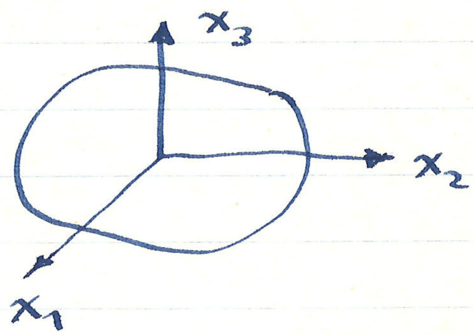
Theory of the Earth's wobble

To begin with we shall treat the ⊕ as a rigid body, $\mu = \infty$ everywhere.

Let A, B, C be the principal moments of inertia about axes $\hat{x}_1, \hat{x}_2, \hat{x}_3$



A possible stable motion of such a body is uniform rotation about axis $\hat{x}_3$ of greatest inertia.

In that case the angular momentum vector $\underline{H}$ is:

$$\underline{H} = C\Omega\,\hat{x}_3$$

We ask the question: are there

other nearby possible motions?
We consider only motions with c.o.m. fixed (we are not concerned with orbit of $\oplus$ about $\odot$ but with motion of $\oplus$ <u>about</u> c.o.m.)

Only possible such motion for a <u>rigid</u> $\oplus$ is a rotation. Thus we look for motions of form:

uniform rotation <u>mean</u> rate Ω

$$\underline{\Omega}(t) = \Omega \hat{x}_3 + \underline{\omega}(t)$$

$$|\underline{\omega}(t)| \ll \Omega$$, <u>nearly</u> a uniform rotation

Angular momentum of this non-uniformly rotating body, as seen by an observer on the body (in the $\hat{x}_1 \hat{x}_2 \hat{x}_3$ frame) is:

$$\underline{H}(t) = \underline{\underline{C}}(t) \cdot \underline{\Omega}(t)$$

inertia tensor

the $\hat{x}_1, \hat{x}_2, \hat{x}_3$ components, hence in <u>body</u> frame

$$= \hat{x}_1 A \omega_1(t) + \hat{x}_2 B \omega_2(t) + \hat{x}_3 C \left[\Omega + \omega_3(t) \right]$$

In <u>inertial</u> or space frame $\underline{C}$ is time dependent, but in body frame it is not and $\underline{H}$ is easy to write.

The equation of motion of such a rigid body is, in <u>inertial frame</u>.

$$\left(d\underline{H} / dt \right)_{\cancel{\text{body}}\ \text{space}} = \underline{N}(t)$$

externally applied torque

Our interest now in <u>free</u> motions (superposed on precession + nutation) so we set $\underline{N} = \underline{0}$. Thus we have

$$\left(d\underline{H} / dt \right)_{\text{space}} = \underline{0}$$

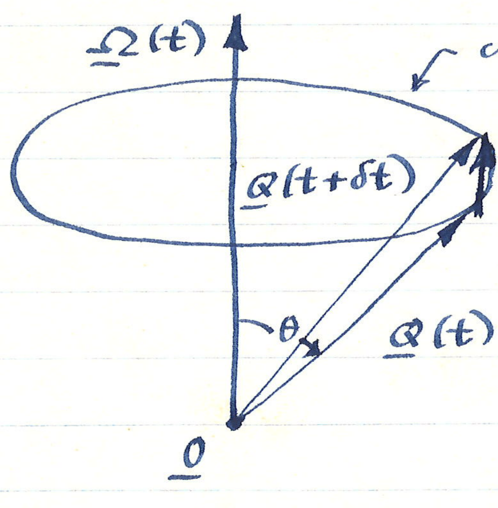
But we know $\underline{H}(t)$ in <u>body</u> frame. Since <u>we</u> view motion in body frame it is convenient to recast above eqn in body frame. For <u>any vector</u> $\underline{Q}(t)$ we have rate of change in body

$$\left(d\underline{Q}/dt\right)_{space} = \left(d\underline{Q}/dt\right)_{body} + \underline{\Omega}(t) \times \underline{Q}$$

total rate of change in space

rate at which it would change in space if fixed (say, painted) in body.

Proof: say $\underline{Q}(t)$ is fixed in body. Body rotating with instantaneous angular velocity $\underline{\Omega}(t)$.



circle of radius $Q \sin\theta$

length of difference: $Q \sin\theta \, (\Omega \, \delta t)$

angle turned through in time δt

Observer in space sees $\underline{Q}$ change its orientation. Net change in a time δt is:

$$|\underline{Q}(t+\delta t) - \underline{Q}(t)| = Q \sin\theta \, \Omega \, \delta t$$

$$\lim_{\delta t \to 0} \frac{|\underline{Q}(t+\delta t) - \underline{Q}(t)|}{\delta t} = \underset{\uparrow \text{ length of } d\underline{Q}/dt}{Q \sin\theta \, \Omega}$$

direction of $d\underline{Q}/dt$ is direction of $\underline{\Omega} \times \underline{Q}$ by right hand rule, also $|\underline{\Omega} \times \underline{Q}| = \Omega Q \sin\theta$, so

$$(d\underline{Q}/dt)_{space} = \underline{\Omega} \times \underline{Q}$$

If $\underline{Q}$ varies in the body as well (if the painted arrow moves) then

$$\boxed{(d\underline{Q}/dt)_{space} = (d\underline{Q}/dt)_{body} + \underline{\Omega} \times \underline{Q}}$$

$$\boxed{\text{Thus: } (d\underline{H}/dt)_{body} + \underline{\Omega}(t) \times \underline{H}(t) = 0}$$

Now $(d\underline{H}/dt)_{body} = \hat{x}_1 A \dot{\omega}_1$

$+ \hat{x}_2 B \dot{\omega}_2 + \hat{x}_3 C \dot{\omega}_3$ where $\dot{} \equiv \frac{d}{dt}$.

$$\underline{\Omega} \times \underline{H} = \begin{vmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 \\ \omega_1 & \omega_2 & \Omega + \omega_3 \\ A\omega_1 & B\omega_2 & C(\Omega + \omega_3) \end{vmatrix}$$

$$= \hat{x}_1 [\, C(\Omega + \omega_3)\omega_2 - B(\Omega + \omega_3)\omega_2 \,]$$
$$- \hat{x}_2 [\, C(\Omega + \omega_3)\omega_1 - A(\Omega + \omega_3)\omega_1 \,]$$
$$+ \hat{x}_3 [\, (B - A)\omega_1 \omega_2 \,]$$

Now since $\omega_i \ll \Omega$ we can neglect terms of order $(\omega_i)^2$, thus

$$\approx \hat{x}_1 (C - B)\Omega\omega_2 - \hat{x}_2 (C - A)\Omega\omega_1$$

We thus get three component equations

$$\boxed{\begin{aligned} A\dot{\omega}_1 + (C - B)\Omega\omega_2 &= 0 \\ B\dot{\omega}_2 - (C - A)\Omega\omega_1 &= 0 \\ C\dot{\omega}_3 &= 0 \end{aligned}}$$

The last eqn just says that a <u>possible</u> free motion is $\omega_3(t) = \text{const}$. uniform rotation at a different rate — this solution is obvious. The other two eqns are more interesting as they describe a <u>non uniform</u> possible free motion.

