

## Rotation of the Earth : introduction

A rich and fascinating subject,  $\exists$  numerous irregularities in  $\oplus$ 's rotation, convenient to divide into 3 categories:

1. changes in length of day (l.o.d.) due to internal causes and due to tidal friction of  $\alpha$  and to lesser extent  $\odot$ .
2. precession : due to external torques, change in orientation of  $\underline{\Omega}$  axis in space.
3. wobble : wobbling of solid  $\oplus$  about its rotation axis, due to internal causes.

Briefly examine nature, cause and measurement of each of these.

1. Changes in l.o.d. : a slowing down or speeding up of rate of rotation. Modern measurements compare star transit times with time measured by uniform clocks. Early astronomers ~~used~~ (20<sup>th</sup> century) used pendulum clocks.

Prior to this I observations based on Ephemeris time, this uses planetary motions to define a uniform time scale, can compare time based on  $\oplus$  rotation with this.

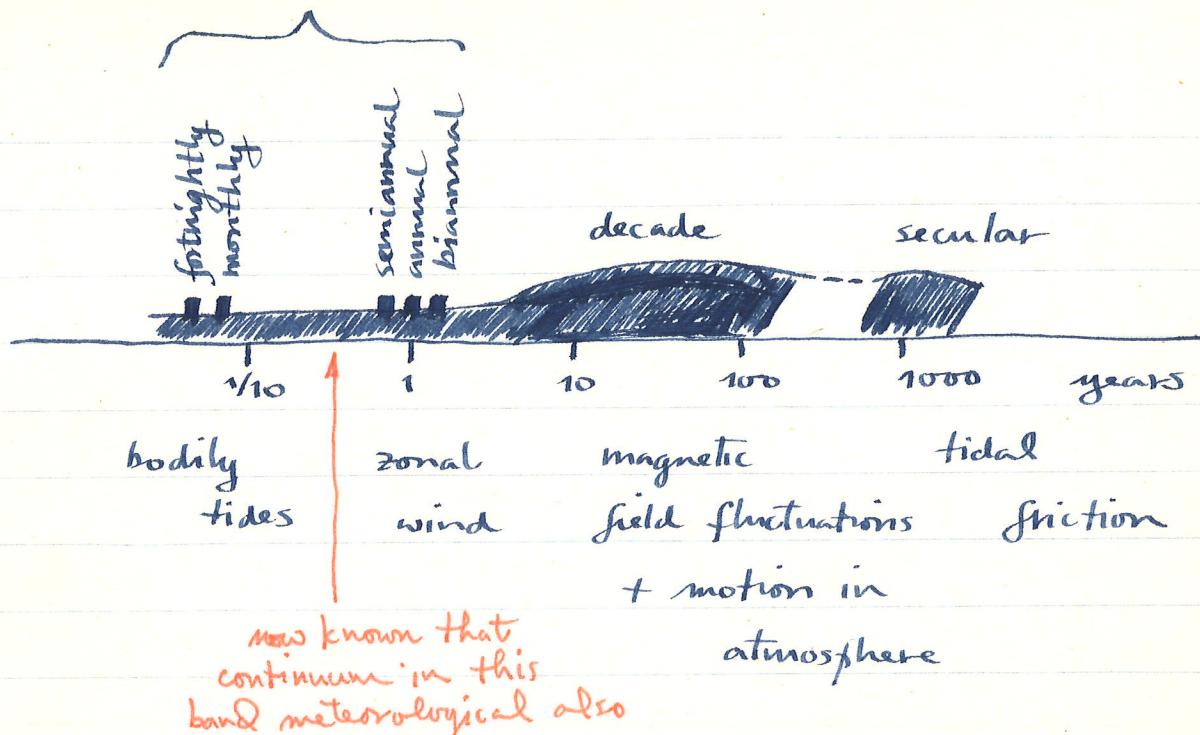
~ 1950's quartz crystal clocks came into use, tend to drift over periods of a few years.

Modern observations use atomic frequency standards (cesium beam clocks) which are drift free and accurate to 1 part in  $10^{10}$ .

Timekeeping and astronomy done by several groups: NBS, US Navy, BIPM in Paris.

Observations best presented in a spectrum, a continuous spectrum with a few lines superposed. Schematically looks like:

lines above continuum



Two common ways of showing the data. Let  $\bar{\Omega} \equiv \underline{\text{mean rotation}}$  rate of  $\oplus \equiv 7.292115 \cdot 10^{-5}$  rad/sec  
 Let  $\Omega(t) \equiv \underline{\text{instantaneous}}$  angular velocity of  $\oplus$ . The quantity often considered is then

$$m_3 \equiv (\Omega - \bar{\Omega}) / \bar{\Omega}, \text{ the fractional change in angular velocity.}$$

This is related to the change in the l.o.d. by

$$m_3 = -\Delta(\text{l.o.d.}) / \bar{\text{l.o.d.}}$$

The quantity actually observed is  $\tau$ ,  
 the amount by which the  $\Theta$   
is slow after a time interval  $t$ ,  
 given by ~~.....~~

$$m_3 = - d\tau / dt \quad \text{or}$$

$$\tau(t) = \tau(0) - \int_0^t m_3(t) dt$$

$\tau$  is an angle but measured in seconds.

The fortnightly, monthly and a fraction of the semiannual variations are due to bodily tides.

Tides deform solid  $\Theta$ , change principal moment of inertia  $C$ .  
 Shape of long-period tides (only) is  $P_2(\cos\theta)$  causes change in size of equatorial bulge.



$H = C\omega_3$  must remain constant (can change only by external torques)  
 i.e. l.o.d. = 24 hours ( $1 \pm 10^{-9}$ )

Tidal changes in l.o.d.  $\sim 1$  part in  $10^{-9}$  so  $\tau$  is of order 0.1 to 0.2 msec ( $10^{-3}$  sec).

Fig. 5.7 of Lambek shows time series of  $m_3(t)$  for 1968 and Fig. 5.8 shows spectrum of same record. Peaks at 14 and 27 days, of right amplitude to be of tidal origin are visible.

Most of semi-annual and all of annual and biannual changes caused by seasonal fluctuations in zonal winds (winds which blow from E to W around the  $\oplus$ ).

These blow against mtns, bushes, buildings, etc. and exert torque on  $\oplus$ . Or, alternatively, can take point of view that  $H_{atm} + H_\oplus$  must be conserved, if  $H_{atm}$  increases  $H_\oplus$  must decrease.



These ~~lines~~ are about 0.4 part in  $10^8$ ,  
~~so  $\tau$  is of order 20-30 ms~~  
 so  $\tau$  is of order 20-30 ms  
 (longer than tidal changes since  $\tau$  is cumulated amount  $\oplus$  is slow).

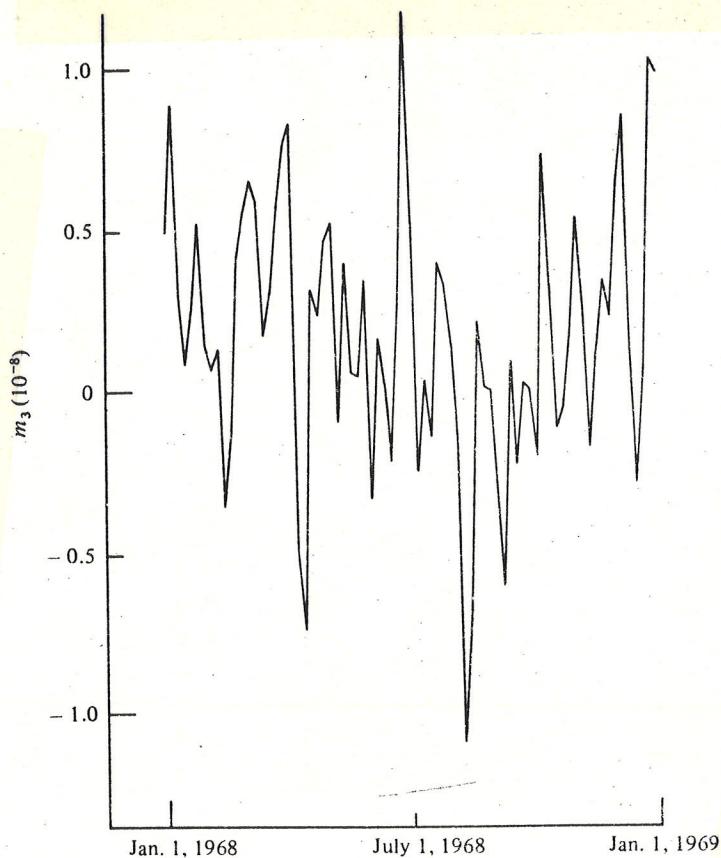


Figure 5.7. Time series of  $m_3$  for 1968 based on the unsmoothed values of  $\bar{\pi}$  published by the BIH.

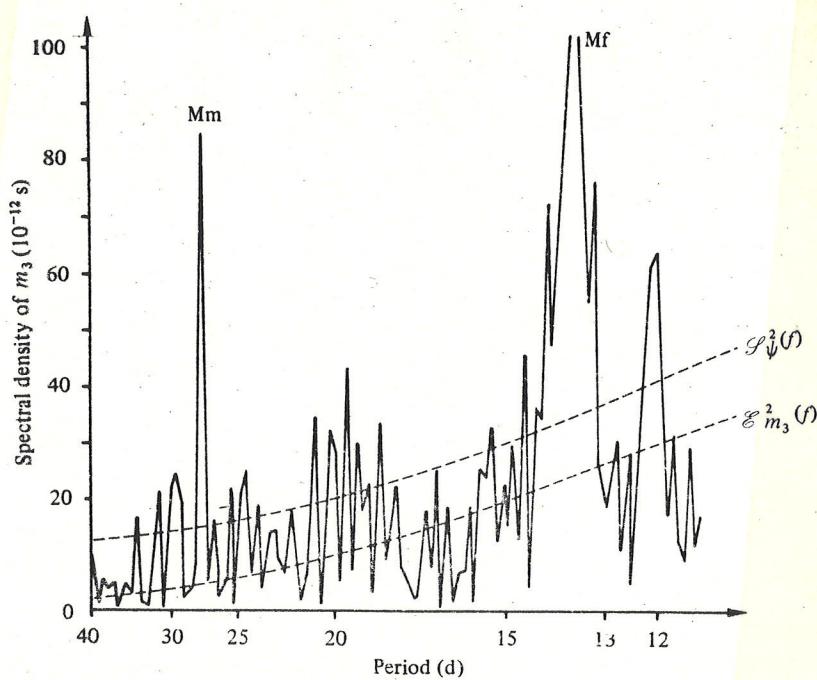


Figure 5.8. Unsmoothed amplitude spectrum of  $m_3$ . Tidal terms occur near 14 d (Mf) and near 27 d (Mm). The estimated error spectrum  $E_{m_3}(f)$  is based on  $\nu_{\bar{\pi}} = 1$  ms.  $S_\phi(f)$  is the estimated continuum of the meteorological excitation spectrum.

Fig 5.6 of Lambeck shows  $m_3(t)$  for period 1962 - 1978, annual and semiannual changes are apparent, Fig 1 from Lambeck + Cazenave shows spectrum of this data, peaks at 6 and 12 mo, plus energy at 2-3 yrs (quasi-annual).

$H_{atm}$  can be determined from measurements of wind velocity vs. altitude + latitude and compared with above.

