ~ same result viz. within Roche limit between 1-2 b.y. ago.

The calculation assumes that the <u>friction</u> has remained constant, only the amplitude of the tides has increased (<u>the lag angle ϕ is fixed</u>)



More specifically we've assumed the ht. of the tides just varies like R^{-3}.

We have seen that the dissipation <u>all</u> occurs in the oceans and the configuration of the oceans has changed with time because of sea-floor spreading.

The most obvious solution to the "time scale problem" is that tidal friction (in particular the lag angle ϕ) must have been less in the past, or rather that ϕ must not have varied just like R^{-3}.
Calculations show that if the current friction is ~ 3 times greater than the average in the past the time scale problem can be avoided, close approach can be pushed back to ~ 4 – 4.5 b.y. ago.

Is there any reason to think this might be the case?
Such a change could be attributed to a change in the configuration of continents. The period of an ocean basin oscillation is about, roughly,

$$\boxed{T_{\substack{\text{natural}\\ \text{period}}} \sim \frac{2L}{\underbrace{\sqrt{gH}}_{\text{speed of deep } H_2O \text{ wave}}}}$$

size of basin

depth

<u>Overall conclusion</u>: essentially all dissipation in oceans by some unknown mechanism. Used to be thought ~ all in shallow seas. Now thought that much of dissipation may occur in deep oceans, perhaps by scattering off topography into internal waves.

It is known that only the $l = 2$ part of the complicated co-tidal charts contributes to the dissipation since it is the inner product of ζ with $\bar{\zeta}$ which counts.

$$\sqrt{gH} \sim \sqrt{10^3 \cdot 4 \cdot 10^5} \sim 2 \cdot 10^4 \text{ cm/sec}$$

200 m/sec

The "typical" scale of current ocean basins is, say,

$L \sim 5000$ km. If so

$$T \sim \frac{2 \cdot 5 \cdot 10^8}{2 \cdot 10^4} \sim 5 \cdot 10^4 \text{ s}$$

$$\sim \frac{5 \cdot 10^4}{3.6 \cdot 10^3} \sim \underline{\underline{12 \text{ hrs}}}$$

Also $T_{forcing} = \frac{\text{l.o.d}}{2}$ shorter in past

The ocean basins are near M_2 resonance at the present time. This near resonance condition leads to amplified tides and as a result amplified friction.

At the time of Pangea, which broke up ~ 200 m.y. ago the scale of the ocean basins was much larger, also the l.o.d. was shorter and resonance would be less. this (tidal periods were shorter) now thought to be main factor.

Brosche and Sundermann have attempted to model the M_2 tide for the <u>Pangea configuration</u> 230 b.y. ago, requires an assumption about paleo <u>depths</u> H, clearly a dubious undertaking but their result was a dissipation rate <u>about 1/3 the present</u> which is consistent with the above reasoning.

Paleontological clocks

There have been attempts by several people, starting with Wells (1963) to extrapolate the astronomical observations into the past by using growth lines in corals and bivalves and stromatolites.

Devonian rugose corals studied by Wells show daily and annual and possibly monthly growth lines. Presumably the feeding cycle is somehow tidally controlled.

Found ~ 400 daily growth increments per annual and ~ 31 per monthly band — these presumed to be no. of days in year and no. of days in month.

Since the length of the year itself changes only slowly 400 d / year ⇒ l.o.d. in Devonian ~ (365/400) · 24 hr.

Thus length of day in Devonian ~ 22 hrs, this ~ 390 m.y. ago.

Many other such studies have been done. Fig. 3 from Munk (1968) compares some of these studies with <u>constant phase lag extrapolations</u> for the three cases of friction.

Data not good enough to distinguish nor really long enough time base to see if $\langle \phi \rangle$ in past $<$ current ϕ.

Attempts have been made to go back into Precambrian using <u>stromatolites</u> "organo-sedimentary structures produced by an interaction between the growth and metabolic activity of microorganisms (mainly blue-green algae) and sedimentation processes". Lambeck (p. 358) says "Possibly we are asking too much of these organic structures if we expect them to lead us to the origin of the ☾; and if only they were aware of what geophysicists are trying to read into them today, surely they would have adopted quite different living habits!"

Figs. 2+3 Lambeck shows no. of days per synodic month (new ☾ to full ☾ to new ☾) from various sources, chiefly corals and bivalves.

also per year

Note no. days / month greater in past because day shorter even though month was shorter too.

Recall $\dot{\Omega} \sim 50''/a$ today

It is not clear how much stock should be placed in any of this paleontological data — the counting is very subjective for one thing — and the underlying assumption of one per day etc. is unproved — studies are ongoing with living specimens.