Differentiate first and plug into second.

$$A\ddot{\omega}_1 + (C-B)\Omega\dot{\omega}_2 =$$

$$A\ddot{\omega}_1 + \frac{(C-B)(C-A)}{B}\Omega^2\omega_1 = 0$$

$$\boxed{\ddot{\omega}_1 + \sigma^2\omega_1 = 0 \quad \text{where} \quad \sigma^2 = \frac{(C-A)(C-B)}{AB}\Omega^2}$$

By differ. second and plugging into first get same eqn for ω_2, viz.

$$\boxed{\ddot{\omega}_2 + \sigma^2\omega_2 = 0}$$

For σ^2 positive this is the equation of a <u>simple harmonic oscillator</u> with angular frequency σ.

We have actually analyzed 3 cases at once:

$\hat{x}_3$ <u>greatest</u> princ. axis	$C > B > A$
$\hat{x}_3$ <u>least</u> princ. axis	$A > B > C$
$\hat{x}_3$ <u>intermediate</u> princ. axis	$C > B$ and $< A$ or vice-versa

The first two are <u>stable</u> since $\sigma^2 > 0$ but for the third $\sigma^2 < 0$ and a solution to $\ddot{\omega}_1 + \sigma^2 \omega_1 = 0$ is $\omega_1 = \text{const} \exp(\alpha t)$ where $\alpha^2 = -\sigma^2$, exponential blowup, steady rotation about <u>intermediate</u> axis is <u>not stable</u>, can be verified experimentally by tossing a book into the air, when slight <u>friction</u> taken into account rotation about least axis also (secularly) unstable, only rotation about greatest axis is stable with (or without) friction. That is of course the axis ⊕ <u>is</u> rotating about.

For the $\oplus$ $A \approx B$, let us take $A = B$ as a good approximation.

Then:

$$\ddot{\omega}_1 + \sigma^2 \omega_1 = 0$$
$$\ddot{\omega}_2 + \sigma^2 \omega_2 = 0$$
$$\sigma = \frac{C-A}{A}\,\Omega$$

Most general form of solution then

phase of oscillation

$$\omega_1(t) = D \cos(\sigma t + \phi)$$
$$\omega_2(t) = D \sin(\sigma t + \phi)$$

amplitude of oscillation

note relative phase of ω_1 and ω_2 to satisfy original coupled eqns.

Note: ω_1 and ω_2 are rotations about equatorial axes $\hat{x}_1$ and $\hat{x}_2$. The above motion corresponds to a wobble of rotation axis $\underline{\Omega}(t)$ about mean axis $\hat{x}_3$.

Can also be shown that ω_1/Ω and ω_2/Ω are direction cosines of instantaneous ang. velocity vector $\Omega(t)$ w.r.t. body axes.

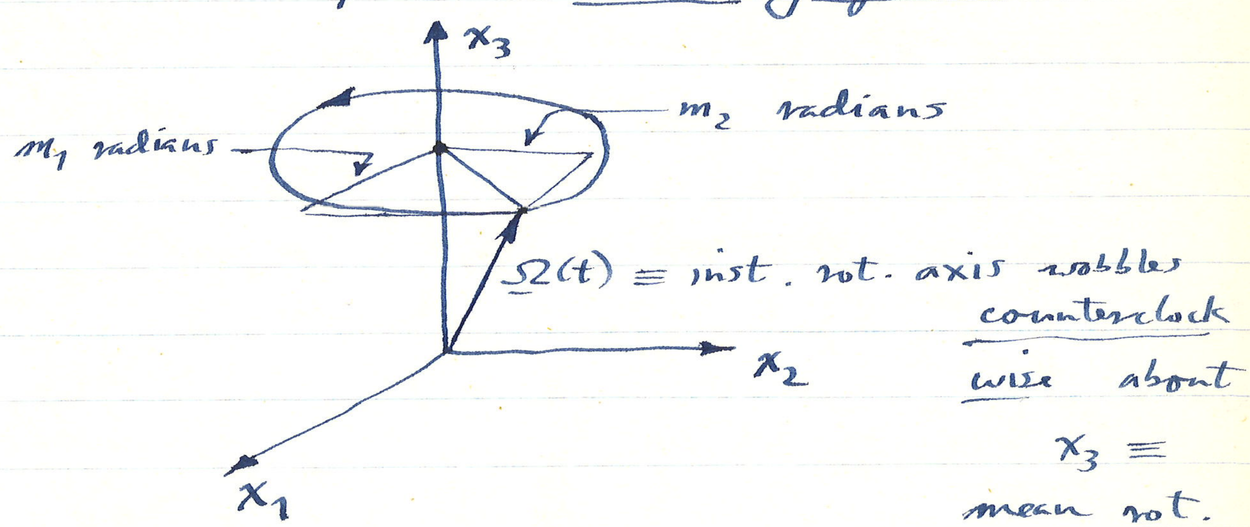
Reason $\underline{\Omega}(t) = \Omega_i \hat{x}_i = \Omega(t)\hat{\Omega}$ so
$\hat{x}_i \cdot \hat{\Omega} = \Omega_i / \Omega(t) \approx \Omega_i / \Omega$

conventional notation for these direction cosines is:

$$\boxed{\begin{aligned} m_1 &= \Omega_1/\Omega = \omega_1/\Omega \\ m_2 &= \Omega_2/\Omega = \omega_2/\Omega \end{aligned}}$$

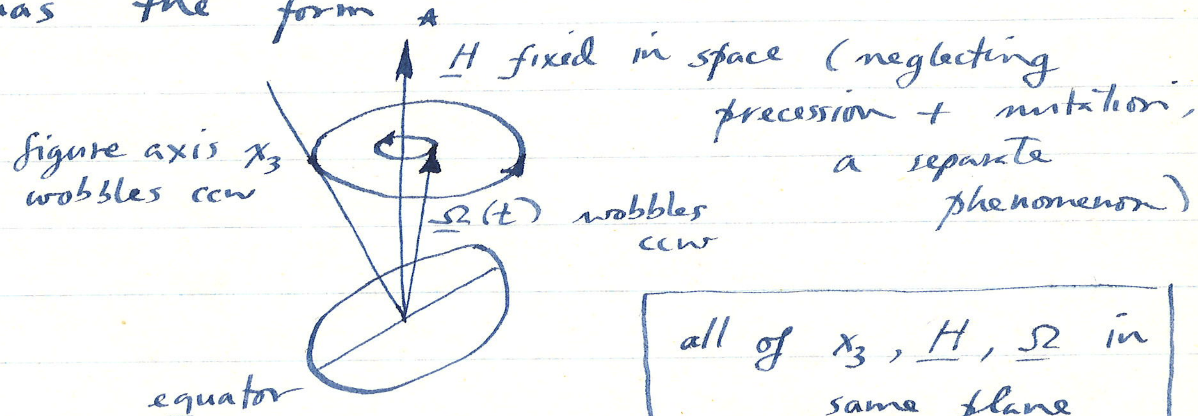
direction cosines measured in radians

So we have picture <u>in body frame</u>



One cycle every $2\pi/\sigma = T =$ period.

Can be shown that from pt. of view of observer <u>in</u> space this motion has the form



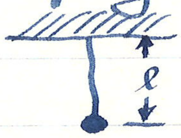
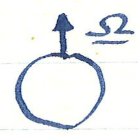
$\boxed{\text{all of } x_3,\ \underline{H},\ \underline{\Omega} \text{ in } \underline{\text{same plane}}}$

$\underline{H}$ fixed <u>in</u> <u>space</u>, both x_3 (C axis of body) and $\underline{\Omega}$ wobble about it.