Comparison for 1958 - 1962 shown Fig. 7.9 of Lambeck. Agreement excellent. Also recent evidence for good agreement with  $H_{atm}$  on shorter time scales (down to a few days). There are also irregular fluctuations with a time scale of decades. Most reliable determination since invention of crystal clocks 1955. Data after removal of annual + semi-annual lines shown Fig 5.5 of Lambeck, see fluctuations periods 6-8 years superposed on longer period (secular) changes.

No effect of  
El Nino in  
mid 1982

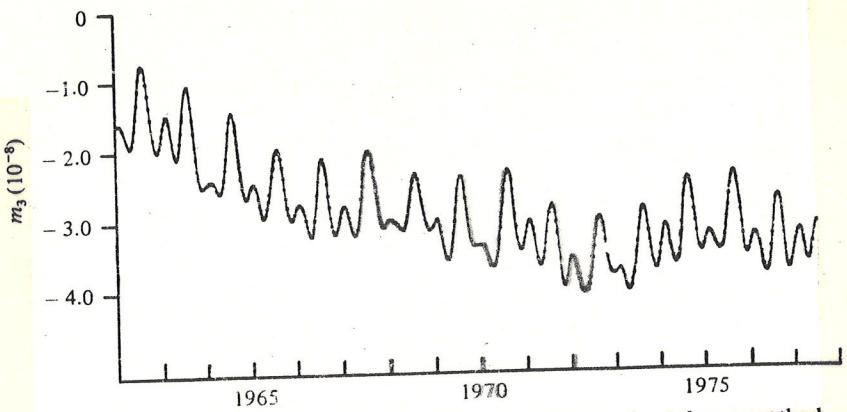


Figure 5.6. Time series of  $m_3$  from 1962 to 1977 based on the smoothed values of  $\bar{J}$  published by the BIH.

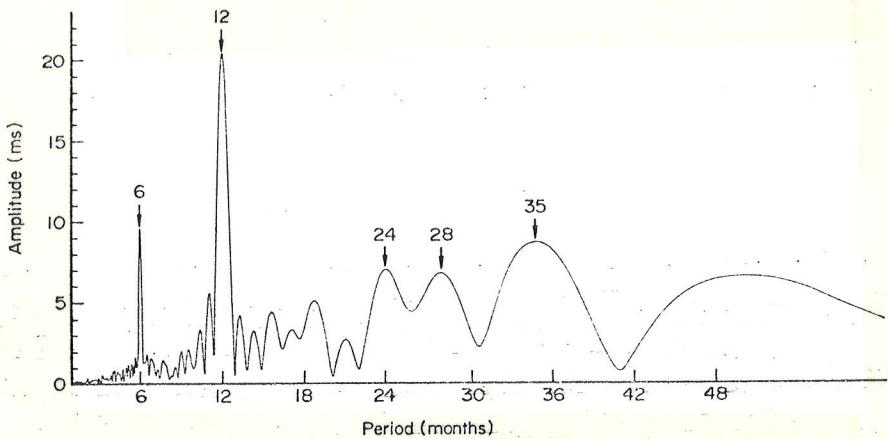
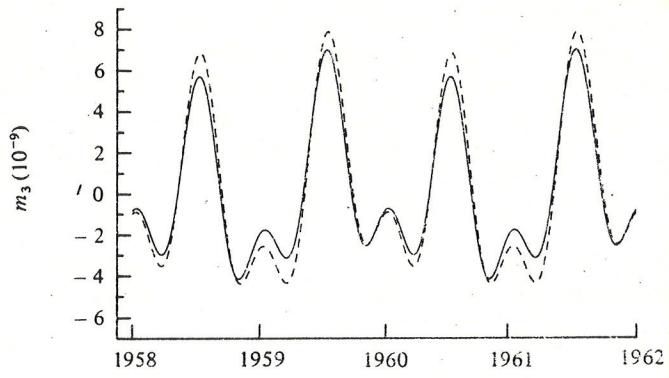


FIG. 3. Periodogram of the  $-\tau$  after the elimination of the long-period variations.

(a)



(b)

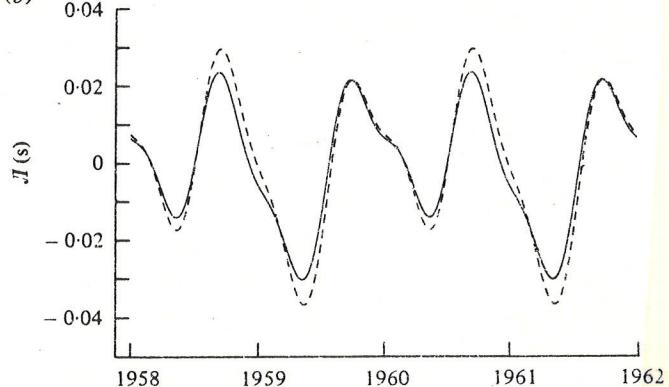


Figure 7.9. (a) Sum of the three seasonal excitation functions, compared with the astronomical estimate of  $m_3$ ; (b) comparison of the meteorological and astronomic estimates of the seasonal amounts by which the Earth is slow. Astronomic estimates are indicated by broken curves.