Everything we know seems to indicate there must have been a Gerstenkorn event some time in the past — very high tides — a close moon — a short day.

Referred to as an event

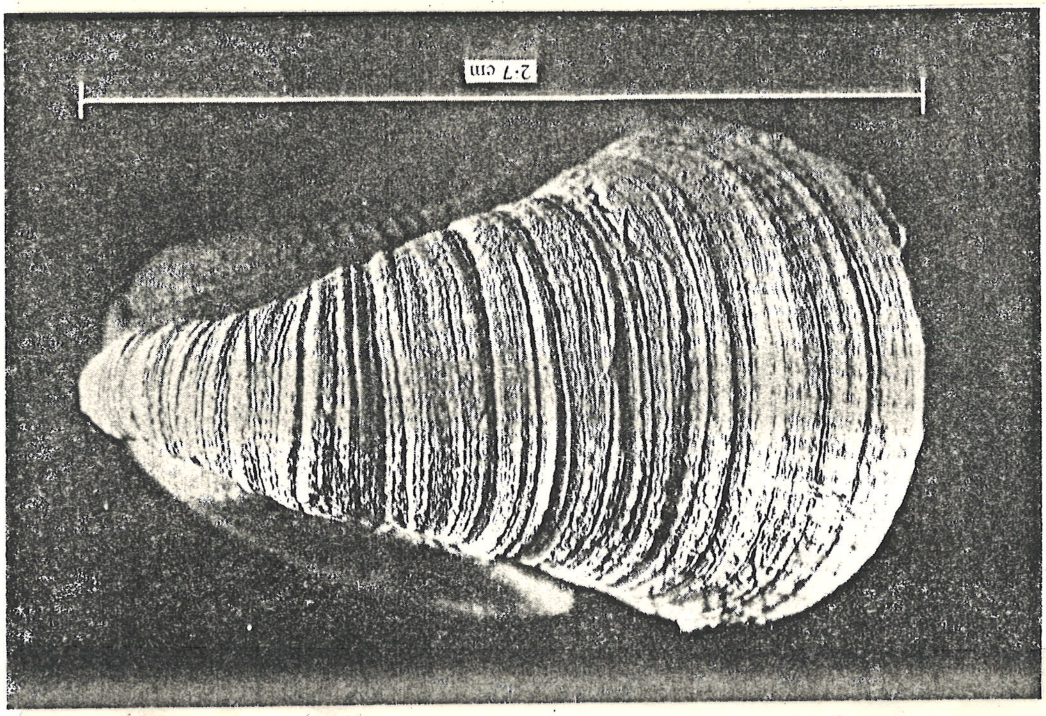


Figure 11.6. Middle Devonian coral epitheca from Michigan, U.S.A., illustrating 13 well-developed bands, each with an average of 30.8 ridges (supplied by C. T. Scrutton).

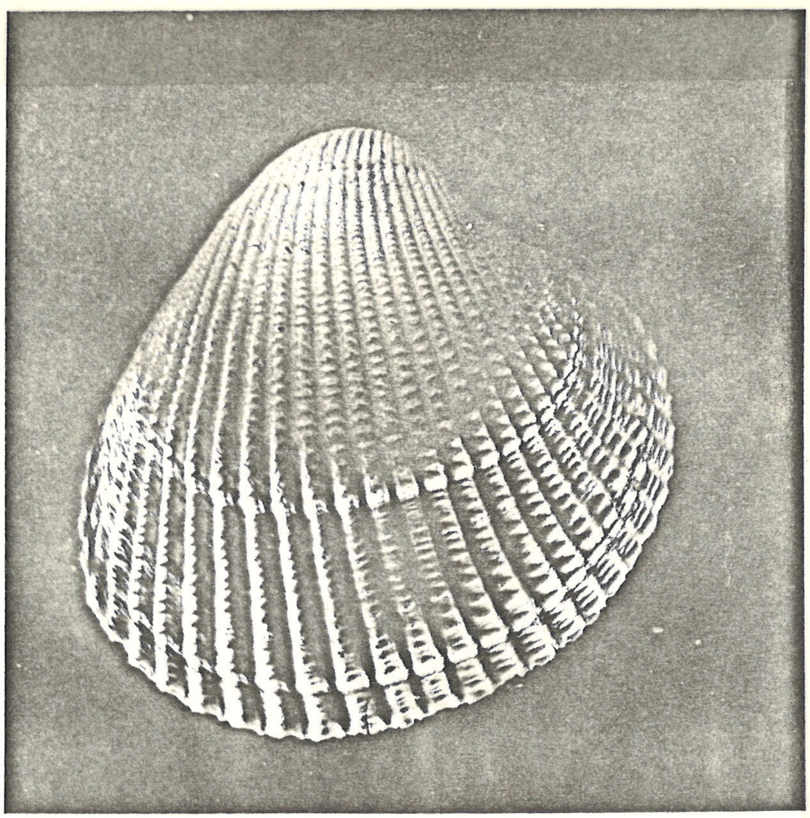


Figure 11.7 Bivalve *Clinocardium nuttalli* showing the external growth ridges (from Evans 1975).