The angle between $\underline{\Omega}$ and $\underline{H}$ can be shown to be $(C-A)/A$ <u>times</u> the angle between $\hat{x}_3$ and H.
For $\oplus$, $(C-A)/A \sim 1/300$ so $\underline{\Omega}$ and $\underline{H}$ are <u>essentially</u> <u>aligned</u>, we may think of $\underline{\Omega}$ as fixed is space and wobble as motion of body w.r.t. $\underline{\Omega}$. In <u>body</u> <u>frame</u> we observe wobble of $\underline{\Omega}(t)$

Can be observed with football, meaning of term "wobbly pass".

Natural frequency σ of this motion depends only on $\oplus$. This a <u>free</u> <u>motion</u>. <u>Analogy</u>: a <u>pendulum</u>.

pendulum	rigid body
equil. state hanging free	equil. state <u>uniform rotation</u> about C axis
	
possible free motion small oscillation	possible free motion, small oscillation, wobble
$\ddot{\theta} + \sigma^2\theta = 0$ $\sigma = (g/\ell)^{1/2}$ depends only on parameters of pendulum (length ℓ)	
	frequency $\sigma = \frac{C-A}{A}\Omega$

σ does <u>not</u> depend on how motion is excited, e.g.

For $\oplus$, period in days = [redacted] $A/(C-A) \approx 300$ days ≈ 10 months.

This calculation first done by Leonard <u>Euler</u>

in ~1758. He suggested ⊕ might be undergoing such a motion, if something were available to excite it. If so could be detected astronomically as a variation in latitude of observatories.

For next 150 yrs astronomers (e.g. Bessel, Maxwell) searched to no avail for 10 mo. periodicity.

Then (1891) S.C. Chandler "a prosperous merchant in Cambridge Mass" analyzed astronomical records going back 200 yrs, finds "many discouraging discrepancies" explainable by a variation of latitude.

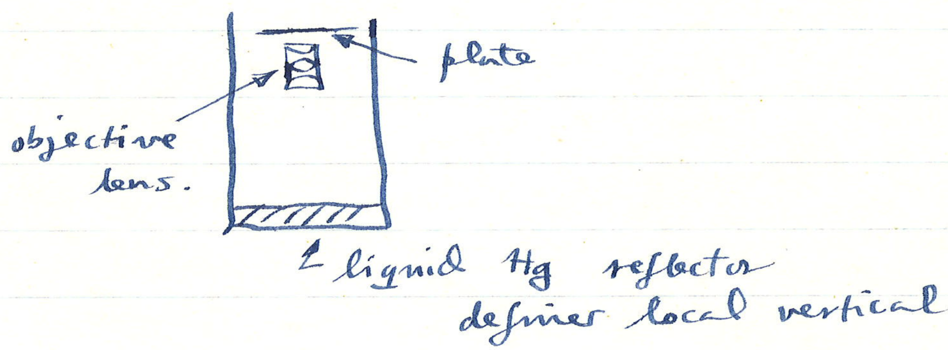
Finds 2 periodicities:

1. 12 months = 1 year
2. 14 months

No evidence of Eulerian period 10 months. Confirmed independently by simultaneous contemporary observations at Waikiki + Berlin (180° apart in longitude, curves of variation thus opposite in phase).

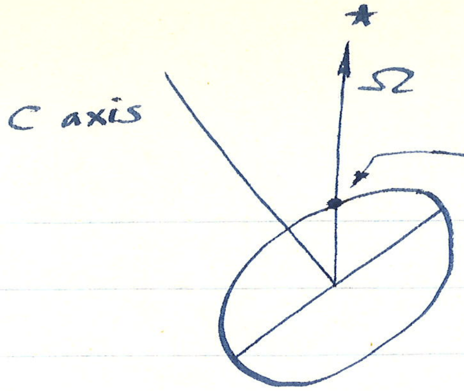
Establishment of ILS (1900): chain of five observatories 39° 08′ N (Gaithersburg, Md is nearest). Can observe same ☆, no error due to proper motions, observe zenith ☆'s, less refraction error.

ILS still exists, oldest international scientific cooperation on record, now ∃ more than 100 observatories dedicated to polar motion observations, commonly use PZT's

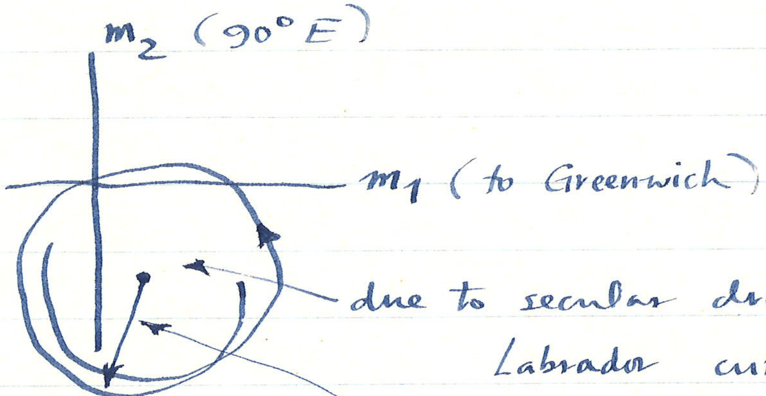


Best modern data comes however from <u>Doppler tracking of satellites</u>.

80 yrs of data confirm Chandler's findings. Data often plotted as <u>polar motion path</u>, can think of as bird's eye view near N pole.



plot this point of intersection with ⊕ surface as fcn. of time (origin = CIO ≅ <u>mean</u> rot. pole in 1900, inception of ILS)



counterclockwise pole path

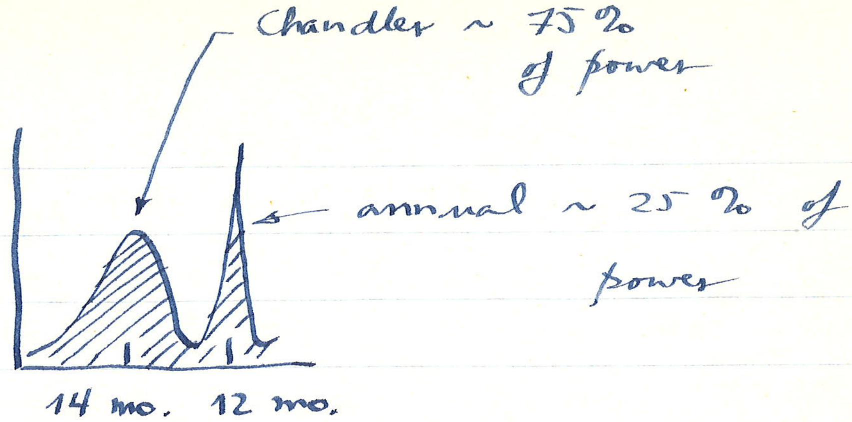
due to secular drift ≈ toward Labrador current path of pole down in this quadrant.

rough amplitude ≈ .14″ ≈ 14 feet or 4 m on ⊕ surface.

Astronomical data very noisy due to irregularities of atmospheric refraction, error of a single observation surpasses measured phenomenon, about 0.2″ to 0.5″ requires many observations for $N^{-1/2}$ reduction of variance.