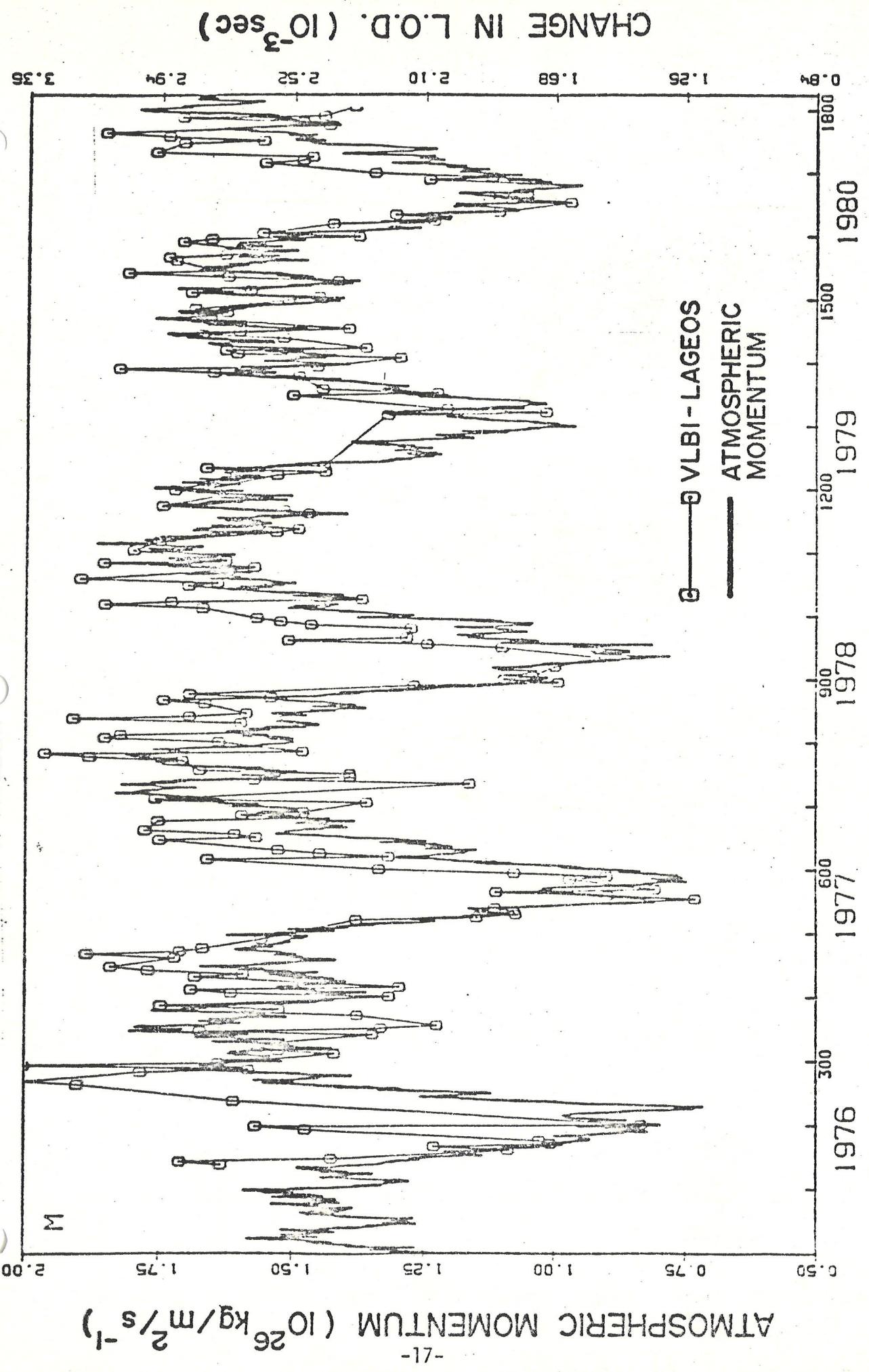


Figure 4

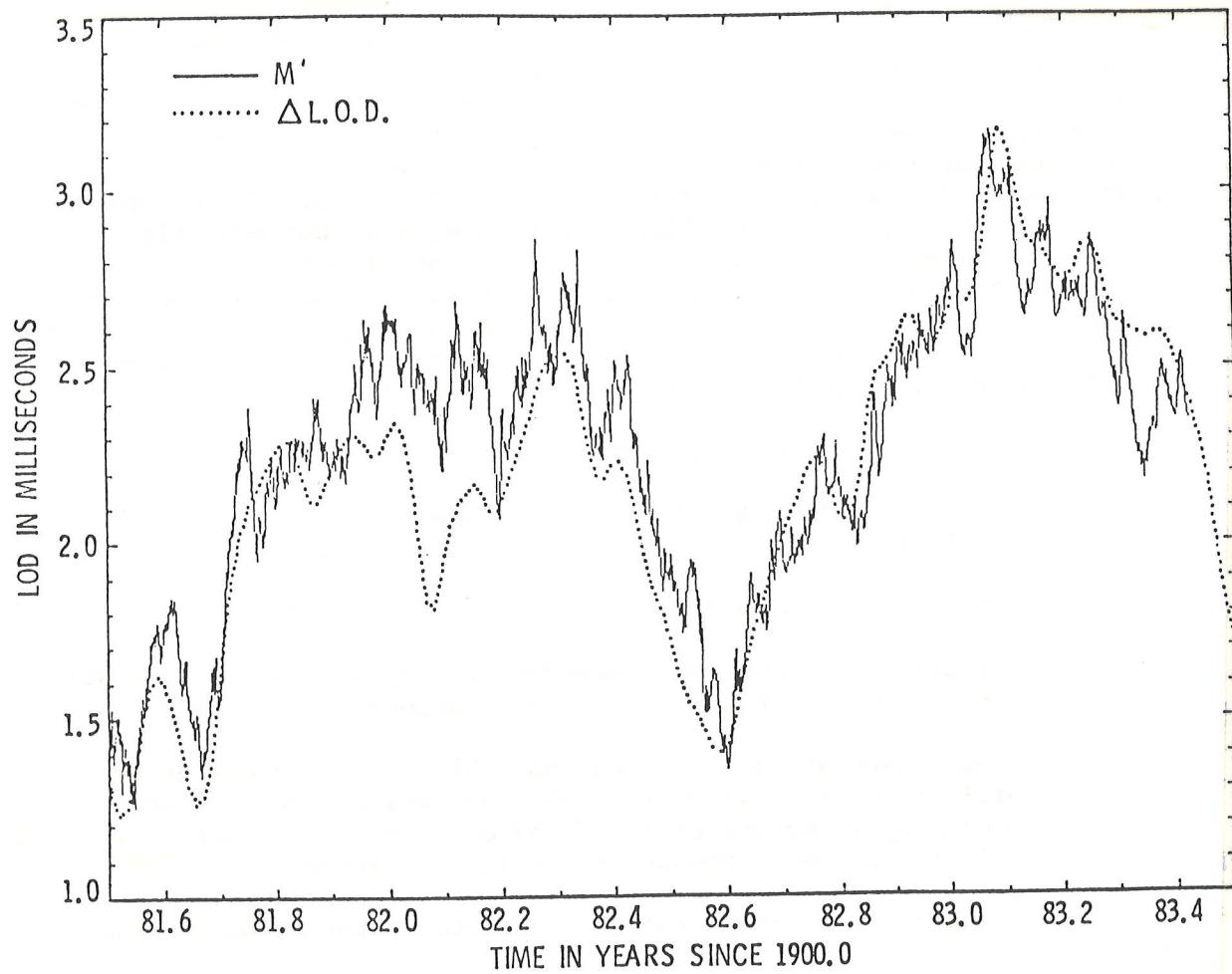


Figure 7. Change in Length of Day

Change in the length of day as determined from BIH data (dotted line) and as calculated on the basis of effects due to changes in the angular momentum of the atmosphere (solid line). (From Rosen, R. D., Salstein, D. A., Eubanks, T. M., Dickey, J. O., and Steppe, J. A. "An El Nino Signal in Atmospheric Angular Momentum and Earth Rotation," submitted to Science, February 1984).

Amplitudes few parts in  $10^{-8}$ .

Prior to 1955 data less reliable,  
based on comparisons with  
Ephemeris time.

$m_3(t)$  since 1820 as determined by  
independent analyses of Brower  
and Morrison Fig. 6 of Cazenave +  
Lambeck.

Variations, amplitude  $\sim 5 \cdot 10^{-8}$ ,  
corresponds to  $\Delta(\text{l.o.d.})$  of a  
few (3-4 ms). The observed  
quantity  $\tau$  is several tens of seconds  
because of the long period of the  
variations, this makes it possible  
to see sans clocks.

Some of this decade variation is  
probably due to decade climate  
variations, but principal cause  
thought to EM torques exerted  
on mantle by changing  $B(t)$   
in core.

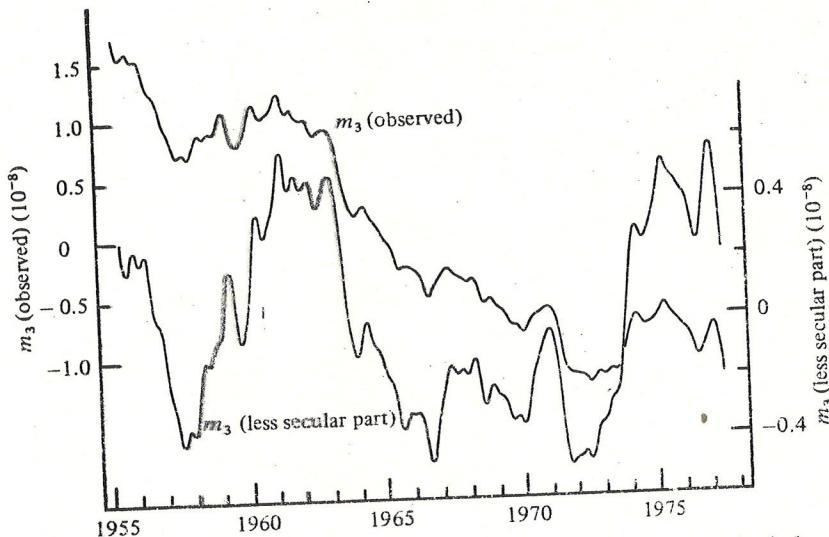


Figure 5.5. Time series of  $m_3$  from 1955 to 1977 after removal of the 6-month and 12-month oscillations and after some smoothing to remove high-frequency ( $>6$  cycle  $\text{yr}^{-1}$ ) fluctuations.

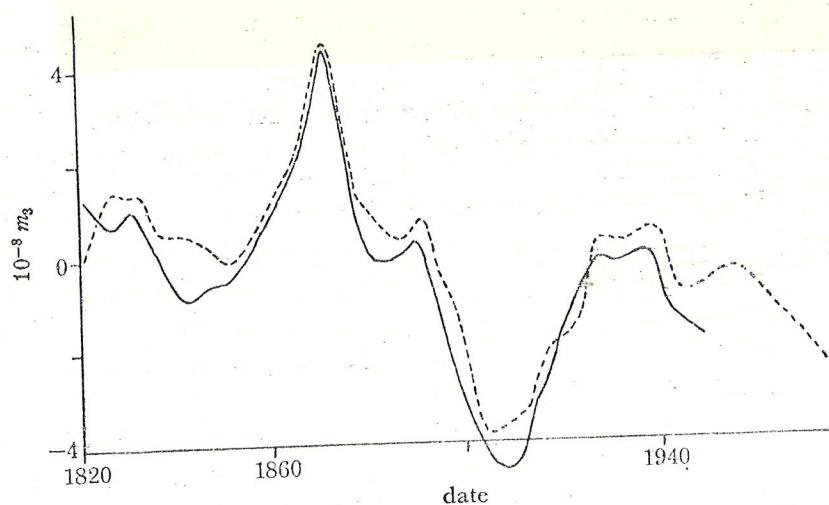


FIGURE 6. Long-period variations observed in  $m_3$  since 1820 as determined by Brouwer (—) and by Morrison (---).

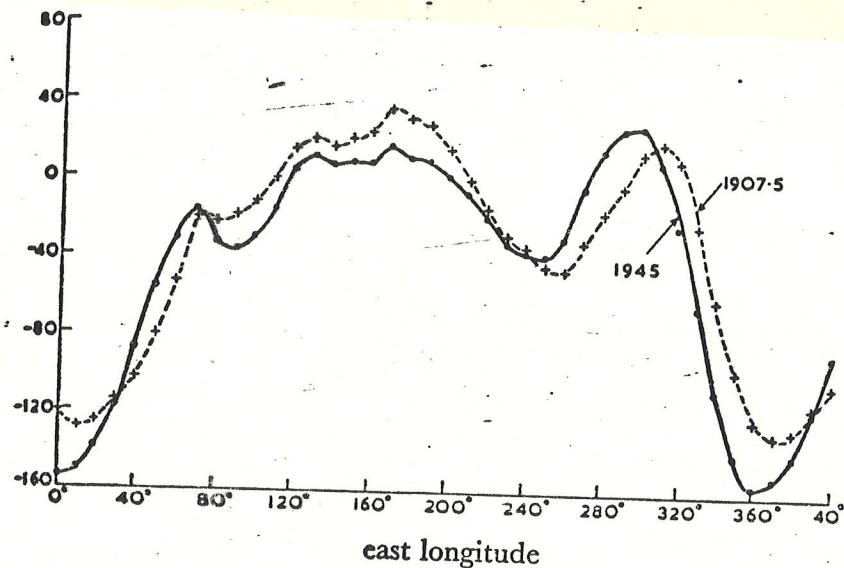


FIGURE 7. Vertical field on the equator.

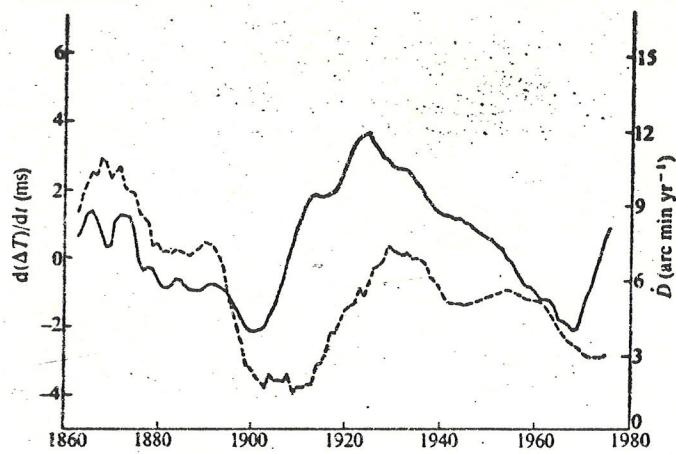


Fig. 1 Plot of the secular variation of declination  $\bar{D}$  (solid curve and right hand scale) and of the excess length of day  $d(\Delta T)/dt$  (dashed curve and left-hand scale) for the period 1861-1978.  $\Delta T$  data are from refs. 18, 19. The relative changes in the Earth rotation rate  $\Delta\Omega/\Omega$  are obtained from the  $d(\Delta T)/dt$  values through multiplication by  $-1.157 \times 10^{-8}$ .

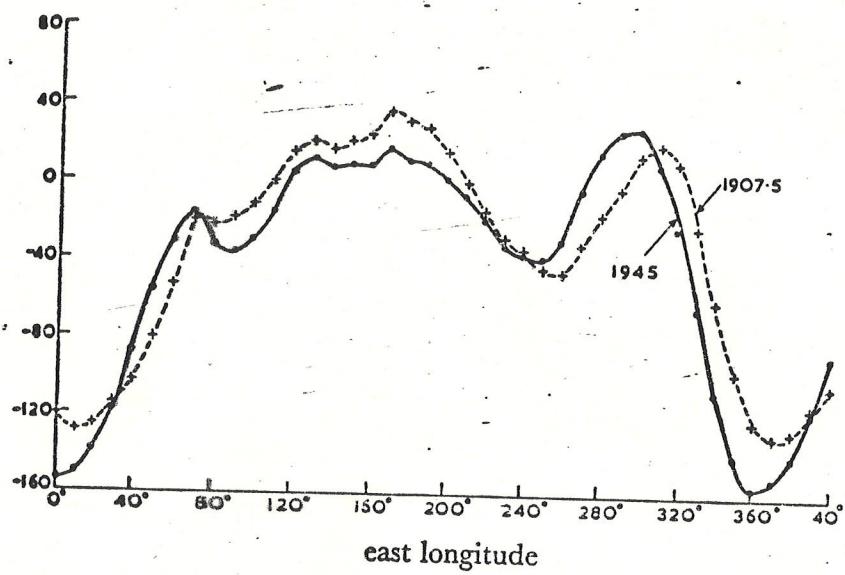


FIGURE 7. Vertical field on the equator.