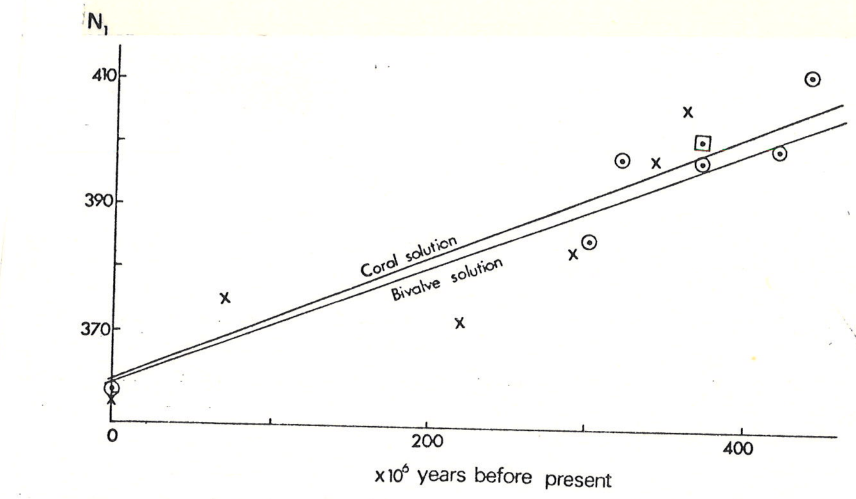


Fig. 2. Estimates of N_1 according to Wells (⊙), Scrutton (⊡), and Pannella (x). One line represents the coral solution (4), the other the bivalve solution (5)

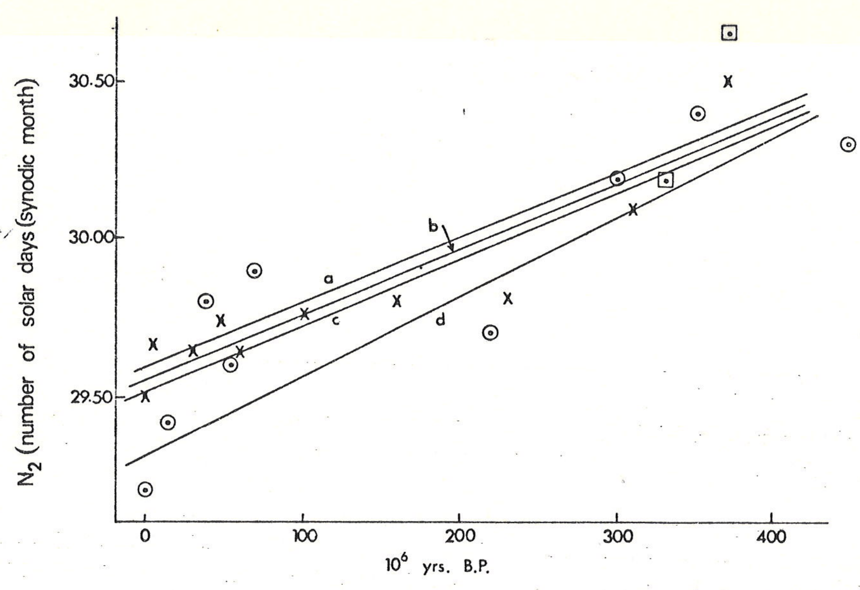


Fig. 3. Estimates of N_2 according to Pannella (⊙), Berry and Barker (x) and the coral data (⊡). The four lines represent $N_2(t)$ based on (*a*) solution (4) of coral data, (*b*) an unweighted best fit to Berry and Barker's observed data, (*c*) an unweighted best fit to Pannella's observed N_2 data, and (*d*) solution (5) from all of Pannella's bivalve data