~ 99% of energy in <u>counterclockwise</u> direction.

Spectrum :



Factors to explain :

1. why no 10 month component as predicted ?
2. what are the two observed components ?

As we have noted 12 month component is a *forced* wobble forced by shifts in mass of atmosphere . It is essentially a single line at 12 months with a phase and amplitude which are fairly well understood. Reason it is predominantly ccw is near resonance to Chandler peak.

Chandler or 14 month wobble is the free motion with period altered by ⊕'s finite rigidity .

The Chandler period

The observed Chandler or free wobble period from ~ 80 years of observation, from Wilson + Haubrich, is:

$$\boxed{T_{obs} = 435.2 \pm 2.6 \text{ days}}$$

1 standard deviation

sidereal days (86164 s)

Qualitative explanation of 14 rather than 10 months given 1892 by Simon Newcomb, american astronomer. Rigidity of ⊕ then known from Darwin-Kelvin measurement to be

$$\bar{\mu} \sim 8 \cdot 10^{11} \text{ dyne/cm}^2 \sim \mu_{steel}$$

Newcomb's physical argument:

$$\boxed{T_{theory} = \frac{A}{C-A} \text{ days}}$$

But part of C−A, the equatorial bulge,

is the purely elastic and instantaneous response of ⊕ to the rotation. Part, on the other hand, is "set".

If rotation pole shifts in ⊕ the elastic bulge will just follow it, has no effect on stability or wobble period

$$\frac{A}{C-A}$$

A ← inertia = eq. moment since wobble equiv. to rotation ω_1, ω_2 about equatorial axis

$C-A$ ← restoring force, responsible for stability of rotation about C axis, more oblate body more stable, period of wobble less

$$(C-A)_{total} = (C-A)_{set} + (C-A)_{inst\ elastic}$$

set or "effective"

Only $(C-A)_{set}$ can contribute to Chandler period i.e. $T_{theory} = \frac{A}{(C-A)_{set}}$ days, actually

A fluid ⊕ would have $(C-A)_{set} \equiv 0$.

If turned off rotation "suddenly" would become spherical. If turned off rotation of real ⊕ only part $(C-A)_{elastic}$ would disappear.

1909 A. E. H. Love and Sir Joseph Larmor showed independently how $(C-A)_{elastic}$ could be expressed in terms of the tidal Love number k. Thus related Chandler period to the tidal deformation of ⊕. The derivation is quite simple:

The centrifugal potential due to the rotation is

$$\Psi(r,\theta) = -\frac{1}{2}\Omega^2 r^2 \sin^2\theta$$

$$= -\frac{1}{3}\Omega^2 r^2 \left[1 - P_2(\cos\theta)\right]$$

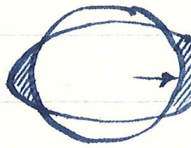
response to this will be purely radial, not of interest here.

$\frac{3}{2}\cos^2\theta - \frac{1}{2}$

Legendre polynomial

It is this part which gives rise to a bulge.

We consider the response of a spherical ⊕ to this potential. Our theoretical period should then be good to better than order $\varepsilon_a \sim 1/300$ (since we are calculating only part of the period, namely $(C-A)_{elastic}$.

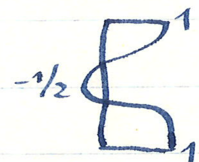


radial displacement $u(\theta)$

The bulge produced can be expressed in terms of Love number h, viz.

$$u(\theta) = -h\left[\tfrac{1}{3}\Omega^2 a^2 P_2(\cos\theta)\right]/g$$

displ. of outer surface $r=a$



plot of $P_2(\cos\theta)$, neg. at equator

More interesting for us is the <u>change in grav. pot.</u> at $r=a$ due to the redistribution of mass in the bulge. It is given by

<u>External</u> grav. pot. due to mass in bulge:

$$\delta V(r,\theta) = \underbrace{k\left[\tfrac{1}{3}\Omega^2 a^2 P_2(\cos\theta)\right]}_{\text{value on surface } r=a}\left(\frac{a}{r}\right)^3$$

must fall off like r^{-3}

Can also (always) express this in terms of <u>MacCullagh's formula</u> which says that

this is $(C-A)_{elastic}$

$$\delta V(r,\theta) = G\left(\frac{C-A}{a^3}\right)\left(\frac{a}{r}\right)^3 P_2(\cos\theta)$$

Comparing the two we find that

$$\boxed{(C-A)_{\substack{elastic\\ part}} = \frac{ka^5\Omega^2}{3G}}$$

Another application of the Love number k. Using

$$(C-A)_{set} = (C-A)_{total} - (C-A)_{elastic}$$

we find the <u>Love-Larmor formula</u>

$$\sigma = \frac{C-A-ka^5\Omega^2/3G}{A}\,\Omega$$

*

$$T_{theory} = \frac{A}{C-A-ka^5\Omega^2/3G} \text{ days}$$

↑ this will lengthen period

It was known in 1909 that $\bar{\mu}_{\oplus} \sim 8 \cdot 10^{11}$ and that together with formula

$$k = \frac{3/2}{1 + \frac{19}{2} \frac{\mu}{\rho g a}}$$

gave a value for $k \sim 0.3$. A modern value for 1066A, B is

$$\boxed{k = 0.300}$$

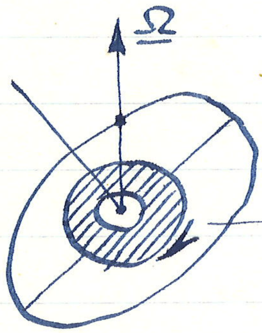
This increases T_{theory} to ~ 14 months, in fair agreement with data.
For $k = 0.300$, $a = 6371$ km and $\Omega = 7.292115 \cdot 10^{-5}\ s^{-1}$ we find in fact

$$T_{theory} = 447.4 \text{ days}$$
103 % of T_{obs}

This good agreement is however misleading. All elementary treatments stop here however.

The formula * would be valid for an everywhere solid ⊕ but not for one with a fluid core and oceans. We must account for both separately.

Consider effect of core: easy if mantle rigid and core-mantle boundary spherical. Then clearly only the mantle will wobble



The period in this case will clearly be

$$T = \frac{A_m}{C_m - A_m} \text{ days}$$

mantle only, but $C_c = A_c$ so $C_m - A_m = C - A$

inertia decreased, this will decrease T_{theory}, not as much of ⊕ is wobbling.

$$T = \frac{A_m}{C - A} \text{ days}$$

To see what will happen with elliptical core-mantle bdry and deformable mantle more complicated. Must consider fluid dynamics of core. Can however be shown that to $O(\varepsilon)$ core still does not participate but decrease of C-A by elasticity is same. Thus

$$T_{theory} = \frac{Am}{C - A - ka^5\Omega^2/3G} \text{ days}$$

This gives 396.9 days. The lack of participation of core in wobble reduces period by 50 days. T_{theory} is now way too low but we have still ignored oceans.

Effect of wobble is to raise a pole tide in oceans ~ 1/2 - 1 cm high just as it raises a solid ⊕ "pole tide" we have calculated above. The ocean pole tide has been observed although in many places it is below noise level.