westward drift of field  $\sim 0.2^\circ/\text{yr}$

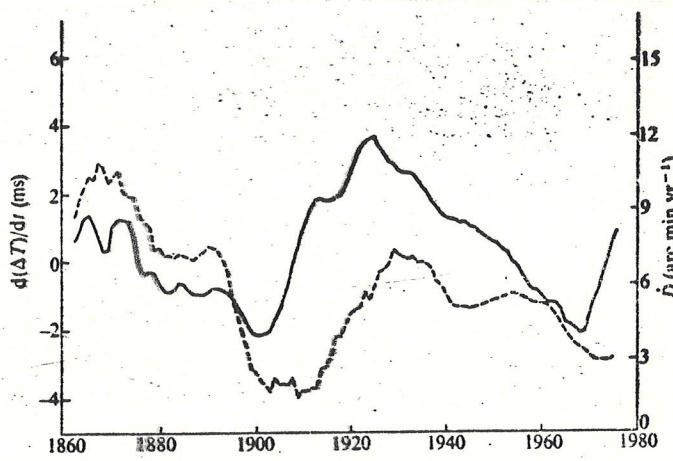


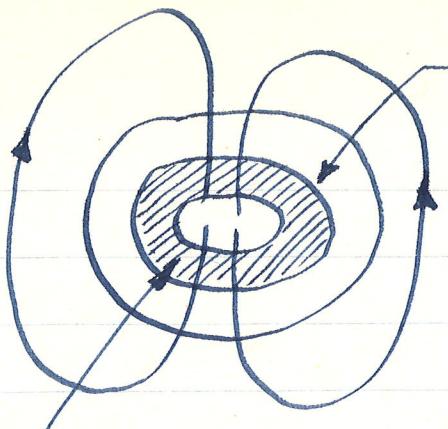
Fig. 1 Plot of the secular variation of declination  $\bar{\Delta}$  (solid curve and right hand scale) and of the excess length of day  $d(\Delta T)/dt$  (dashed curve and left-hand scale) for the period 1861–1978.  $\Delta T$  data are from refs. 18, 19. The relative changes in the Earth rotation rate  $\Delta\Omega/\Omega$  are obtained from the  $d(\Delta T)/dt$  values through multiplication by  $-1.157 \times 10^{-8}$ .

last:  
 note general  
 slowing  
 down  
 trend  
 since  
 1860;  
 to find long  
 time changes  
 a new approach  
 needed;  
 study  
 of  
 ancient  
 eclipses

magnetic  
 variations  
 lag behind  
 spin variations  
 because they  
 must diffuse  
 thru the  
 mantle

declination : direction of compass  
 needle

$\dot{\Delta} \approx$  rate of westward drift of field  
 mantle spin follows average core; when  
 upper part decelerates mantle  
 accelerates; fast westward drift  
 (backward) correlated with fast spin



$B(t)$  magnetic field generated in core

lower mantle fairly good electrical conductor

(high temp. compressed silicate)

$$\sigma \sim 10 - 100 \text{ mho/m}$$

$$\sigma_{\text{Cu}} = 10^8 \text{ mho/m}$$

$$\sigma_{\text{distilled H}_2\text{O}} = 10^{-4} \text{ mho/m}$$

Secular variation of  $B(t)$  on decade time scale induces currents in lower mantle; these interact (Lenz's law) to exert torques on mantle.

All the above changes produced by indigenous events. In all,

$$\frac{H_\oplus}{\Phi} = \frac{H}{\text{mantle}} + \frac{H}{\text{core}} + \frac{H}{\text{atm}} \text{ is conserved.}$$

There is also a steady slowing down of  $\oplus$ 's rotation due to tidal friction, an external influence. This secular change has been measured since  $\sim 1700$ . Without clocks by observing occultations of  $\alpha$ ,  $\odot$  and  $\varnothing$  Mercury. Also modern determinations using clocks and also satellite determinations

of tidal bulge lag, also determined by lunar laser ranging. All agree that secular deceleration due to tidal friction ~~is~~ (and other causes) is:

$$\dot{\gamma}_\text{total} \approx -5 \cdot 10^{-22} \text{ rad/sec}^2$$

corresponds to 2.7 ms / century change in l.o.d.

& however is substantial because of

secularity - responsible for occasional leap second.

There also appears to be a slight secular acceleration due to internal causes, possibly response to deglaciation, so that  $\dot{\gamma}_\text{total}$  is somewhat larger (more negative) than  $\dot{\gamma}_\text{tidal}$ .

Telescope data covers too short an interval to get  $\dot{\gamma}_\text{total}$  very accurately and ancient eclipse data is used, an interesting combination of astro-geophysics and classical studies.

$$\dot{\gamma}_\text{total} \sim -5 \cdot 10^{-22} \text{ rad/s}^2$$

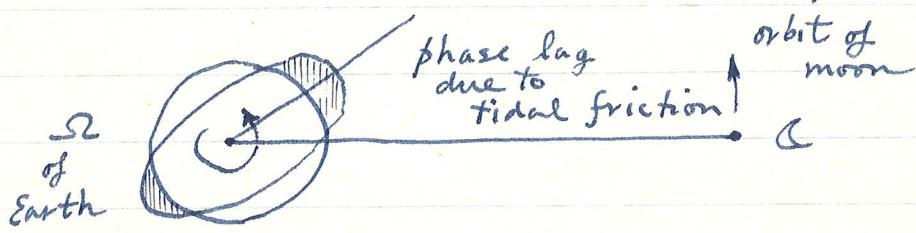
$$\dot{\gamma}_\text{tidal} \sim -7 \cdot 10^{-22} \text{ rad/s}^2$$

$$\dot{\gamma}_\text{nontidal} \sim 1-2 \cdot 10^{-22} \text{ rad/s}^2$$

Both  $\dot{\gamma}_\text{total}$  and  $\dot{\gamma}_\text{tidal}$  known to only 10-20 % so  $\dot{\gamma}_\text{nontidal}$  is of doubtful significance.

$\dot{\Omega}_{\text{total}}$  and  $\dot{\Omega}_{\text{tidal}}$  can be separated because the latter affects the lunar orbit also, one observes lunar occultations.

Cause of tidal deceleration is torque exerted (mostly) by  $\Omega$  on lagged tidal bulge (first proposed 1754 by Kant to explain known lunar discrepancies)



$H_{\oplus}$  changes but  $H_{\oplus} + H_a$  remains roughly constant (ignoring sun). Rotation rate of  $\oplus$  slows down and  $a$  slows down in its orbit and moves away from  $\oplus$  at a few cm/yr.

A fascinating problem: we'll discuss in more detail, many questions (where and how is energy dissipated, what is extrapolated past history of  $\oplus a$  system, can one extrapolate to future, etc? ).

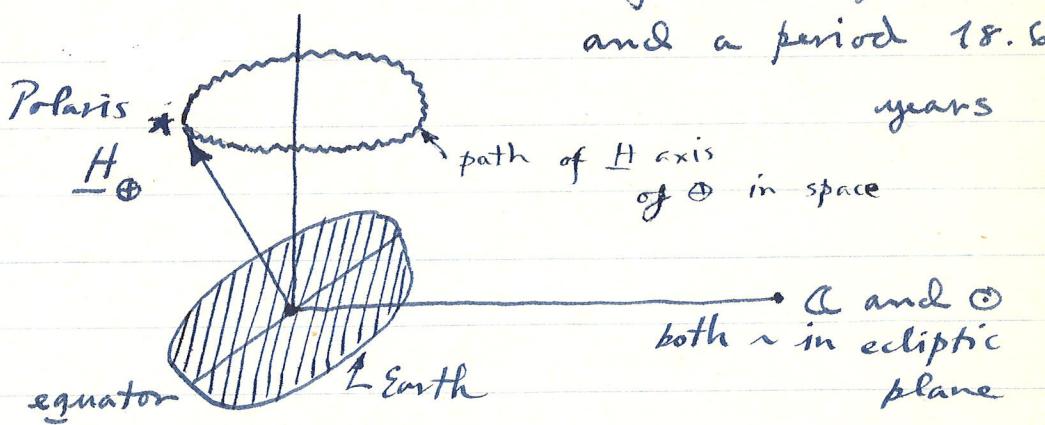
## 2. precession and forced nutation

A consequence of action of external torques on permanent bulge of  $\oplus$ , would occur in absence of friction.

Principal phenomenon is precession of equinoxes but complex orbit of  $\alpha$  causes small nutations (nodding) to be superposed on this.

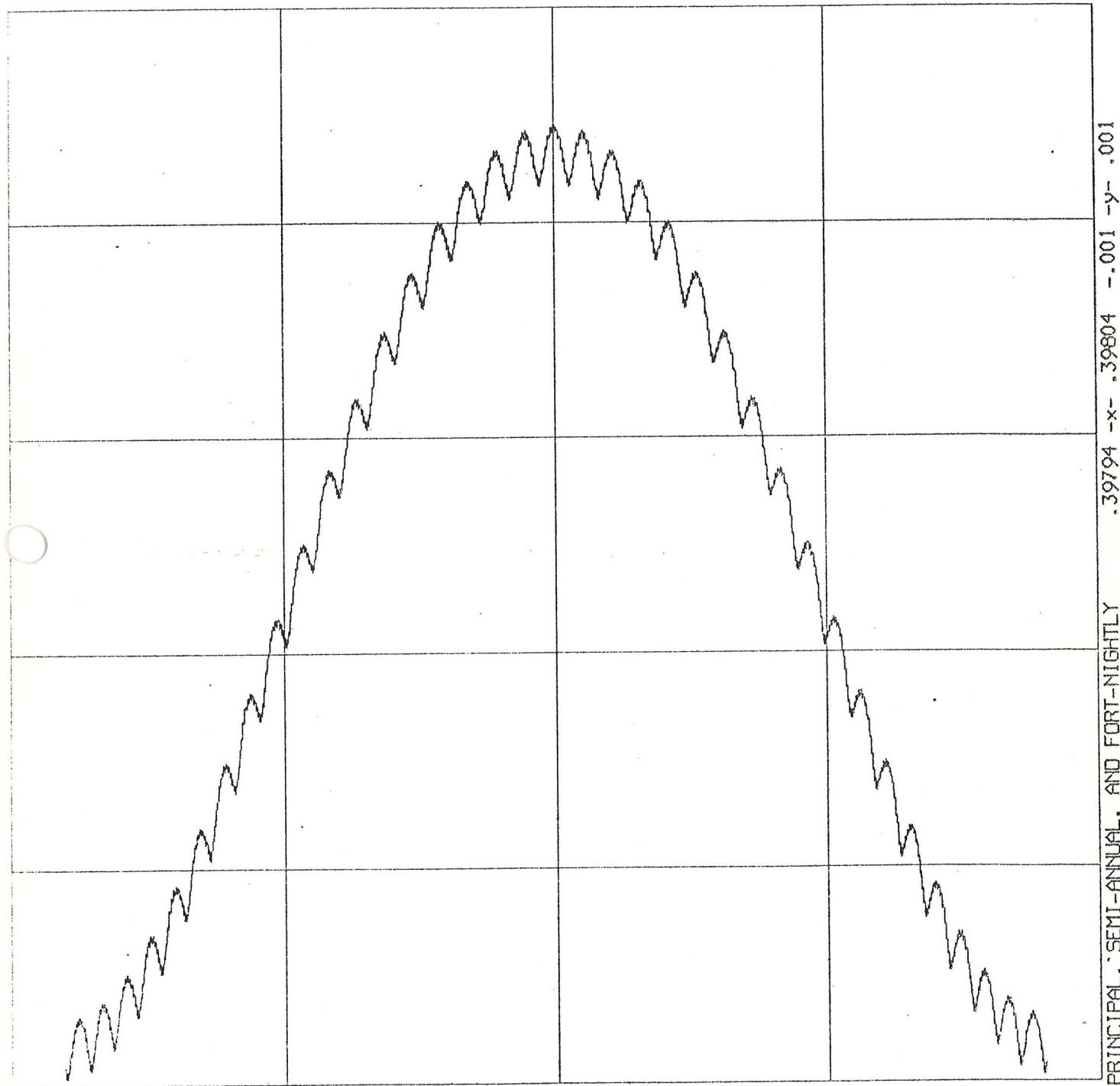
All these are motions of  $\oplus$ 's  $H$  vector in space (its orientation changes).