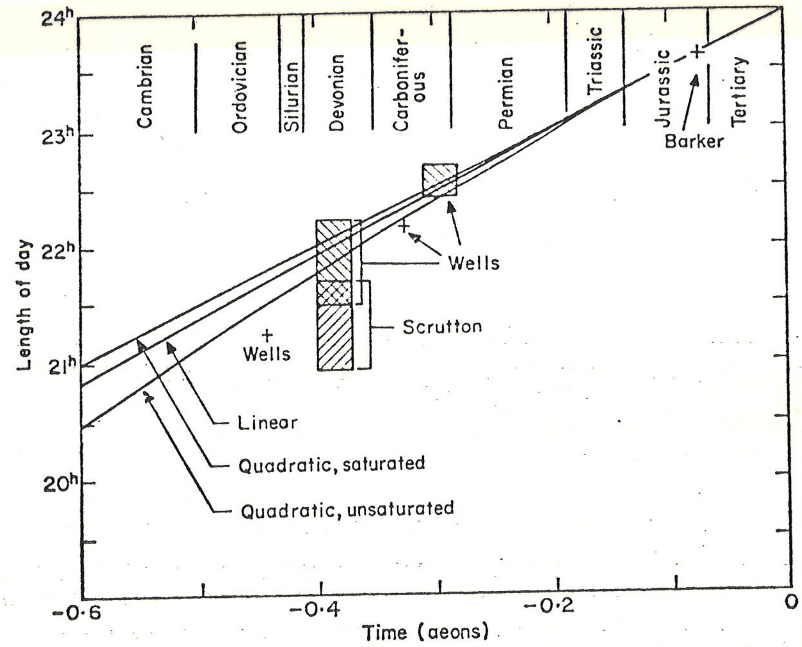


FIG. 3. The paleo length-of-day from coral growth lines. Rectangles are adapted from Lamar & Merifield (1967), based on observations by Wells (1963) and Scrutton (1964), as discussed by Runcorn (1964). Crosses show preliminary results of recent work by Wells and by Berry & Barker (1968). Lines are drawn for the cases of saturated (j=4·5) and unsaturated (j=9) quadratic friction, and for linear friction (j=6).

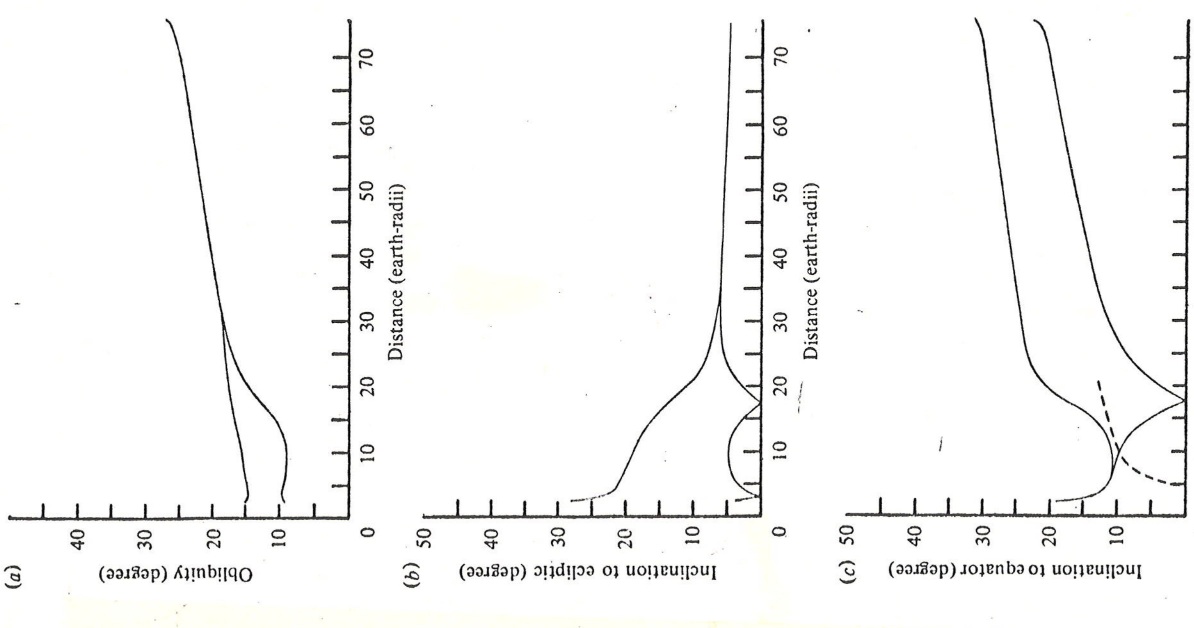


Figure 11.4. Variations in (a) obliquity of the Earth's equator to the ecliptic, (b) inclination of Moon's orbit to the ecliptic, and (c) inclination of the Moon's orbit to the Earth's equator, for ε_{2mpq} = constant (from Goldreich 1966). Rubincam's results are indicated by the broken line (section 11.4).

because of extreme dependence of rates on distance R, things were happening very quickly.