Can be shown that effect of <u>equilibrium</u> pole tide is to change k by an amount δk due to increased "elastic + fluid" bulge

$$\boxed{k \to k + \delta k}$$

where $\delta k = 0.045$

so $k_{eff} = 0.345$

only mantle wobbles

Then $$T_{theory} = \frac{A_m}{C - A - k_{eff}\, a^5 \Omega^2 / 3G} \text{ days}$$

yielding of mantle + pole tide reduces "restoring force" and lengthens period

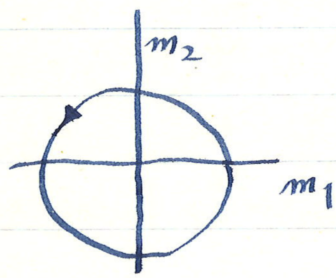
This gives $T_{theory} = 426.7$ days, 8.5 days too low or ~ 3 std. deviations, which is significant.

oceans increase period by <u>~30 days</u>

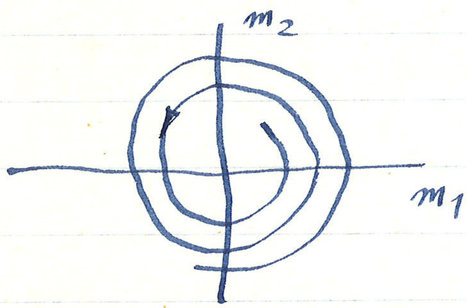
T_{theory} for a <u>realistic</u> <u>perfectly</u> <u>elastic ⊕</u> with <u>actual ocean-cont. config.</u> should be good to 1d.

Dissipation and dispersion

The continued deformation of ⊕ as the elastic bulge follows the wobble ⇒ the presence of dissipation. Looking down on pole



A rigid body with no dissipation if ever excited initially would wobble in a circular (if $A = B$) path forever, like a frictionless pendulum.



Wobble of real ⊕ would spiral in slowly due to dissipation.

Say the amplitude of the free wobble decays by e^{-1} in a time τ.

Then

$$m_1(t) = A \cos \sigma t \; e^{-t/\tau}$$
$$m_2(t) = A \sin \sigma t \; e^{-t/\tau}$$
} a spiral path.

Often written in terms of Q of mode.
dimensionless parameter defined by

$$\boxed{2\pi Q^{-1} = \text{fractional energy dissipated by friction in one cycle}}$$

Since energy ~ (amplitude)^2 and

$$\text{ampl} \sim e^{-t/\tau}$$
$$\text{energy} \sim e^{-2t/\tau}$$

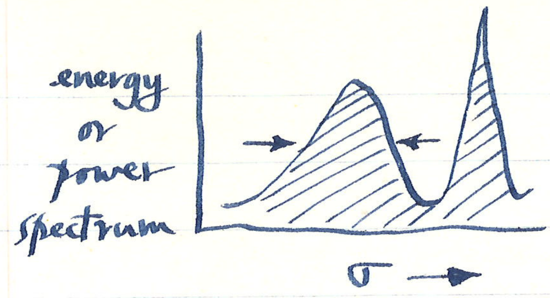
Then if we write $\boxed{\tau = \frac{2Q}{\sigma}}$ so

relation between decay time and Q

$$\boxed{\text{energy} \sim e^{-\sigma t/Q}}$$

Q = fractional energy dissipated / cycle

In the frequency domain it can be shown that with a white input near the resonance peak that



Q^{-1} = width of resonance peak at half power level

Measured Q of Chandler wobble uncertain because of noise and because of unknown irregular excitation.

Wilson + Haubrich find <u>$Q = 100$</u> with a 1σ range of 50 – 400.
Wilson + Vicente using the homogenized ILS data find <u>$Q = 170$</u> and claim the snr is higher in the newly reduced series.

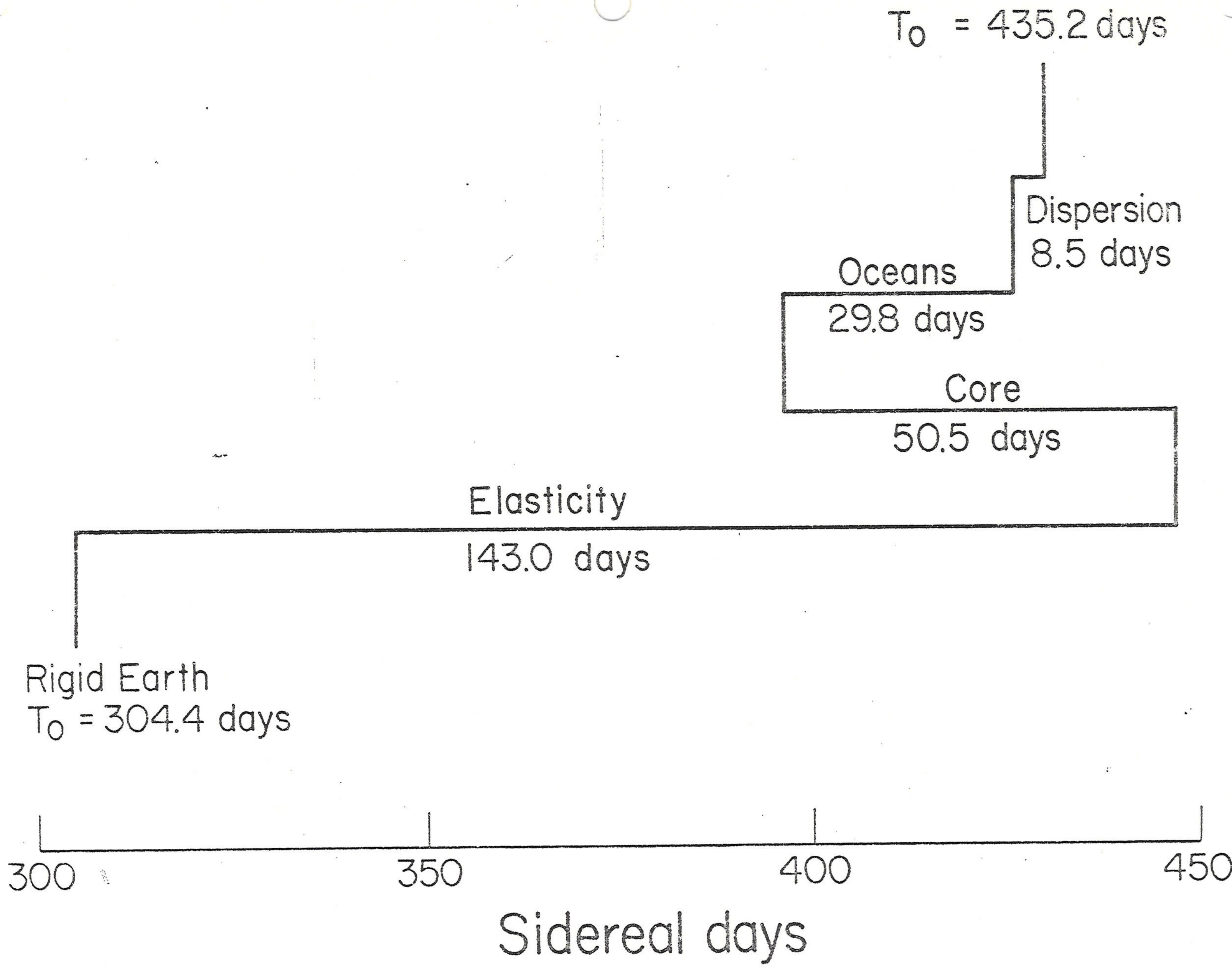
$\boxed{\text{Q = 100 corresponds to a decay time } \tau \sim 35 \text{ years}}$

So Chandler wobble is a <u>damped free motion</u>, if not continually re-excited would die away in a few times 35 years.