Largest or principal nutation has an amplitude of  $8''$  of arc and a period 18.6



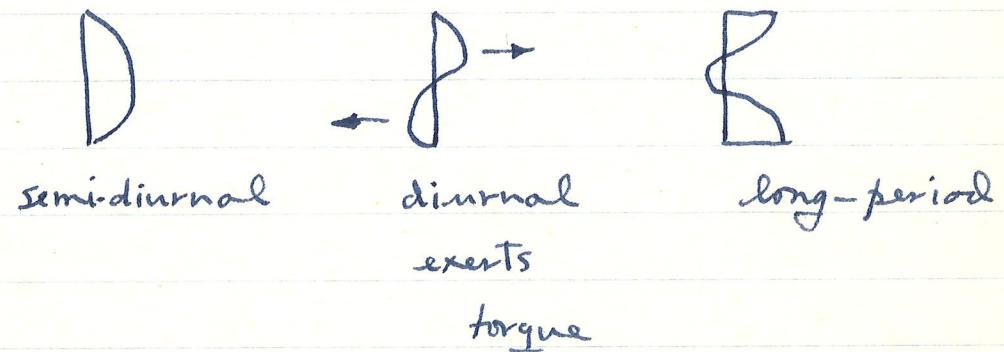
Precession period 25,800 years.

Next largest nutations semi-annual and fortnightly



PRINCIPAL, SEMI-ANNUAL, AND FORT-NIGHTLY .39794 -x- .39804 -.001 -y- .001

Intimate connection of nutations with diurnal tides, no time to explore here but note that only diurnal tides exert net torque on  $\oplus$

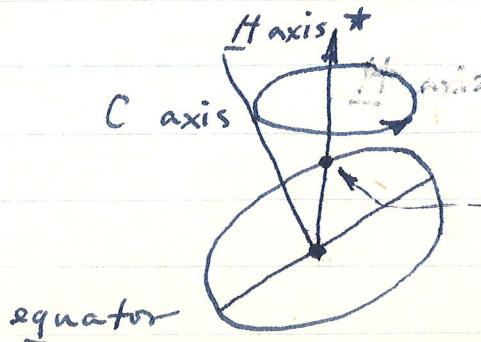


Path of  $H$  axis in 18.6 yrs from MLS.

### 3. wobble of $\oplus$ about its rotation axis

This due strictly to events on  $\oplus$ , not caused by external torques.  
To be distinguished from precession.

Wobble:  $H_{\oplus}$  is constant,  $\oplus$ 's principal axis of inertia displaced from rotation axis.



$H$  and  $\underline{S_2}$  are (essentially) aligned

pt. rotation axis pierces  $\oplus$  changes with time.

To an observer on  $\oplus$ , the rotation axis appears to move, ~~wobble~~ wobble also called polar motion.

A way to measure polar motion + distinguish from precession, nutation, in principle.

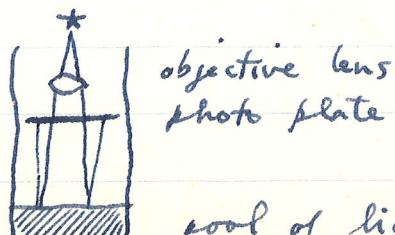
Time exposure of star trails on a camera pointed straight up (w.r.t. local vertical). At one point on  $\oplus$  surface center \* in photo appears in center of photo itself.

That point is where rotation axis of  $\oplus$  skewers photographer.  
Up near N pole.



Point found to change slowly with time, this is wobble (\* in center is same)  
Precession + nutation : the \* changes.

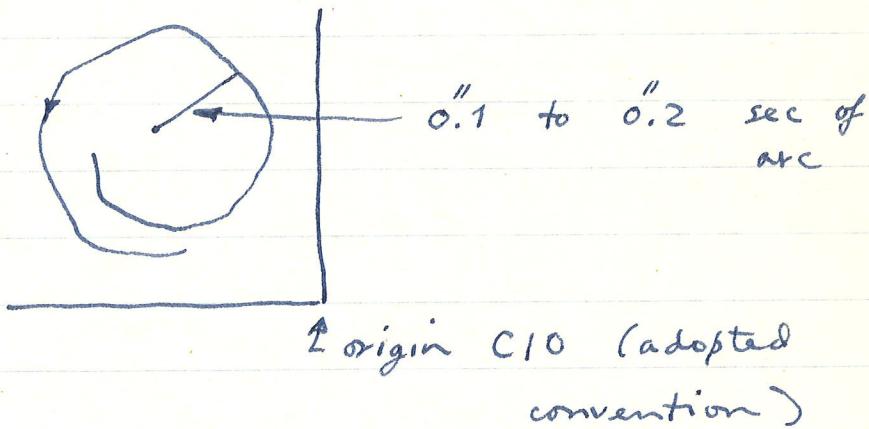
Actual observations made using so-called PZT's (photographic zenith tubes) at many locations on  $\oplus$ 's surface.



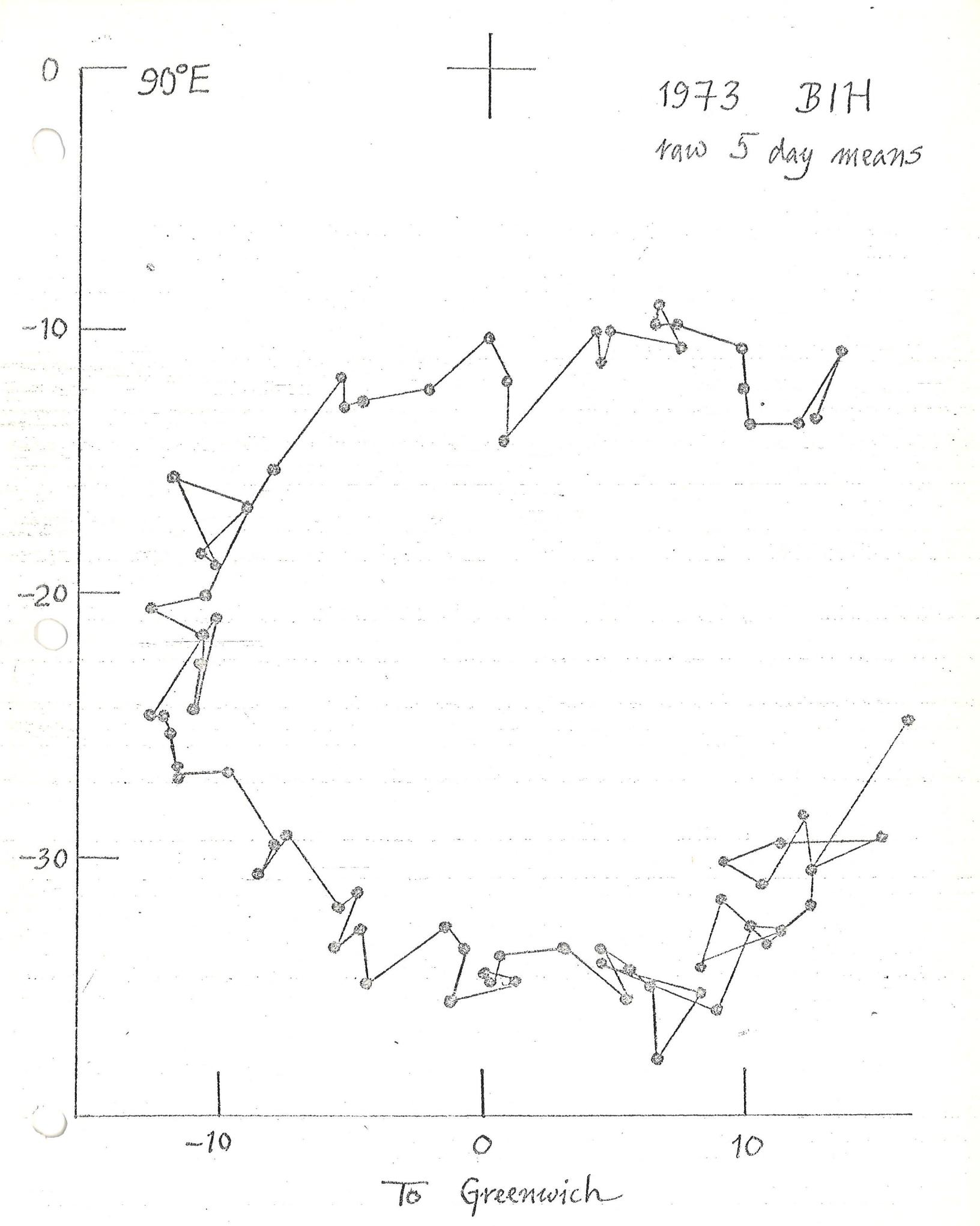
pool of liquid Hg reflecting mirror + define local vertical.

Best recent determinations from  
 Doppler tracking of artificial  
 satellites. See determinations from LAGEOS  
<sup>See also</sup>  
<sup>1974</sup>  
<sup>companion</sup>  
<sup>of LAGEOS</sup>  
<sup>VBI.</sup> Fig. 5.10 of Lambeck shows  
 path of pole determined by two  
 organizations ILS and BIH  
 for 1968 - 1970. Data noisier  
 than optimistic "error bars"  
 indicate.

