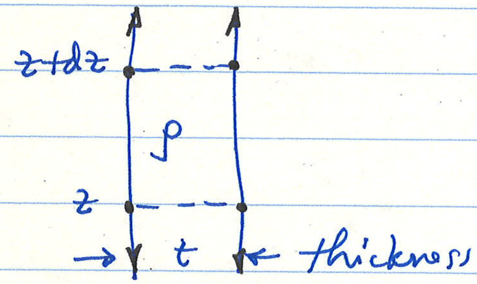
Vertical soap ~~film~~ film

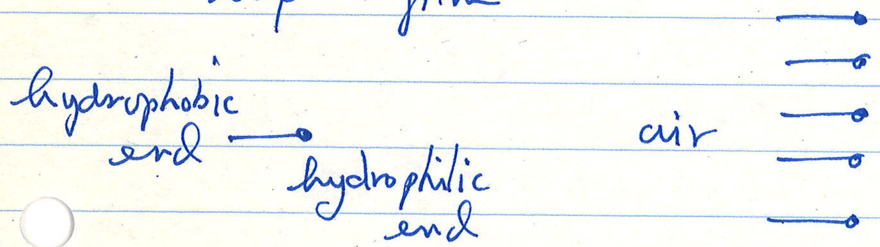


$$\rho g t \, dz = 2\left[\gamma(z+dz) - \gamma(z)\right]$$

$$\boxed{\frac{d\gamma}{dz} = \frac{1}{2}\rho g t}$$

γ increases upward

soap film



the mutual repulsion of the soap molecules lowers γ

adding soap decreases $\gamma = 72$ dyne/cm for pure H_2O — why — detergent

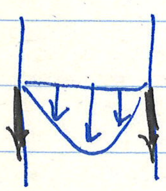
more soap molecules in film near bottom

What about eqn ⋆?

$\mu = 0$, $Q = 0$, $f_s = 0$, $\nabla \cdot \hat{n} \cong 0$

$\nabla\gamma = \gamma' \hat{z}$

$$\boxed{T_{nz}^{\text{water}} = \frac{d\gamma}{dz}}$$



viscous drag on sidewalls ind. of viscosity:

$$\boxed{T_{nz} = \frac{d\gamma}{dz} = \frac{1}{2}\rho g t}$$

Batchelor eqn 4.2.10 page 182

flow velocity $u_z = \frac{G}{2\mu}(n)(t-n)$

$$\varepsilon_{zn} = \frac{1}{2}\partial_n u_z = \frac{G}{4\mu}(t-2n) = \frac{Gt}{4\mu} \quad \text{at } n=0$$

$$\varepsilon_{zn} = \pm\frac{Gt}{4\mu} \quad \text{at } n=0,\ n=t.$$

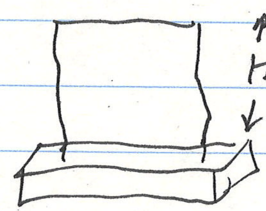
$$T_{nz} = 2\mu\,\varepsilon_{nz} = \frac{Gt}{2\cancel{\mu}}$$, independent of μ

$$\boxed{G = \frac{dP}{dz} = \rho g} \longleftarrow$$ driving pressure gradient

Typical value for $t \approx \lambda_{light} \approx 400-700$ nm

say $t \sim 10^3$ nm $= 10^{-6}$ m $= 10^{-4}$ cm

Then $\frac{d\gamma}{dz} = \frac{1}{2}(10^{-4})(1)(10^3) = .05 \ \frac{\text{dyne}}{\text{cm}^2}$

In a museum  $H \approx 2$ meters $= 2\cdot 10^2$ cm

$\gamma_{top} - \gamma_{bottom} \sim 10$ dyne/cm^2 $\sim \frac{1}{7} \times \gamma_{pure\ water}$

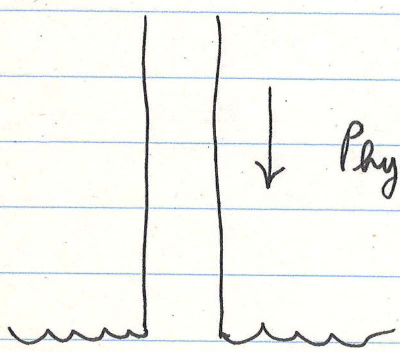
The pressure in the water column is $\sim$hydrostatic, $dP/dz = \rho g$

Something wrong —

Aha! The $G = \frac{dP}{dz}$ in Batchelor's eqn 4.2.10 is the modified pressure gradient.

The fact that it is hydrostatic $\Rightarrow$

$$\boxed{\frac{dP}{dz} = \text{constant in the column}}$$



$P_{hydrostatic}$ goes up as we go down

but $P_{driving}$ goes down as we go down — after all, it is a driving pressure.

What is the maximum height to which one can draw such a column?

$$\frac{d\gamma}{dz} H_{max} = \gamma_{pure\, H_2O} = 72 \text{ dyne/cm}$$

$$H_{max} = \frac{2\gamma_{pure}}{\rho g t} \qquad H_{max}\ (\text{cm}) = \frac{144}{10^3 t}$$

$$\boxed{\begin{array}{c} H_{max} = .14\, t^{-1} \\ \text{both in cm} \end{array}}$$

What is the downward drain velocity?

$$\boxed{u_{max} = \frac{1}{8}\frac{G}{\mu}t^2}$$

$$= \frac{1}{8}\frac{\rho g}{\mu}t^2 = \frac{1}{8}\frac{10^3}{10^{-2}}10^{-8}$$

$$\sim 10^{-4} \text{ cm/sec}, \text{ very slow}$$

For a water column 10 time thicker:

$$t \sim 10^{-3} \text{ cm}$$

$$\frac{d\gamma}{dz} = 0.5 \frac{\text{dyne}}{\text{cm}^2}$$

$$\gamma_{top} - \gamma_{bottom} \sim 100 \text{ dyne/cm} > 72 \text{ dyne/cm}$$

for a 2m high column

$$u_{max} \sim 10^{-2} \text{ cm/sec}$$

How long does it take to drain?

$$\boxed{\text{Call the thickness } h \text{ rather than } t}$$

$$\bar{u} = \frac{1}{h}\int_0^h \frac{G}{2\mu}\, n(h-n)\, dn = \frac{1}{12}\frac{G}{\mu}h^2$$

$\bar{u} h$ = volume loss per sec per unit distance $\perp$ to page

$= - H \dot{h}$

$$\dot{h} = -\bar{u}\frac{h}{H} = -\frac{1}{12}\frac{G}{\mu H} h^3$$

$$h^{-3}\, dh = -\frac{1}{12}\frac{G}{\mu H}\, dt$$

$$h^{-2} - h_0^{-2} = \frac{1}{6}\frac{G}{\mu H} t$$

$$\boxed{h = \sqrt{\frac{1}{h_0^{-2} + \frac{1}{6}\frac{G}{\mu H} t}}} = h_0 \sqrt{\frac{1}{1 + \frac{1}{6}\frac{G h_0^2}{\mu H} t}}$$

Will be well drained when, say,

$$\frac{G}{\mu H} t \sim \frac{10}{h_0^2} \qquad t \sim \frac{10 \mu H}{G h_0^2}$$

$$t \sim \frac{10 \cdot 10^{-2} \cdot 200}{10^3\, 10^{-8}} \sim 2 \cdot 10^7 \text{ sec} \sim 1 \text{ yr!}$$

Observed drain time obviously much faster — something seems wrong!