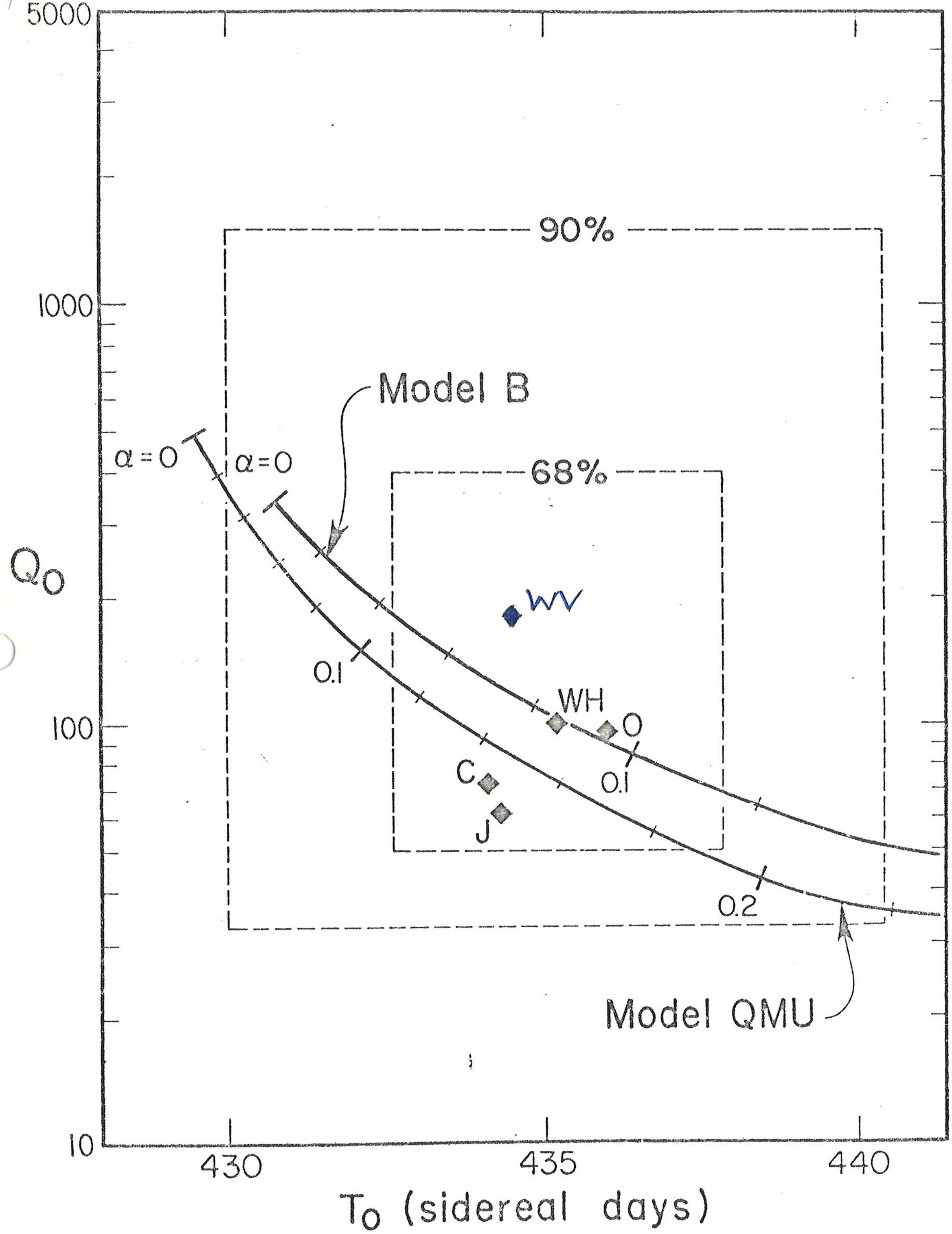
Chandler wobble problems :

1. what is reason for 8.5 day discrepancy between T_{obs} and T_{theory} ?
2. what is mechanism of dissipation ?
3. what is mechanism of excitation ? Observed to be some irregular process, like boys discharging peashooters at a pendulum.

Answers to first two questions linked. Now known that any dissipative mechanism in an anelastic solid must be associated with dispersion, a frequency dependence of elastic parameters and in particular the rigidity μ. When this taken into account, bodily friction in mantle can account for both the period discrepancy and the observed Q if Q_μ of mantle has a slight frequency dependence, parameterized for purposes of comparison by $Q_\mu \sim \sigma^\alpha$, α small < 0.2. See Figure in T_0 vs. Q_0 space from Smith + Dahlen paper.



T_0 = 435.2 days
Dispersion
8.5 days
Oceans
29.8 days
Core
50.5 days
Elasticity
143.0 days
Rigid Earth
T_0 = 304.4 days
300
350
400
450
Sidereal days



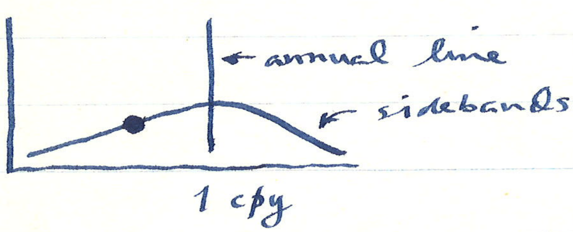
5000
1000
100
10
Q_0
90%
68%
Model B
$\alpha = 0$
$\alpha = 0$
0.1
0.1
0.2
WV
WH
O
C
J
Model QMU
430
435
440
T_0 (sidereal days)

The excitation of the Chandler wobble

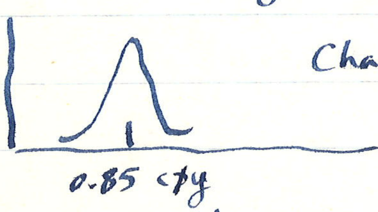
This the major outstanding problem in study of $\oplus$'s rotation, second is cause of decade fluctuations in l.o.d.

Two different possible mechanisms have been extensively investigated.

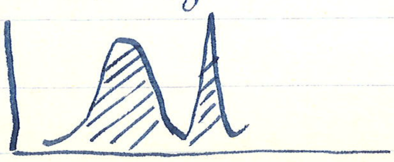
The atmosphere is known to be responsible for most if not all of the nearby annual wobble. But motions of mass of atmosphere are not a pure annual line, $\exists$ both FM modulation (some winters shorter) and AM modulation (some winters milder). This will produce sidebands in freq. domain



spectrum of meteorological input



Chandler resonance peak – response of $\oplus$ polar motion to any input



resulting output = product of input $\times$ filter response

Obvious test of this hypothesis is to compare integrated atmospheric data at 0.85 cpy with wobble data, there needs to be adequate power in the sideband at • so that when resonantly amplified output ~ 0".14, the observed amplitude of wobble

Wilson and Haubrich find the so-called coherence between the two series (meteorology + astronomy) to be ~ 0.5 .

the coincidence of max. coherence with zero phase suggests the two are correlated

Also the two are in phase as expected ; see Fig. 8.8 Both series are very noisy and the agreement may be improved in the future.