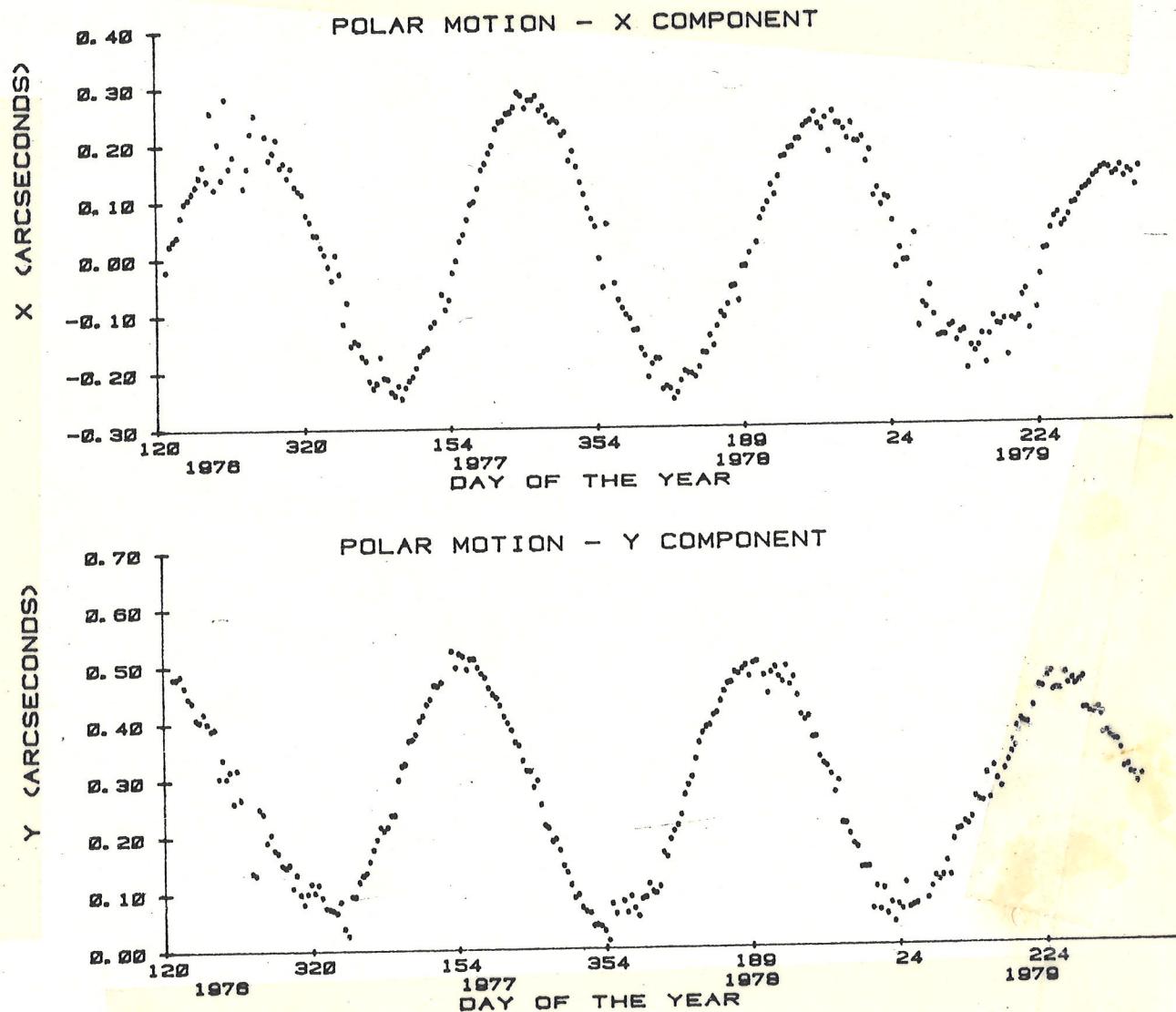
### Amplitude of motion



1 centisecond =  $0.01'' \approx 1$  ft.  
 subtended on  $\oplus$ 's surface so  
 amplitude is 10-20 feet or 3 to  
 6 meters. Very small, hard to  
 detect, obscured by atmospheric  
 refraction effects, requires hundreds  
 of observatories.

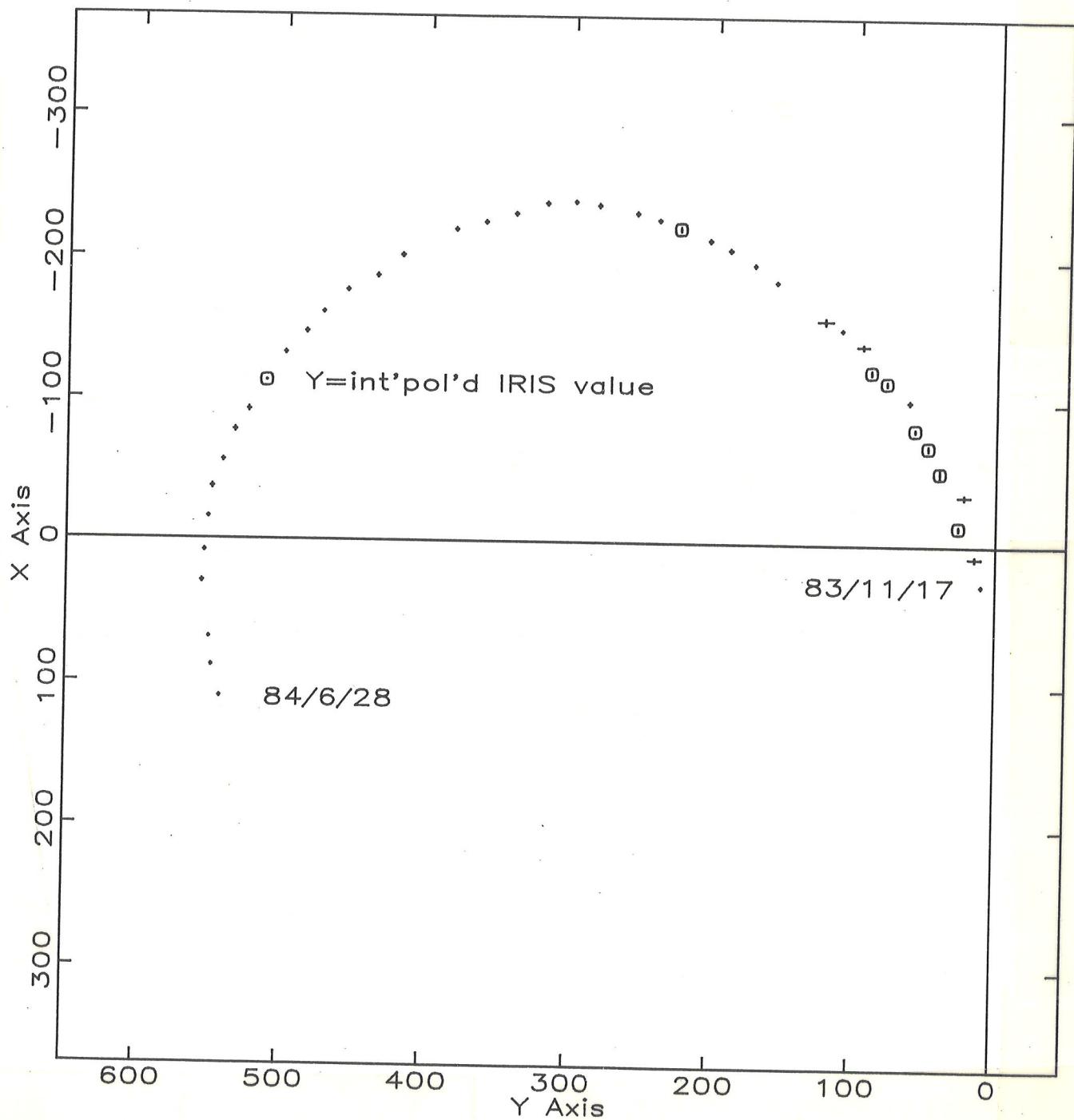


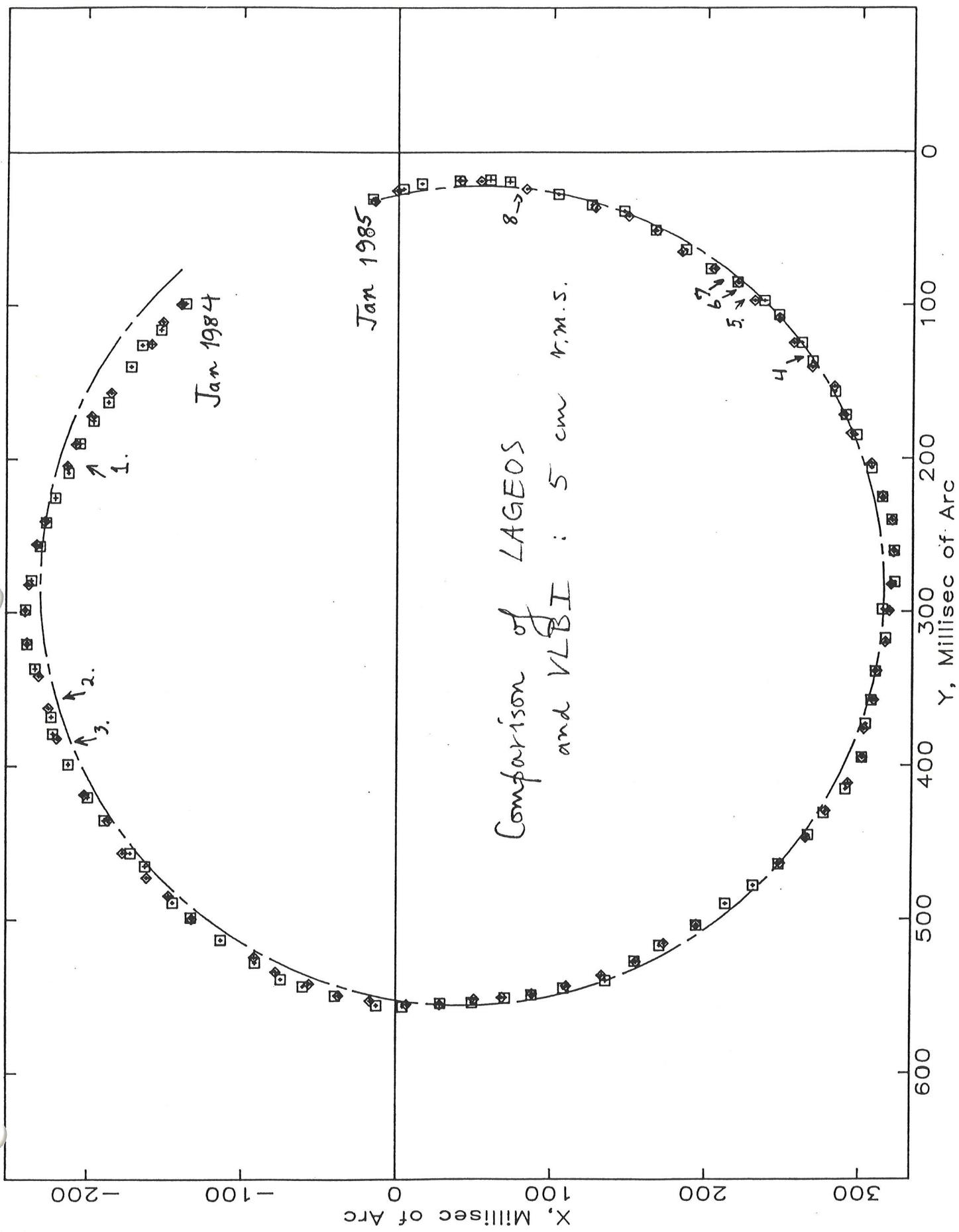
Polar motion determined by LAGEOS



# POLE POSITION

From Nov 17, 1983 to Jun 28, 1984  
at 5-day intervals  
(Circled pts: Y=BIH value)





## 5.3 POLAR MOTION

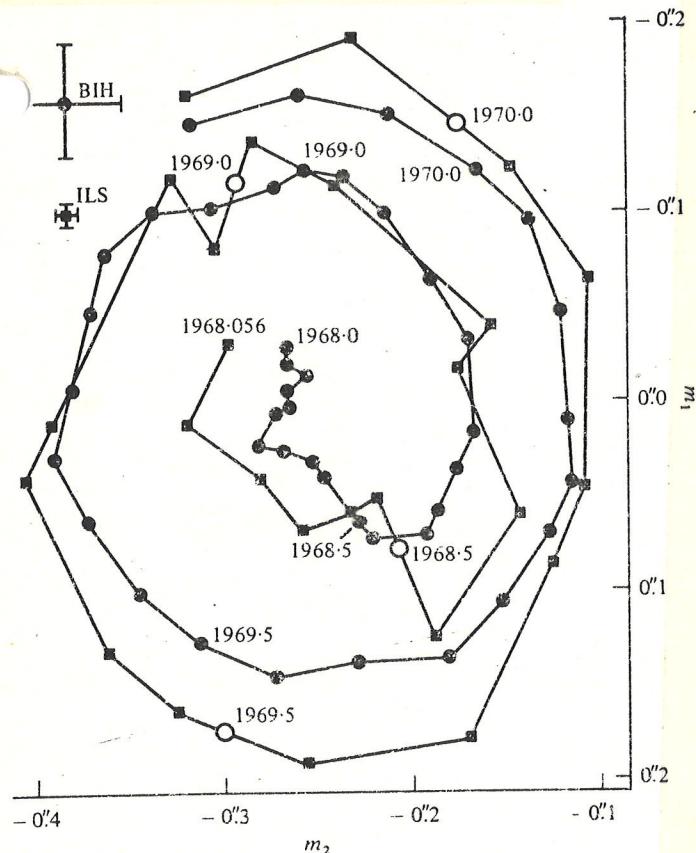


Fig. 5.10. Pole paths as determined by the ILS (■) and by the BIH (●) from 1968.0 to 1970.0. The ILS data are the unsmoothed values given at successive intervals of 0.0833 yr. For convenience in comparing with the BIH results, interpolated values at intervals of 0.5 yr are also indicated. BIH values are the unsmoothed values at intervals of 0.05 yr. Error estimates for the two data sets are indicated at top left-hand side.