When the extrapolations into the past assuming $\phi =$ const. are done including changes in the inclinations and obliquities one always finds that at close approach the inclination of the lunar orbit $\rightarrow$ polar, must have been non-equatorial at say 10 $\oplus$ radii, at least. This a significant constraint on theories of lunar origin. Difficult to envision fission from $\oplus$ as origin given this.

Giant impact theory is now in favor.

Fig. 11.4 Lambeck shows Goldreich's integration plotted against distance R. Generally thought that such curves meaningful in spite of time-scale problem, but this not clear. The two branches are limits of oscillation during a precession period, 25,500 yrs.

From Munk: "Will you attempt to visualize circumstances surrounding the Gerstenkorn event? A heavy hot atmosphere over a darkened earth. Giant tides on a 5 hr day, with steaming tidal bores following the moon on a 7 hr polar orbit. Mr. Gerstenkorn may not appreciate having his name attached to this era."

When this may have been is a matter of speculation. That it was is less speculative but not certain.

Giant impact theory now more popular

A popular hypothesis for lunar origin is capture from a high inclination close approach orbit followed by subsequent evolution to current state. Chief argument against: apparent difficulty in slowing an object down enough to capture it.

What about the future evolution?

We consider again the conservation of angular momentum. For simplicity let us make use of $M_☾ \ll M_\oplus$. Let $t = 0$ be now. Then

$$H_{\oplus ☾} = C\Omega + M_☾ R^2 n_☾ = \text{const.}$$

Current value of const. can be written as

$$H_{\oplus ☾} = (1 + K)\, C\Omega_0$$

current spin rate

ratio of current orbital ang. mom. to $\oplus$ ang. mom.

$$K = \frac{M_☾ R_0^2 n_{☾0}}{C\Omega_0}$$

$$K = 4.912$$

$$\boxed{H_{\oplus ☾} = 5.912\, C\Omega_0}$$

Thus

$$C\Omega + M_☾ R^2 n_☾ = 5.912\, C\Omega_0$$

Let us consider the situation where $\Omega = n_☾$. The lag angle will in that case vanish ($\oplus$ will keep same "face" toward ☾) and tidal friction + evolution will cease.
From Keplers third law $n_☾^2 R^3 = GM_\oplus$

$$R^2 = (GM_\oplus)^{2/3}\, n_☾^{-4/3}$$

Thus

$$\left[C + M_{☾} \frac{(GM_\oplus)^{2/3}}{\Omega^{4/3}} \right] \Omega = 5.912\, C\, \Omega_0$$

$$\Omega\,(5.912\,\Omega_0 - \Omega)^3 = \frac{(GM_\oplus)^2 M_{☾}^3}{C^3}$$

$$\frac{(GM_\oplus)^2 M_{☾}^3}{C^3 \Omega_0}$$

$$\boxed{\frac{\Omega}{\Omega_0}\left(5.912 - \frac{\Omega}{\Omega_0}\right)^3 = 4.240}$$

↑ a quartic for Ω

Two solutions which are real:

$$\boxed{\begin{array}{ll} \Omega/\Omega_0 = 4.96 & (4.8 \text{ hrs}) \\ \Omega/\Omega_0 = 0.021 & (48 \text{ days}) \end{array}}$$

currently 60

These occur at 2.4 and 86.4 $\oplus$ radii. The former is inside the Roche limit of ~ 3 $\oplus$ radii and is probably not meaningful. The latter would in the absence of the sun be the final state of the $\oplus$☾ system. With ϕ = current value it would occur ~ 14 b.y. in the future.

Actually the ⊙ will continue to exert tidal friction transferring ⊕☾ <u>orbital</u> ang. mom. to orbital motion about ⊙ and ☾⊕ distance will very slowly approach ⊕ and coalesce. This however will require several b.y. more.

This scenario may be of interest for other planets, may be reason for absence of Mercurian or Venusian satellites.

Reason ☾ keeps same face toward ⊕ is tidal friction there in past (recall 200. greater than tides on ⊕) has despun any initial rotation completely. Same true for Galilean satellites of Jupiter, Io etc. The extreme tidal heating of Io is due to the eccentricity of its orbit.

Mercury is also interesting:

$$\frac{2\pi}{\Omega} = 59 \text{ days}$$

$$\frac{2\pi}{n} = 88 \text{ days} = \frac{3}{2}(59 \text{ days})$$