At present, however, the atmosphere alone seems inadequate. The power at • appears to be too low by at least a factor of two and maybe more. The comparison of the two power spectra in Fig. 8.9 shows a disagreement at 0.85 cpy of ~ factor of 6 ($2.2 \cdot 10^{-16}$ rad^2/cpy vs. observed wobble power ~ $14 \cdot 10^{-16}$ rad^2/cpy)

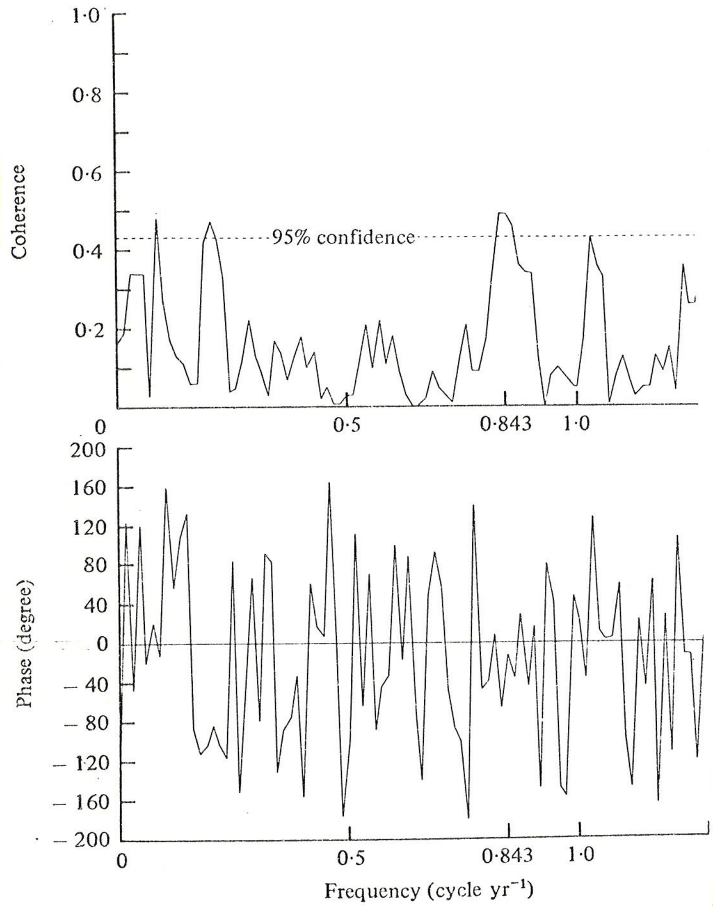


Figure 8.8. Coherence and phase between the atmospheric excitation function ψ_A and the astronomically deduced excitation ψ in the frequency range 0–1.2 cycle yr^{-1} (from Wilson & Haubrich 1976*a*).

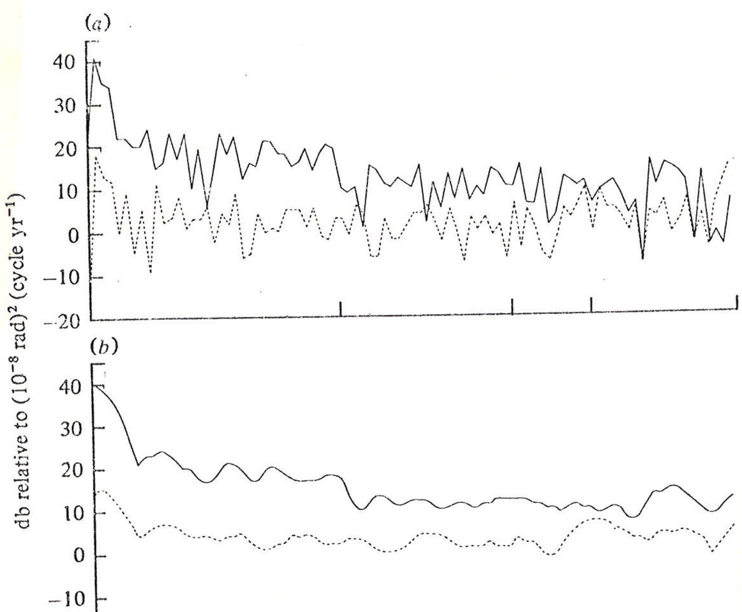
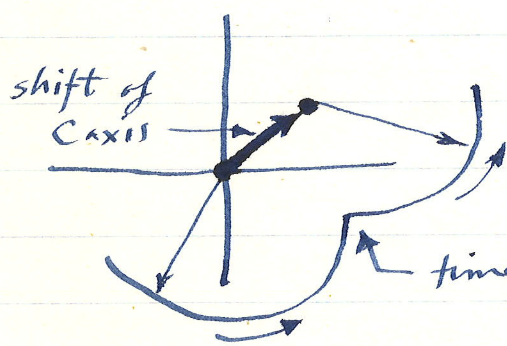


Figure 8.9. (*a*) Comparison of power spectra of the astronomically deduced excitation $\hat{\psi}$ (solid curve) and the atmospheric excitation ψ_A (broken curve) (from Wilson & Haubrich 1976*a*). (*b*) Smoothed spectra.

Second hypothesis has received much attention: excitation by <u>earthquakes</u>. Mechanism: earthquake faulting ⟹ redistribution of ⊕'s mass ⟹ change in location of axis of principal inertia.

Chandler motion is a wobble of inst. rot. axis about axis C of greatest inertia. Thus at time of an earthquake



Net effect would be a continual re-excitation. This very much like boys with peashooters.

A reason for believing this mechanism may be viable: tantalizing correlation between ampl. of Chandler wobble and ergs/yr released in <u>quakes</u>, see Fig. 8.2 of Lambeck.

Biggest earthquake ever recorded is 1960 Chilean quake, seismic moment $M_0 \sim 3 \cdot 10^{30}$ dyne-cm. Smylie + Mansinha fitted circular arcs to smoothed BIH data before and after, found a break in about right direction and magnitude, not however seen in newly homogenized ILS data by Wilson + Vicente. Observers of polar motion are now awaiting a large quake to look for its effect.

Real question is however: is cumulative effect of all quakes adequate to account for observed Chandler power, one can calculate effect of past quakes from seismic catalogue, difficulty is knowing seismic moments (better measure of size than Richter magnitude) of historical quakes, old instruments not well suited to measure moments.