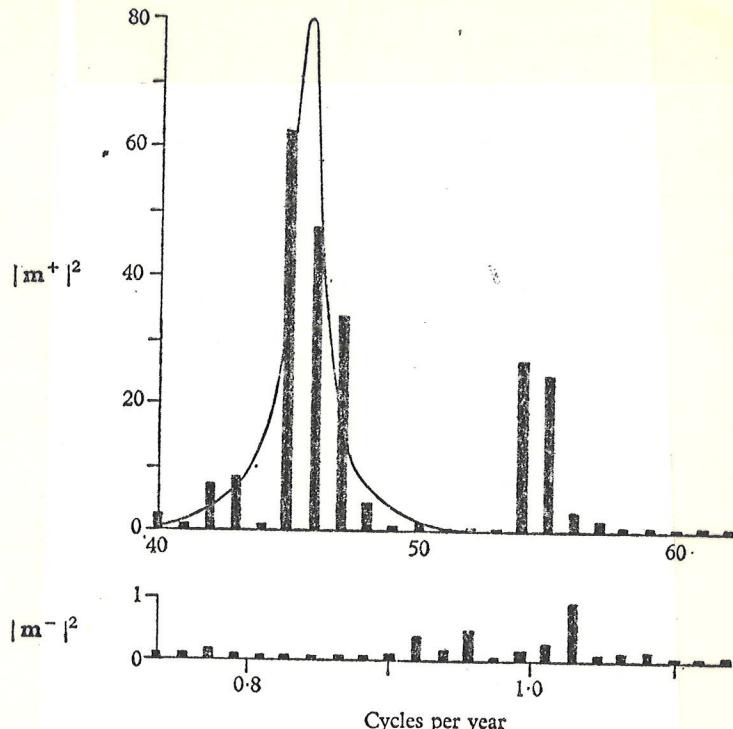


Fig. 7.5. The spectrum of variation in latitude, according to Rudnick (1956). The upper figure refers to the positive (west-to-east) motion, the lower figure to the negative motion of the pole of rotation (see § 6.7). For both motions the harmonics 40 to 62 are shown, with the corresponding frequency scale, in cycles per year, indicated below. The length of the spectral lines gives the contribution per harmonic toward the mean square radius arm (in units of  $(0^{\circ}01)^2$ ). The scale for negative motion is ten times that for positive motion. The curve has been fitted by the method of maximum likelihood (see Appendix A.2).

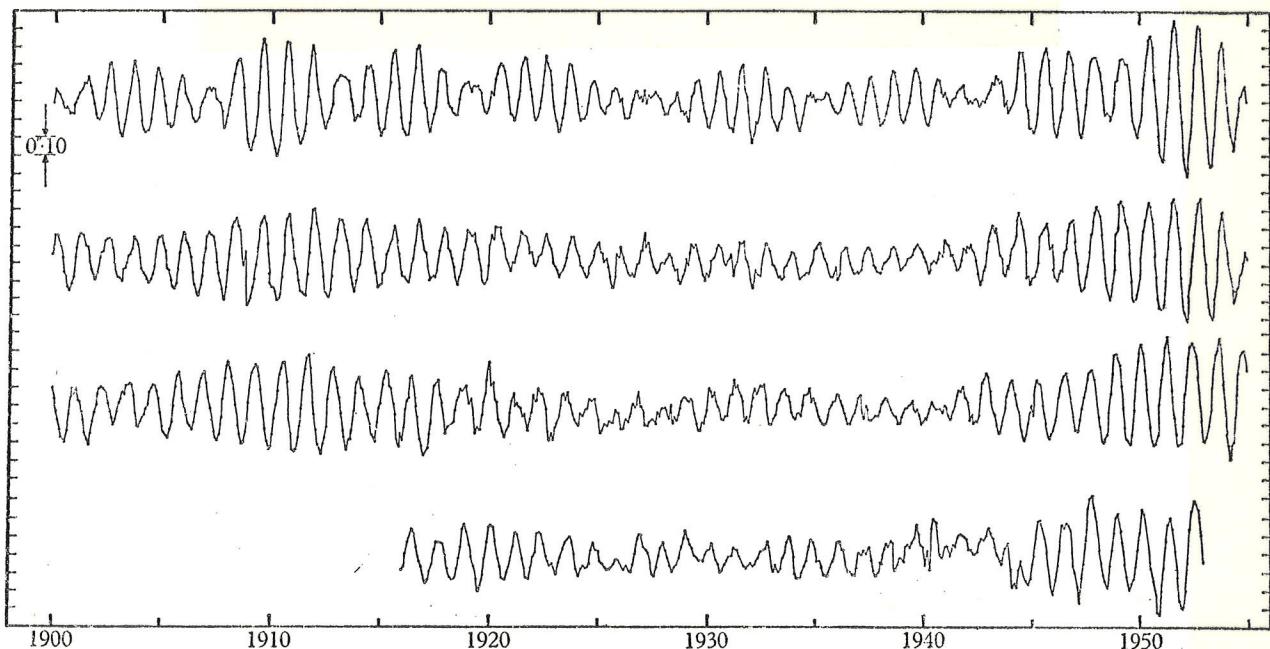
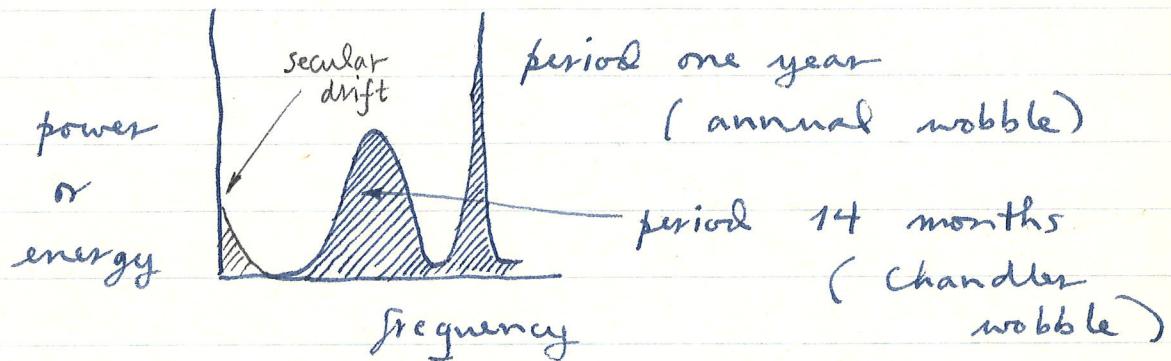


Fig. 7.4. The component,  $m_1$ , of the unsmoothed ILS observations, before (top) and after (second curve) removal of the seasonal variation; the component,  $-m_2$ , of the unsmoothed ILS observation after removal of the seasonal variation (third curve) and the corresponding non-seasonal variation in the latitude of Washington, as obtained with the PZT (bottom).

Spectrum of wobble looks like :



Chandler wobble  $\sim 75\%$  of total power  
both counterclockwise

Annual wobble is a forced oscillation  
driven by annual shifts in mass  
of atmosphere.

High pressure area over Siberia in  
winter, low in summer, a  
major atmospheric phenomenon,  
responsible for monsoon climate  
of SE Asia, an annual shift  
in mass of atmosphere from N to  
S hemisphere forces a shift in  
solid  $\oplus$  to conserve angular  
momentum, keep  $H_\oplus$  in fixed  
direction.

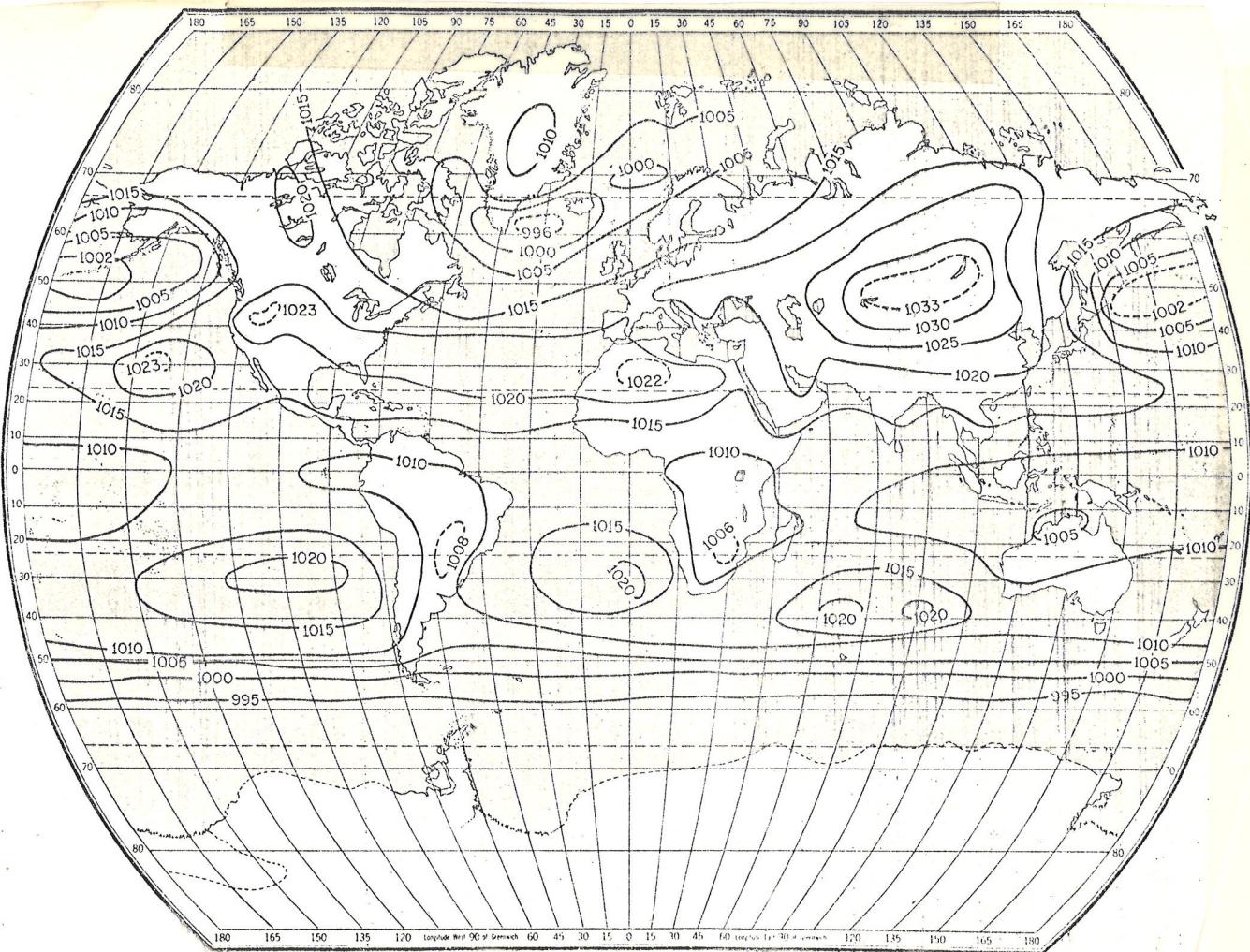
First quantitative discussion of annual wobble 1916 Sir Harold Jeffreys, allowed for response of oceans to shifting highs and lows by inverted barometer response, integrate only over land.

\* Hydrological factors also play a role, precipitation, melting of snow + ice etc.

Jeffreys considered also sap rising in trees, leaves on branches, almost but not quite significant.

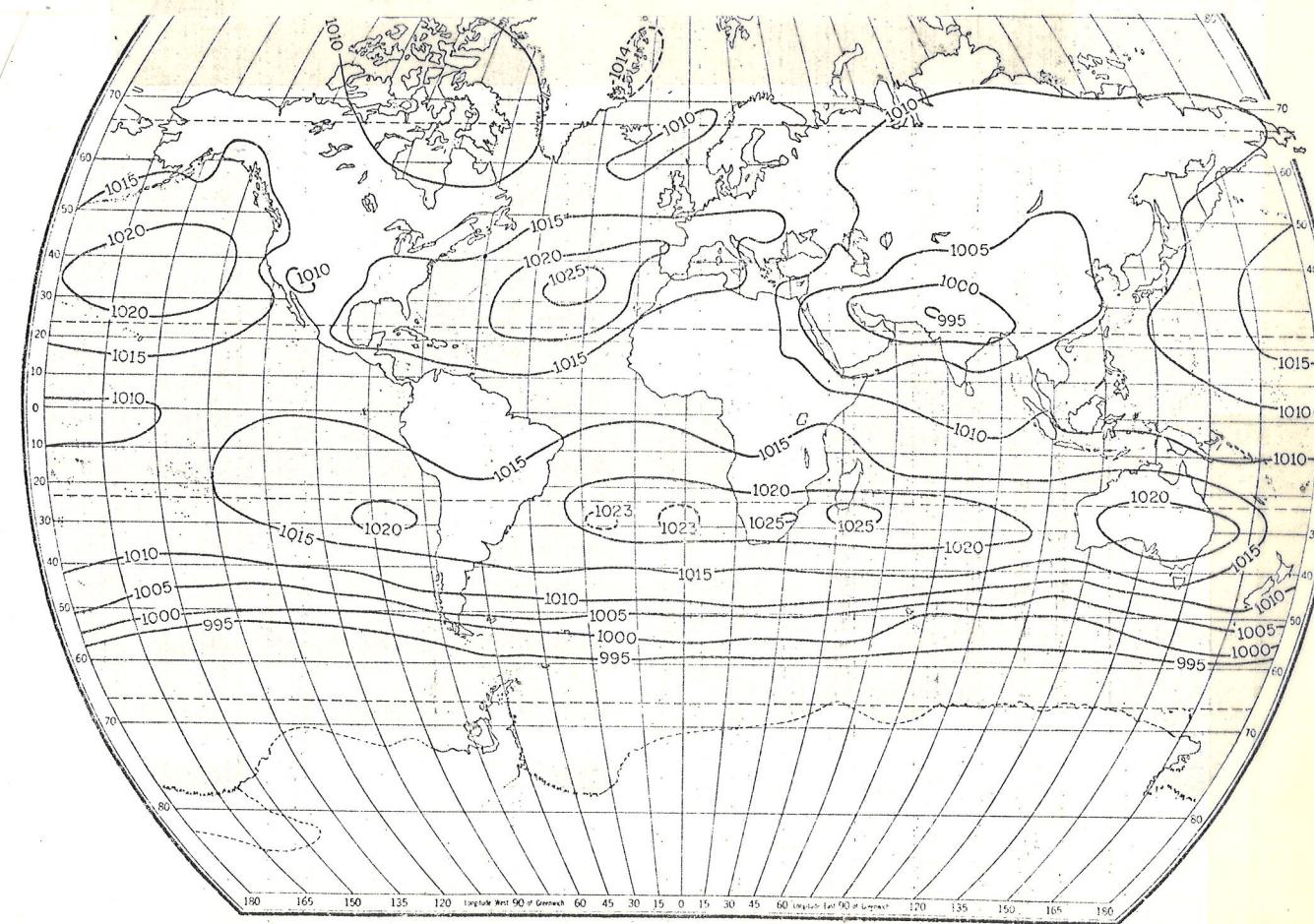
Agreement of astronomy with meteorology + hydrology is fair considering difficulties of estimating latter. Both amplitude and phase of annual line in fair agreement. See Fig. 4 of Wilson and Hanbrick.

~~The annual C.W. is a mere oscillation of the period over pretty much undisturbed by energy source or excitation till it reaches a greater detail.~~



Van der Grinten projection, courtesy A. J. Nystrom & Co., Chicago.

Fig. 4-5 World distribution of mean sea level pressure in January.



Van der Grinten projection, courtesy A. J. Nystrom & Co., Chicago.

Fig. 4-6 World distribution of mean sea level pressure in July.

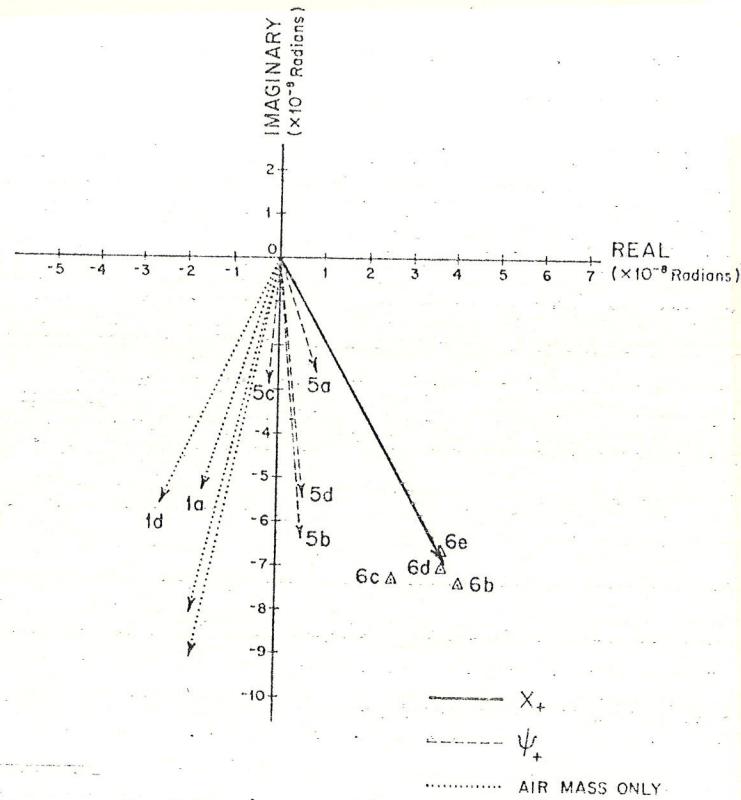


FIG. 4. Positive annual frequency excitations.

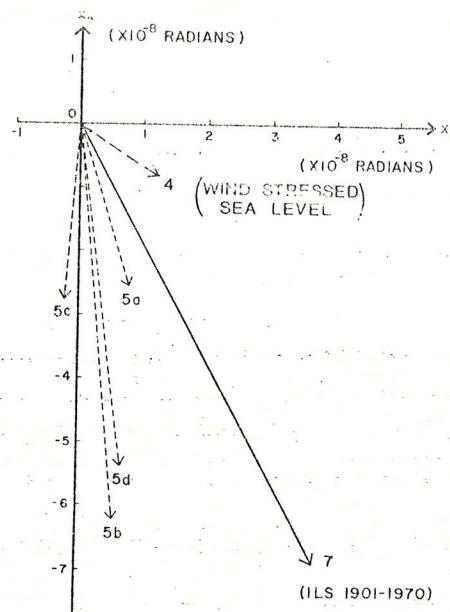


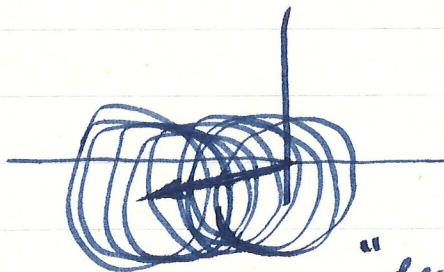
Figure 4. Positive circular annual excitation vector from geophysical estimates of air mass redistribution, continental water storage, and mountain torque (Wilson & Haubrich 1976a, Fig. 4), ILS data and the wind stressed sea level (this study). Numbers refer to lines in Table 3.

Points labelled 1 : shifts in air mass alone.

5 : air mass + hydrological factors gives better agreement with  
6 (astronomy)

Fig. 4 from O'Connor shows 4:  
additional contribution due to simple  
model of non-isostatic oceanic  
response to winds, improves  
agreement when added to 5.

There has also been a secular  
shift of mean pole since  
observations began  $\sim$  1900



"drift" of center of  
annual and Chandler  
wobbles

25 feet  $\sim$  8m

Amount  $\sim$  0.25" in 70 years or  
about 0". 003 /yr toward  
Labrador, roughly.

10 cm /yr.  
3/10 foot /year

This corresponds to a polar wander of about  $0.8^\circ$  per million years which agrees with the observed rate from paleomagnetic data using Jason's hot spot reconstruction.

This supports idea that observed secular drift in ILS data is just recent measurement of same long-term polar wander which has prevailed for last several m.y.

Also has been suggested that it's due to Pleistocene deglaciation - it's certainly in right direction, as figure from Sabadini and Peltier shows

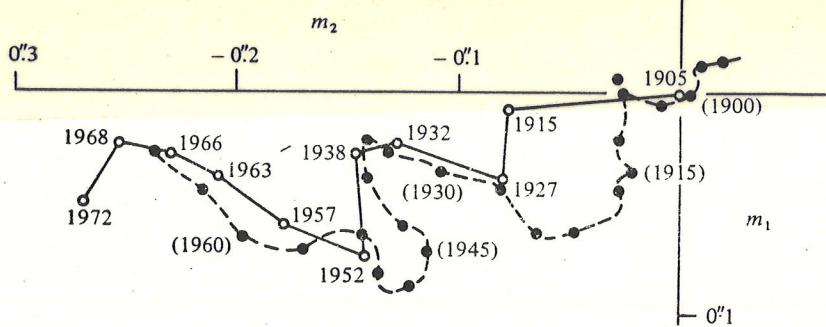


Figure 5.12. Motion of the mean pole according to Stoyko (●, dates in parentheses). Pole positions represent running means based on 6-yr intervals. Positions every 3 yr are indicated. The motion of the mean pole according to Markowitz is indicated by ○; these positions also represent 6-yr means.