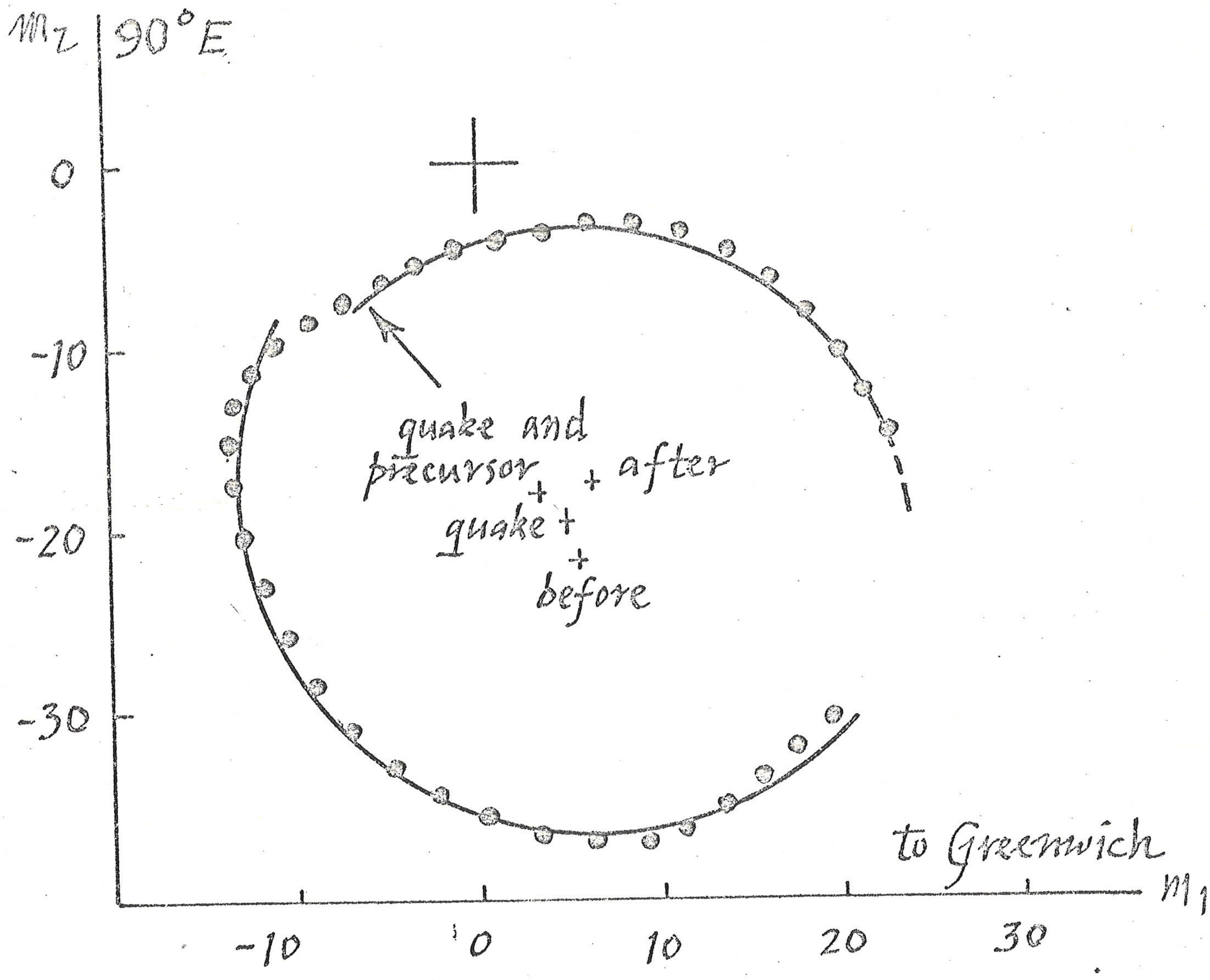
This causes considerable uncertainty in estimates. Fig. 8.6 from Lambeck shows synthetic wobble curve due to quakes calculated by Dziewonski + O'Connell using Q of wobble = 100 (the higher Q is the less input power is required to excite to observed level).

They find quakes ~ a factor of two too low but the variation in Chandler amplitude with time is reproduced. Kanamori feels however they have overestimated the moments of many older earthquakes by as much as a factor of 10.

A speculative suggestion is that slow precursor to quakes and "silent" quakes may play a major role.

The problem is definitely not solved, as final comparison of meteorology ~~[illegible]~~ + seismology vs. astronomy (Fig. 8.10) shows.

The Chilean precursor



Total moment ~ $6 \cdot 10^{30}$ dyne-cm.

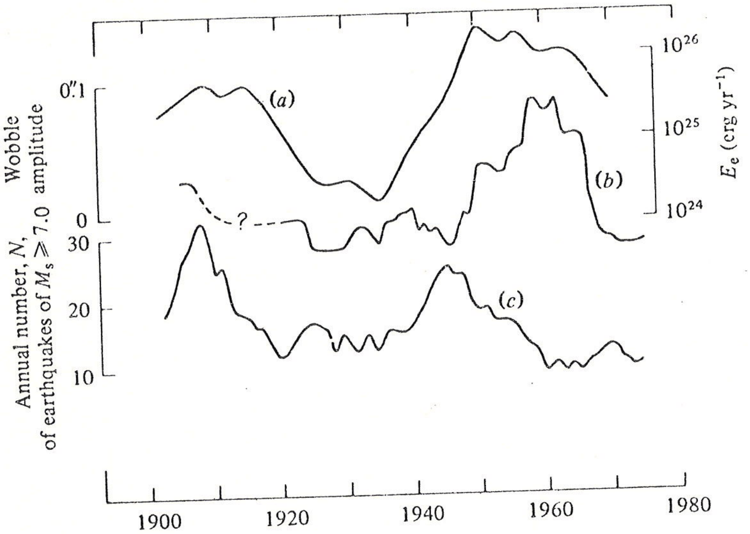


Figure 8.2. Comparison of (a) the amplitude of the Chandler wobble, (b) the elastic energy, E_e, released by earthquakes (5-yr running averages), and (c) the annual number, N, of earthquakes of $M_s \geq 7.0$ (5-yr running averages). From Kanamori (1977a).

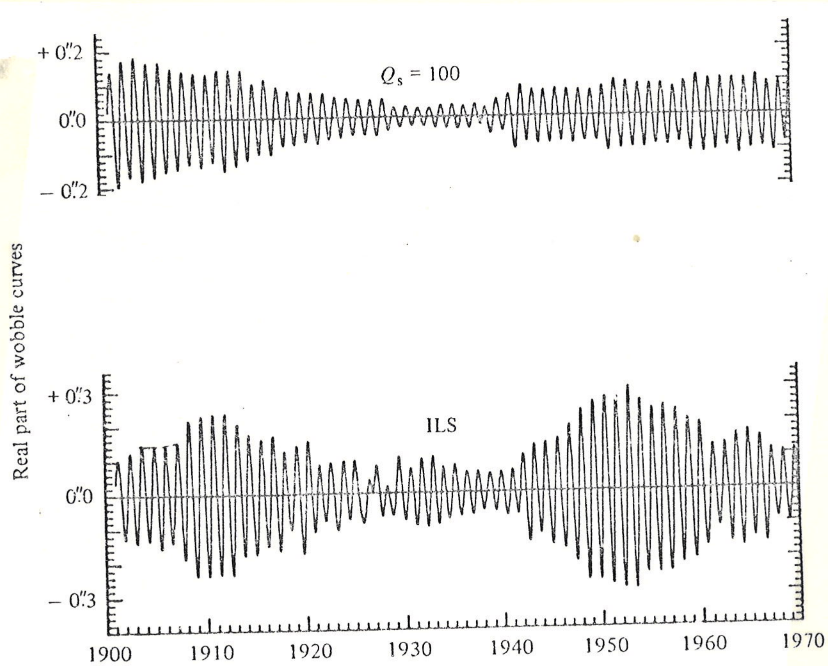


Figure 8.6. Changes of amplitude of the Chandler wobble from 1900 to 1970 due to the seismic excitation function of O'Connell & Dziewonski for $Q_w = 100$, compared with the astronomical results (lower curve).